A GRAMMAR SCHOOL ARITHMETIC

BY

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PREFACE.

THIS Arithmetic is designed to give pupils of the grammar-school age an intelligent knowledge of the subject and a moderate power of independent thought.

Whether Arithmetic is studied for mental discipline or for practical mastery over the every-day problems of common life, mechanical processes and routine methods are of no value. Pupils can be trained to logical habits of mind and stimulated to a high degree of intellectual energy by solving problems adapted to their capacities. They become *practical* arithmeticians, not by learning special business forms, but by founding their knowledge on reasoning which they fully comprehend, and by being so thoroughly exercised in logical analysis that they are independent of arbitrary rules.

The book contains a great number of well-graded and progressive problems, made up for youths from ten to fourteen years of age. Definitions and explanations are made as brief and simple as possible. It is not intended that definitions shall be committed to memory, but that they shall be simply discussed by teacher and pupils. Every teacher, of course, will be at liberty to give better definitions, and to make a better presentation of methods, than those given in the book. In short, the chief object in view will be gained if pupils are trained to solve the problems by neat and intelligent methods, and are kept free from set rules and formulas.

A great many number-problems are given in the first pages of the book, so that the necessary facility and accuracy in computing

under the four fundamental rules may be acquired; as want of accuracy and rapidity in mere calculations distracts the attention which should be given to the investigation and correct statement of clothed exercises. The pupil should be required to do only so many of these number-problems as are found to be necessary to give him facility and accuracy in the four fundamental operations; and he should be allowed to omit some of the harder-clothed problems until he reviews the book; as, for example, the problems on pages 28–30, 43–45, 58–60, 76–79, 97–104, 115, 116, 160–162, 205–210, 243, 244, 292, 293.

The chapter on the Metric System is put near the end of the book because many grammar-school pupils have no time for it, while those who have time can as well learn the system at this stage of their progress as earlier.

The chapter on Mensuration is suited to the ability of beginners. The intention is not to give a *system of Geometry*, but to render familiar the notions of geometry that are indispensable for practical purposes. The whole subject has been illustrated and enforced by many practical examples.

The chapter on Miscellaneous Problems and Examination Papers is intended as a *review* of the subject-matter of Arithmetic and as a *test* of the learner's knowledge.

The author is under obligations to many teachers who have given valuable suggestions and assistance in the preparation of this work.

G. A. WENTWORTH.

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VOCABULARY.

Abstract number. This phrase is employed to designate numbers used without reference to any particular unit, as 8, 10, 21. But *all numbers are in themselves abstract whether the kind of thing numbered is or is not mentioned.*

Addition. The process of combining two or more numbers so as to form a single number.

Aliquot part. A number which is contained an integral number of times in a given number. Thus, 5, $6\frac{1}{4}$, $12\frac{1}{2}$, $16\frac{2}{3}$, are aliquot parts of 100.

Amount. The sum of two or more numbers. In Interest, the sum of principal and interest.

Analysis. The separation of a question into parts, to be examined each by itself.

Antecedent. The first of the two terms named in a ratio.

Area of a surface. The ratio of the surface to another surface assumed as the unit of measure; usually the square of the linear unit.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property belonging to an estate, individual, or corporation.

Average. The mean of several unequal numbers, so that, if substituted for each, the aggregate would be the same.

Bank. An establishment for the custody, loaning, and exchange of money; and often for the issue of money.

Bank discount. An allowance received by a bank for the loan of money, paid at the time of lending as interest.

Bonds. Written contracts under seal to pay specified sums of money at specified times, issued by national governments, states, cities, and other corporations.

Cancellation. The striking out of a common factor from the dividend and divisor.

Commission. Compensation for the transaction of business, reckoned at some per cent of the money employed in the transaction.

Common denominator. A denominator common to two or more fractions.

Common factor. A factor common to two or more numbers.

Common multiple. A multiple common to two or more numbers.

Complex fraction. A fraction that has a fraction in one or both of its terms.

Composite number. The product of two or more integral factors, each factor being greater than unity.

Compound denominations. Several denominations used to express parts of one quantity.

Compound interest. When the interest due is left unpaid, and considered as an increase made to the principal, the whole interest, accruing in any time, is called compound interest.

Compound fraction. A fraction of another fraction.

Concrete number. A phrase used to denote numbers applied to specified things; as 6 horses, 8 desks.

Consequent. The second of the two terms named in a ratio.

Consignee. The person or firm to whom goods are sent.

Consignor. The person or firm who sends goods to another.

Corporation. An association of individuals authorized by law to transact business as a single person.

Couplet. The two terms of a ratio taken together.

Coupon. A certificate of interest attached to a bond, to be cut off when due and presented for payment.

Creditor. A person or firm to whom money is due.

Cube root. One of the three equal factors of a number.

Customs. Duties or taxes imposed by law on merchandise imported, and sometimes on merchandise exported.

Debtor. A person who owes money to another.

Decimal fractions. Fractions of which only the numerators are written, and the denominators are ten or some power of ten.

Decimal point. A dot placed after the *units'* figure to mark its place.

Decimal system. The common system of numbers founded on their relations to *ten*, *ten tens*, etc.

Denominator. The number which shows into how many equal parts a unit is divided.

Difference. The number which, added to a given number, makes a sum equal to another given number.

Discount. Allowance made for the payment of money before it becomes due. Also, the difference between the market value and the face value when the market value is *below* the face value.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and required factor (called quotient). In business, the share of profits which belongs to each owner of stock, on his proportion of the capital.

Division. The operation by which, when a product and one of its factors are given, the other factor is found.

Divisor. The number by which a given dividend is to be divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a draft. The person to whose order the sum of money named in a draft is to be paid.

Drawer of a draft. The person who signs the draft.

Duties. Taxes required by the government to be paid on goods imported, exported, or put on the market for consumption.

Equation. A statement that two expressions of number are equal.

Equation of payments. The finding of an average time at which several payments may be justly made.

Exchange. A system of paying debts, due to persons living at a distance, by transmitting drafts instead of money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last terms of a proportion.

Evolution. The process of finding the root of a number.

Factors. The factors of a number are a set of numbers whose product is the given number; they are assumed to be integral, except in the extraction of roots. In commerce, agents employed by merchants to transact business.

Figures. Symbols used to represent numbers in the common system of notation. Also diagrams used to represent geometrical forms.

Firm. The name under which a company transact business.

Fractions. One or more of the equal parts into which the unit is divided.

Grace. An allowance of three days, after the date a note becomes due, within which to pay the note.

Gram. The unit of weight in the metric system.

Greatest common measure. The greatest number which is a common factor of two or more given numbers.

Improper fraction. A fraction whose numerator equals or exceeds the denominator.

Index. A figure written at the left and above the radical sign to show what root of the number under the radical sign is required. A fraction written at the right of a number, of which the numerator shows the required power of that number, and the denominator the required root of that power.

Instalment. A payment in part.

Insurance. A guarantee of a specified sum of money in the event of loss of property by fire, storm at sea, or other disaster; or of loss of life.

Integral number. A number which denotes whole things.

Interest. Money paid for the use of money.

Involution. The process of finding a power of a number.

Latitude of a point. The angle made by the vertical line at that point with the plane of the equator.

Least common multiple. The least number which is a common multiple of several given numbers.

Liability. A debt, or obligation to pay.

Line. Length without breadth or thickness. The path of a moving point.

Liter. The unit of capacity in the metric system equal in volume to a cube each edge of which is one-tenth of a meter.

Long division. The method of dividing in which the processes are written in full.

Longitude of a point. The angle between two planes supposed to pass through the centre of the earth and to contain, the one the meridian of that point, and the other the standard meridian.

Loss. The excess of the cost price above the selling price.

Maturity of a note. The date at which a note legally becomes due.

Mean proportional. A number which is both the second and third terms of a proportion.

Means. The terms of a proportion between the extremes.

Meter. The unit of length in the metric system.

Minuend. The given number in subtraction which is equal to the sum of another given number called the subtrahend, and a required number called the difference or remainder.

Mixed number. A number that expresses both entire things and parts of things taken together.

Multiple of a number. The product obtained by taking the given number an integral number of times.

Multiplicand. The number to be multiplied by another.

Multiplication. The operation of finding a number bearing the same ratio to the multiplicand which the multiplier bears to unity.

Multiplier. The number by which the multiplicand is multiplied.

Net proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written agreement to pay a specified sum of money at a specified time.

Number. The answer to the question, How many?

Numeration. A system of naming numbers.

Obligation. A debt, or liability to pay.

Order of units. A name used to designate the number of things in a group, as *tens*, *hundreds*, *thousands*, etc.

Partial payment. Part payment on a note.

Partnership. An association of two or more persons to carry on business.

Par value. Face or nominal value.

Pendulum. A body suspended by a straight line from a fixed point, and moving freely about that point as a centre.

Percentage. A part of any given number reckoned at some rate per cent.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll tax. A tax levied by the head or poll.

Power. The product of two or more equal factors.

Premium. Money paid for insurance computed at some rate per cent of the value insured. Also the excess of market value above par value.

Present worth. The present value of a debt due at some future day.

Prime number. A number which has no integral factors except itself and one.

Principal. Money drawing interest.

Problem. A question to be solved.

Product. The result obtained by multiplying the multiplicand by the multiplier.

Profit. The excess of selling price above cost.

Proof The evidence by which the accuracy of any result is established.

Proper fraction. A fraction, the numerator of which is less than the denominator.

Proportion. A statement that two ratios are equal.

Quantity. The answer to the question, How much?

Quotient. The number sought in division.

Rate per cent. Rate by the hundred.

Ratio. The *relative magnitude* of two numbers or of two quantities.

Reciprocal of a number. *One* divided by that number.

Reduction. The process of changing the *unit* in which a quantity is expressed without changing the *value* of the quantity.

Remainder. The number which, added to the subtrahend, gives a sum equal to the minuend.

Root of a number. One of the equal factors of the number.

Rule. The statement of a prescribed method.

Security. Property used to guarantee the payment of any debt.

Share. One of a certain number of equal parts into which the capital of a company is divided.

Short division. The method of dividing in which the operations of multiplying and subtracting are performed mentally.

Solid. A magnitude which has length, breadth, and thickness.

Solution. The process by which the answer to a question is obtained.

Specific gravity of a substance. The ratio of the weight of a given volume of it to that of an equal volume of water.

Square root. One of two equal factors.

Stock. Capital invested in business.

Subtraction. The process of finding a number which added to one of two given numbers will produce the other.

Sum. The number which results from combining two or more numbers by addition.

Surd. An indicated root the value of which cannot be exactly expressed in figures.

Surface. That which has only length and breadth.

Thermometer. An instrument for measuring heat.

Unit. A single thing. Also, an arbitrary length, adopted as a standard of measure, in terms of which all measurements are expressed.

Verify. To establish, by experiment, the truth of any statement.

Volume of a solid. The ratio of a solid to an assumed unit of measure; usually a cube of the linear unit.

A

GRAMMAR SCHOOL ARITHMETIC.

CHAPTER I.

PRELIMINARY DEFINITIONS.

1. A COLLECTION of several similar objects (as a collection of apples) or the repetition of the same event (as successive peals of thunder) gives the idea of **Number.**

2. The idea of number presents itself also when we wish to express the values of quantities in terms of some well-known value.

3. A **Unit** is a fixed value with which we compare all quantities of the same kind. Each kind of quantity has its own unit. Thus:

The unit of length	is the yard.
The unit of surface	is the square yard.
The unit of capacity	is the quart.
The unit of weight	is the pound.
The unit of money	is the dollar.

4. To measure a quantity is to find the number of times the quantity contains its unit.

5. Number results from measuring a quantity. If the unit is contained in a quantity several times without remainder, the result is an **Integral Number.** Thus, the integral number *three* will represent the length of a line, if the line contains the yard, three times exactly.

6. If the quantity to be measured is less than the unit, we divide the unit into equal parts and find how many times one of these parts is contained in the given quantity. Thus, to measure a line less than a yard, we can apply to this line a third part of a yard, and if this third is contained twice in the line exactly, the length of the line is expressed by two-thirds. The expression *two-thirds* is called a **Fraction.**

7. If a line contains the yard five times and one-fourth of a yard three times, the length of the line is expressed by five and three-fourths. The expression *five and three-fourths* is called a **Mixed Number.**

8. **Arithmetic** comprises all questions that can be proposed upon numbers.

NOTATION AND NUMERATION.

9. The first numbers have special names, as follows:

one, two, three, four, five, six, seven, eight, nine, ten.

10. The first nine of these numbers are called **Simple Units,** or *units of the first order.*

11. The group of ten units has received the name of a **ten,** or *a unit of the second order;* and we count by tens as by units; thus:

one ten, two tens, three tens ... nine tens, ten tens.

12. The group of ten tens has received the name of a **Hundred,** or *a unit of the third order;* and we count by hundreds, as by tens and units; thus:

one hundred, two hundreds ... ten hundreds.

13. A group of ten hundreds is called a **Thousand,** or *a unit of the fourth order.*

14. From ten units of the fourth order is formed *a ten thousand,* or *a unit of the fifth order;* and from ten units of the fifth order is formed *a hundred thousand,* or *a unit of the sixth order.*

15. Units of the *seventh order* are called **Millions**; of the *eighth order, ten millions;* of the *ninth order, hundred millions.* Finally, units of the *tenth order* are called **Billions**; units of the *thirteenth order,* **Trillions**; and so on.

16. The table of units of different orders is as follows:

First order,	*simple units,*	
Second order,	*tens of units,*	first class.
Third order,	*hundreds of units,*	
Fourth order,	*thousands,*	
Fifth order,	*tens of thousands,*	second class.
Sixth order,	*hundreds of thousands,*	
Seventh order,	*millions,*	
Eighth order,	*tens of millions,*	third class.
Ninth order,	*hundreds of millions,*	
Tenth order,	*billions,*	
Eleventh order,	*tens of billions,*	fourth class.
Twelfth order,	*hundreds of billions,*	
Thirteenth order,	*trillions,*	
... ...	... ...	fifth class.
... ...	... ...	

17. The group of the first three orders is called the first class of units, and the group of the three following orders, the second class, and so on.

18. The unit of the second class is equal to a thousand units of the first class, and a unit of the third class is equal to a thousand units of the second class, and so on.

19. To read a number we decompose it into units of the different orders, and state how many groups there are of each kind, commencing with the highest order. Thus, for example, two millions, three thousands, five hundreds, seven tens, and four units.

20. It is clear that the names of all numbers up to a billion are formed by combining the names of the first nine numbers with the words ten, hundred, thousand, million.

21. Usage sanctions the following irregularities:

I. Instead of saying two tens, three tens, four tens, five tens, six tens, seven tens, eight tens, nine tens, we say twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety.

II. The names of the numbers between ten and twenty are eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

22. The names of the numbers between twenty and a hundred are:

twenty-one, twenty-two, twenty-three ... twenty-nine,
thirty-one, thirty-two, thirty-three ... thirty-nine,
... ...
ninety-one, ninety-two, ninety-three ... ninety-nine.

23. The names of the numbers between a hundred and a thousand are:

hundred one, hundred two ... hundred ninety-nine,
two hundred one ... two hundred ninety-nine,
... ...
nine hundred one ... nine hundred ninety-nine.

24. The **common system of notation** employs ten figures or digits:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

The first nine of these figures represent the first nine numbers; the last, which is called *Zero*, *Naught*, or *Cipher*, is used to denote the *absence of units* of the order in which it stands. It is possible to express all numbers by these ten digits *by making the value of each figure increase tenfold for every place that it is moved to the left.*

25. If we have given a number written in figures, the position of each figure counting from the right indicates the order of units that the figure represents. If we divide the number into **periods** of three figures each, the first period on the right will be the period of simple units, the second period will be the period of thousands, the third will be the period of millions, and so on. In each period the first figure on the right expresses the units of that class, the second figure the tens, and the third the hundreds. Thus:

MILLIONS.		THOUSANDS.			UNITS.		
Tens.	*Units.*	*Hundreds.*	*Tens.*	*Units.*	*Hundreds.*	*Tens.*	*Units.*
2	1	3	3	4	3	3	4

Thus, the number 21,334,334 means and is read 21 millions, 334 thousands, 334 units. If the number is applied to dollars, it means and is read 21 million, 334 thousand, 334 dollars. The next period is the **billions'** period.

NOTE. The fundamental principle of forming and expressing numbers should be illustrated by making little bundles of wooden toothpicks, ten in each bundle, and then making bundles of hundreds by taking for each hundred ten bundles of ten each. When the pupil has become familiar with forming and expressing numbers consisting of hundreds, tens, and units, he should be shown that the method of forming and expressing numbers of hundreds, tens, and units of thousands is precisely the same, the only difference being that the unit of this period is not a single toothpick, but a pile of ten bundles of a hundred each, which is a thousand.

26. To write a number in figures we write successively the number of units of each order from left to right, beginning at the highest order and taking care to supply by zeros orders of units that may be lacking.

27. To read a number written in figures we divide the number into periods of three figures each from right to left: this done, we begin to read at the left-hand period and read *as if the figures of that period stood alone*, adding the *name* of the period; then the next period to the right is read with the name of that period, and so on.

28. The number 1256 may be read *one thousand two hundred fifty-six*, or it may be read *twelve hundred fifty-six*. The number 5004 may be read *five thousand four*, or it may be read *fifty hundred four*. *The shortest method is the best method of reading any number.* Twelve hundred fifty-six is shorter than one thousand two hundred fifty-six; five thousand four is shorter than fifty hundred four.

29. It will be seen that the value of each figure, in any number expressed in figures, depends on two things:

First, the value attached to the figure without regard to its position.

And, secondly, the value it acquires from the place it holds in the number.

The value of a figure, without regard to its position, is called its **absolute** value; and the value it acquires by its position is called its **local** value.

30. The art of expressing numbers by means of figures is called **Notation**, and the art of expressing in words a number written in figures is called **Numeration.**

31. The unit of money is the dollar. Instead of writing the word *dollars*, this mark $ is used, which is called the

sign for dollars, or the "dollar mark." Thus, if we wish to write five dollars, we write it $5.

It takes ten ten-cent pieces to make a dollar; that is, a ten-cent piece is **one-tenth** of a dollar. It takes ten single cents to be equal in value to a ten-cent piece. If we have one dollar and one ten-cent piece, we write it $1.10. If we have one dollar, one ten-cent piece, and two cents, we write it $1.12.

The dot which is placed after the one dollar is called the **Decimal Point**. Figures to the left of the decimal point denote whole units. Figures to the right of the decimal point denote parts of a unit, and are called **Decimal Fractions.** The expression $1.10 is read "one dollar and ten cents"; and the expression $ 1.12 is read "one dollar and twelve cents."

Ex. 1.

Write in figures:

1. Two hundred thirty-six, one hundred forty, five hundred two, seven hundred three.
2. Five hundred fourteen, three hundred seventy-six, four hundred thirty, eight hundred two, nine hundred twenty-seven.
3. One hundred ninety, four hundred six, eight hundred ten, two hundred seven.
4. Three hundred ten, two hundred thirteen, six hundred twenty-three, two hundred nineteen.
5. Five hundred fifty, four hundred four, four hundred twenty-five, eight hundred sixty.
6. Eight hundred sixteen, seven hundred eight, nine hundred, seven hundred three.
7. Nine hundred ninety-five, eight hundred eighty, seven hundred, eight hundred seven.
8. Two hundred seventeen, four hundred twelve, four hundred eight, one hundred two.

9. Four hundred seventeen, six hundred nineteen, three hundred six, one hundred eighteen.

Ex. 2.

Read (or write in words):

1.	500,	700,	300,	200,	900,	100.
2.	830,	709,	506,	350,	819,	703.
3.	607,	312,	918,	810,	103,	560.
4.	752,	698,	405,	536,	121,	514.
5.	973,	356,	703,	409,	211,	713.
6.	225,	64,	970,	49,	83,	674.
7.	106,	170,	380,	759,	921,	538.
8.	481,	360,	593,	32,	296,	551.
9.	182,	802,	555,	705,	649,	630.
10.	314,	97,	613,	384,	992,	516.

Ex. 3.

Write in figures:

1. Eight thousand seven hundred three, four thousand forty-five, six thousand three hundred eight, forty-eight hundred.
2. Five thousand forty-eight, nineteen hundred ninety, seven thousand eighty-two, eight thousand fifty.
3. Seven thousand two hundred forty, nine thousand nine hundred nineteen, six thousand seven, eight thousand seven hundred seventy-six.
4. Seven thousand one hundred seven, six thousand eight hundred four, nine thousand one hundred ten, five thousand five hundred fifty.
5. Six thousand eighty-six, four thousand forty, one thousand ten, nine thousand ninety-nine.
6. Eight thousand eighty, seventeen hundred fifty-seven, eleven hundred one, seven thousand seven, forty-five hundred forty-five.

7. Two thousand four hundred ninety-six, eighteen hundred eighty-three, three thousand ninety-five, one thousand eleven.
8. One thousand thirteen, one thousand one, fourteen hundred, thirty-three thousand fourteen.
9. Seventeen hundred thirty-six, three thousand forty-nine, eight thousand eighteen, nine thousand seventy.
10. Four thousand seven hundred nine, fifteen hundred ten, one thousand sixty-nine, sixteen thousand sixteen.

Ex. 4.

Read (or write in words):

1.	8,000,	5,000,	2,000,	6,000,	1,000,	9,000.
2.	9,210,	6,907,	7,402,	9,998,	4,060,	7,210.
3.	5,068,	4,020,	1,400,	7,031,	1,290,	1,010.
4.	8,808,	6,006,	8,482,	3,096,	4,720,	11,973.
5.	12,002,	11,101,	5,812,	1,739,	6,760,	6,903.
6.	4,085,	1,169,	2,615,	5,007,	1,110,	1,460.
7.	4,760,	4,190,	2,607,	5,180,	1,200,	3,746.
8.	9,008,	8,300,	6,804,	2,977,	6,202,	9,620.
9.	6,322,	7,450,	8,673,	2,603,	2,518,	1,508.
10.	7,080,	1,009,	8,070,	5,068,	1,397,	5,782.

Ex. 5.

Write in figures:

1. Twelve and twelve hundredths, twenty-two and eight tenths, three hundred twenty-five and six tenths, one hundred one and one hundred one thousandths.
2. Seventy-five and seventy-five hundredths, eighty-three and twenty-six thousandths, ninety-six and seven hundred four thousandths, one thousand ten and two tenths.
3. Five hundred seventy-three and five hundred seventy-three thousandths, eleven thousand four and sixteen hundredths, three hundred sixty-five and eight tenths, seventy-two and ninety-six hundredths.

4. Three and nineteen thousandths, six hundred fifty-eight and two hundredths, eight hundred and eight hundredths, thirty-seven and five thousandths.
5. Seventy-one and seven tenths, seven and seventeen hundredths, seven hundred and seventeen thousandths, eight hundred ten and one tenth.
6. Eighty-one and one hundredth, eight and one hundred one thousandths, nine hundred sixty-three and two tenths, ninety-six and thirty-two hundredths, nine and six hundred thirty-two thousandths.
7. Six hundred and five tenths, sixty and five hundredths, six and five thousandths.
8. Nine hundred eighty-three and three tenths, ninety-eight and thirty-three hundredths, nine and eight hundred thirty-three thousandths.
9. One hundred twelve and one tenth, eleven and twenty-one hundredths, one and one hundred twenty-one thousandths.
10. Eleven thousand and sixty-three thousandths, twenty-three and eighty-six hundredths, one hundred ten and eleven hundredths.

Ex. 6.

Read (or write in words):

1.	3010.3,	477.12,	60.206,	698.97,	778.15,	84.510.
2.	903.9,	413.9,	17.918,	113.94,	14.613,	204.12.
3.	234.5,	8010.8,	59.106,	43.136,	380.21,	361.73.
4.	6187.8,	785.33,	90.849,	92.294,	27.989,	28.012.
5.	291.59,	29.645,	30.081,	299.07,	30.190,	35.257.
6.	360.4,	3605.9,	361.16,	39.041,	468.64,	463.59.
7.	47.828,	59.184,	600.65,	601.19,	60.108,	52.466.
8.	510.14,	51.028,	580.35,	5804.7,	641.97,	6409.8.
9.	65.002,	69.949,	602.17,	6020.6,	64.058,	76.343.
10.	770.85,	6994.9,	712.06.	719.66,	883.87,	83.493.

Ex. 7.

Write in figures:

1. Fifty thousand three dollars, eighty thousand nine hundred ninety dollars.
2. Twenty-eight million seven hundred forty-four thousand one hundred sixty-nine dollars.
3. Five hundred sixteen dollars and ten cents, twenty-five hundred fifty dollars and sixty-nine cents.
4. Sixteen hundred million thirty thousand three hundred eight dollars and fifty cents.
5. Twenty-seven hundred million one thousand one dollars and eighty-seven cents.
6. Five hundred thousand two hundred one dollars and seventy-five cents.
7. Eight million fourteen thousand three hundred twenty-five dollars and twenty-five cents.
8. Ninety-seven million two hundred thousand one hundred two dollars and five cents.
9. Ten million ten thousand ten dollars and ten cents.
10. Eleven hundred ten thousand dollars and eleven cents.

Ex. 8.

Read (or write in words):

1.	$259,132.10,	$27,186.25.
2.	$1,213,062.50,	$2,763,001.75.
3.	$3,675,321.12,	$3,500,005.15.
4.	$17,360,502.20,	$27,132,857.33.
5.	$55,333,263.36,	$58,785,587.09.
6.	$116,001,556.40,	$275,363,750.11.
7.	$660,878,640.69,	$594,340,000.94.
8.	$600,241,560.02,	$124,271,000.01.
9.	$768,301,520.20,	$802,631,516.73.
10.	$505,631,880.04,	$1,555,676,410.62.

CHAPTER II.

ADDITION.

32. If you put 2 cents with 3 cents, how many cents have you? Answer, 5 cents.

How can you express this operation on your slate?

You can write the figure 2; then the figure 3 beneath it; draw a line underneath, and below the line write the figure 5. The work is shown in the margin.

$$\begin{array}{r} 2 \\ 3 \\ \hline 5 \end{array}$$

Or, you can express it thus: $2 + 3 = 5$.

The sign $+$ is called **plus**, and means that the numbers between which it is placed are to be counted together; and the sign $=$ means **equals**. So that $2 + 3 = 5$ is read 2 plus 3 equals 5.

33. The operation of finding a number equal to two or more numbers taken together is called **addition**; and the result is called their **sum**. The numbers to be added are called **addends.**

Name the sums of the following numbers, and practice naming them until you can name each sum the instant your eye rests upon the numbers to be added.

Ex. 9. (*Oral.*)

$1+1=$	$3+1=$	$1+0=$	$1+7=$	$1+5=$
$2+1=$	$8+1=$	$1+4=$	$6+1=$	$9+1=$
$2+2=$	$2+0=$	$1+2=$	$8+2=$	$2+7=$

2+5=	6+2=	3+2=	2+4=	2+9=
3+4=	2+3=	5+3=	1+3=	3+7=
8+3=	3+0=	3+6=	3+3=	9+3=
0+4=	5+4=	4+7=	9+4=	4+1=
8+4=	4+3=	2+4=	4+4=	4+6=
5+5=	5+7=	3+5=	0+5=	5+9=
2+5=	5+1=	4+5=	5+6=	8+5=
6+3=	1+6=	5+6=	6+0=	2+6=
6+6=	4+6=	6+9=	7+6=	6+8=
5+7=	5+6=	7+3=	7+1=	0+7=
8+7=	7+2=	4+7=	7+7=	7+9=
8+1=	5+8=	2+8=	8+0=	7+8=
8+3=	8+8=	4+8=	8+6=	9+8=
9+0=	9+9=	9+2=	1+9=	3+9=
4+9=	9+6=	8+9=	9+5=	7+9=

5	6	8	7	4	3	6	6	6	7
3	7	3	5	5	6	4	8	5	5

7	7	7	7	7	8	5	8	8	8
7	2	9	4	8	2	8	7	9	8

9	9	9	9	6	2	3	8	3	6
1	4	7	8	8	5	4	2	6	5

8	7	5	3	7	8	9	5	4	3
7	2	4	8	3	3	3	3	3	3

Ex. 10.

Find the sums of the following numbers:

1. 28 5	9. 29 7	17. 32 9	25. 47 8	33. 49 5
2. 43 6	10. 56 8	18. 58 9	26. 65 9	34. 67 9
3. 54 7	11. 78 5	19. 79 4	27. 27 8	35. 43 7
4. 63 8	12. 78 9	20. 57 8	28. 27 6	36. 35 9
5. 74 3	13. 32 7	21. 63 8	29. 42 9	37. 14 6
6. 18 9	14. 27 9	22. 36 8	30. 12 9	38. 73 8
7. 85 7	15. 37 5	23. 59 4	31. 50 5	39. 13 9
8. 19 8	16. 37 9	24. 93 9	32. 89 7	40. 79 6

Copy the following, and fill the blanks:

$42+9=$	$19+8=$	$17+9=$	$23+8=$
$71+9=$	$26+7=$	$35+8=$	$47+6=$
$85+8=$	$18+7=$	$17+3=$	$18+9=$
$29+6=$	$38+5=$	$15+7=$	$14+8=$

34. Since 10 in any place is equal to 1 in the next place to the left, if the sum of the digits of *any* column exceeds 9, write the units' figure of the sum under the column added and *carry the number of tens* to the next column.

Thus, in the following example:

$$\begin{array}{r} 872 \\ \underline{991} \\ 1863 \end{array}$$

The sum of the digits in the right-hand column is 3. The sum of the digits in the second column is 16; the 6 is written under this column and the 1 is carried to the third column. The sum of the digits of the third column, together with the 1 carried to it, is 18; the 8 is written under this column and the 1 is carried to the place of thousands.

Ex. 11.

Add:

1. 497 + 735	6. 689 + 297	11. 9535 + 9675	16. 56902 + 94876
2. 840 + 869	7. 477 + 335	12. 5557 + 5763	17. 93689 + 60086
3. 997 + 289	8. 449 + 483	13. 8284 + 7998	18. 59857 + 84556
4. 643 + 937	9. 857 + 816	14. 8956 + 7694	19. 83897 + 50799
5. 958 + 294	10. 842 + 863	15. 3448 + 4876	20. 59988 + 99939

35. Practise the following additions until you can name the results as rapidly as you can count 1, 2, 3, 4, 5, etc.

Ex. 12. (*Oral.*)

Add by twos to 50, beginning 0, 2, 4, 6, 8. Add by twos to 51, beginning 1, 3, 5, 7, 9.

Add by threes to 102, beginning 0, 3, 6. Add by threes to 100, beginning 1, 4, 7. Add by threes to 101, beginning 2, 5, 8.

Add by fours to 100, beginning 0, 4, 8. Add by fours to 101, beginning 1, 5, 9. Add by fours to 102, beginning 2, 6, 10. Add by fours to 103, beginning 3, 7, 11.

Add by fives to 100, beginning 0, 5, 10. Add by fives to 101, beginning 1, 6, 11. Add by fives to 102, beginning 2, 7, 12. Add by fives to 103, beginning 3, 8, 13. Add by fives to 104, beginning 4, 9, 14.

Add by sixes to 102, beginning 0, 6, 12. Add by sixes to 103, beginning 1, 7, 13. Add by sixes to 104, beginning 2, 8, 14. Add by sixes to 105, beginning 3, 9, 15. Add by sixes to 100, beginning 4, 10, 16. Add by sixes to 101, beginning 5, 11, 17.

Add by sevens to 105, beginning 0, 7, 14. Add by sevens to 106, beginning 1, 8, 15. Add by sevens to 100, beginning 2, 9, 16. Add by sevens to 101, beginning 3, 10, 17. Add by sevens to 102, beginning 4, 11, 18. Add by sevens to 103, beginning 5, 12, 19. Add by sevens to 104, beginning 6, 13, 20.

Add by eights to 104, beginning 0, 8, 16. Add by eights to 105, beginning 1, 9, 17. Add by eights to 106, beginning 2, 10, 18. Add by eights to 107, beginning 3, 11, 19. Add by eights to 100, beginning 4, 12, 20. Add by eights to 101, beginning 5, 13, 21. Add by eights to 102, beginning 6, 14, 22. Add by eights to 103, beginning 7, 15, 23.

Add by nines to 108, beginning 0, 9, 18. Add by nines to 100, beginning 1, 10, 19. Add by nines to 101, beginning 2, 11, 20. Add by nines to 102, beginning 3, 12, 21. Add by nines to 103, beginning 4, 13, 22. Add by nines to 104, beginning 5, 14, 23. Add by nines to 105, beginning 6, 15, 24. Add by nines to 106, beginning 7, 16, 25. Add by nines to 107, beginning 8, 17, 26.

36. Practise adding columns of three digits until you can name the sum of any three digits the instant you see them.

Ex. 13. (*Oral.*)

Find the sums of:

1.	3	2	4	5	4	5	6	7	4	3
	5	1	2	3	3	2	3	2	3	2
	4	3	3	4	2	1	4	3	2	7
2.	3	4	5	6	5	7	8	3	4	5
	4	2	4	2	4	2	1	2	3	2
	1	3	2	3	2	3	0	4	2	3
3.	4	5	3	5	4	3	8	6	7	8
	2	2	4	3	1	5	1	2	9	6
	3	1	3	2	3	2	3	5	4	9
4.	1	8	7	6	7	9	5	7	9	6
	3	6	5	7	2	8	6	3	8	7
	5	1	7	9	9	9	4	9	1	8
5.	7	6	5	9	7	6	5	3	9	5
	2	4	5	0	6	6	9	8	9	8
	3	3	4	8	4	6	8	7	9	7

6.	3	5	7	6	8	9	7	6	9	8
	9	6	2	3	5	6	5	3	8	8
	7	8	5	4	4	4	7	5	8	8
7.	6	6	7	7	4	4	3	5	3	6
	2	5	8	7	9	4	8	5	7	8
	5	3	9	7	4	4	7	5	7	7
8.	8	3	2	3	5	5	9	4	2	2
	4	3	2	4	4	6	3	7	9	2
	4	3	9	8	8	6	6	5	8	8
9.	9	5	7	8	9	6	7	4	5	8
	9	6	7	4	5	7	4	3	4	2
	7	6	5	7	5	6	8	9	7	9
10.	3	2	2	3	6	8	7	8	9	6
	8	9	2	3	7	5	2	2	8	9
	7	8	9	7	4	4	3	2	7	7

37. The quickest way to add columns of four or more digits is to train the eye to see at a glance sums of 20, and simply add these sums. If you add the column given in the margin by single digits, you say to yourself, *ten, thirteen, seventeen, twenty-two, twenty-eight, thirty-seven, forty-five;* if you add by taking two digits at a time, you say *ten, seventeen, twenty-eight, forty-five;* if you add by taking three digits at a time, you say *thirteen, twenty-eight, forty-five;* if you add by 20's, you say *twenty* (separating 5 into 3 and 2), *forty-five.*

8
9
6
5
4
3
2
8
—

Ex. 14.

Find the sums of:

1.	5	9	6	4	5	7	1	7	8	2
	4	5	4	6	8	3	4	3	4	6
	8	7	9	5	7	8	5	6	3	9
	6	3	7	2	3	6	3	8	2	7
	4	8	5	8	4	9	8	5	7	4
	7	4	8	6	8	2	6	4	8	5
	3	9	2	9	5	7	7	3	2	3
	5	7	6	4	9	3	9	6	8	5

2.	8	5	4	4	5	2	3	5	3	6
	2	3	2	2	3	6	8	3	8	4
	4	6	5	5	2	4	6	8	4	7
	9	8	9	7	8	5	7	9	9	3
	3	2	7	6	4	3	9	5	7	5
	7	8	6	9	8	5	5	7	3	8
	6	5	9	4	6	3	4	6	8	6
	5	3	4	3	2	7	8	9	6	7
	2	4	1	8	1	9	2	4	2	9

3.	7	5	4	2	4	8	4	6	9	7
	3	6	4	6	8	7	9	8	3	9
	5	7	9	8	6	3	6	0	8	6
	8	3	6	4	7	5	7	4	2	3
	2	9	7	7	4	5	4	5	8	4
	8	4	9	9	5	4	3	3	7	5
	6	2	3	8	4	7	7	6	3	8
	3	8	7	2	6	9	6	9	4	6
	4	1	7	3	2	9	2	7	6	8

4.	6	5	9	8	5	9	8	7	9	6
	8	8	5	6	9	6	4	9	3	8
	5	3	3	7	4	7	5	6	8	3
	3	7	4	9	3	5	9	3	9	5
	7	9	9	5	7	9	7	4	8	3
	4	4	6	8	2	8	6	5	7	6
	6	8	8	4	9	3	8	8	3	9
	8	3	2	3	6	9	3	6	4	5
	2	6	1	2	8	4	2	1	0	2

5.	8	5	3	8	6	3	5	1	3	4
	5	9	9	6	5	4	5	3	3	3
	5	6	1	4	7	7	6	5	3	2
	1	1	1	3	4	3	1	8	2	6
	1	1	8	4	9	1	5	5	9	1
	9	7	1	0	3	4	1	7	6	6
	7	7	9	4	9	3	1	4	2	7
	1	1	3	3	1	1	1	0	9	8
	6	4	7	9	2	9	8	9	8	9

Ex. 15.

Find the sums of:

1.	**2.**	**3.**	**4.**	**5.**	**6.**
50	40	60	30	10	80
20	80	50	10	70	90
70	20	80	90	10	30
60	30	20	40	90	80
30	70	50	20	40	60
90	80	30	50	70	30
80	60	40	70	30	40
10	50	70	80	20	50

7.	40	8.	80	9.	52	10.	30	11.	42	12.	60
	21		70		60		23		40		40
	90		31		70		52		50		32
	50		42		30		91		80		54
	83		60		81		70		70		90
	70		51		42		33		34		82
	62		90		50		80		90		91

13.	51	14.	56	15.	48	16.	36	17.	25	18.	17
	46		63		31		42		52		82
	30		72		45		50		49		25
	25		81		82		81		38		13
	32		17		19		14		41		80
	47		26		21		35		57		45

19.	18	20.	57	21.	15	22.	44	23.	19	24.	91
	24		31		8		21		27		42
	91		28		23		36		48		36
	33		63		70		8		39		82
	64		90		61		14		7		71
	75		9		55		27		9		54
	37		81		83		59		87		65

25.	48	26.	52	27.	8	28.	16	29.	33	30.	54
	9		61		43		48		52		46
	17		26		52		85		27		8
	29		28		67		7		38		19
	83		83		9		26		41		92
	75		94		17		35		9		57
	21		77		84		54		94		83

31.	32.	33.	34.	35.	36.
55	68	9	13	48	35
67	5	23	99	6	42
78	43	25	7	51	57
9	67	68	85	9	64
4	25	79	64	23	49
18	14	7	39	88	87

Ex. 16.

Add:

1.	2.	3.	4.	5.
123	516	321	225	871
205	341	75	716	215
310	236	184	348	64
79	110	769	519	371
118	196	815	96	296

6.	7.	8.	9.	10.
123	205	310	79	118
516	341	236	110	196
321	75	184	769	815
225	716	348	519	96
871	215	64	371	296

11.	12.	13.	14.	15.
213	421	85	231	526
327	87	222	624	448
98	116	376	785	379
716	615	584	923	87
825	399	972	84	999

16.	17.	18.	19.	20.
213	327	98	716	825
421	87	116	615	379
85	222	376	584	972
231	624	785	923	84
526	448	379	87	999

Ex. 17.

Add:

1.	2.	3.	4.
1234	4321	2345	345
368	6450	3456	2783
5721	378	4567	1497
1050	4291	5678	5840
4862	5782	689	9010
9215	6431	7890	2709

5.	6.	7.	8.
5207	3426	2358	9210
3584	783	7291	1029
2671	5279	5946	291
987	1085	7368	3587
3512	9270	5492	2785
6705	876	876	8899

Ex. 18.

Add:

1.	2.	3.	4.
12345	23456	5	92583
3275	72564	23	4620
4721	3785	936	973
371	23584	6543	25
51028	987	92840	9
61234	96	72104	17

5.	6.	7.	8.
23504	358	56789	123456
4368	9246	3587	258071
25	14376	296	589347
9	845	89	258923
36	29	7	720145
378	7	12345	396012

9.	10.	11.	12.	13.
580921	654321	5	345	584321
42364	41058	24	6197	92047
527913	3792	358	52718	3681
80235	589	1497	6904	927
726048	75	36725	871	1078
4386	9	187348	89	92569

Ex. 19.

Add:

1.	2.	3.
5203461	2587609	1357924
9350472	3582764	6804281
1456849	1357908	5975325
2604030	4670253	7101584
5876543	8492056	9276432
1234567	4759841	6789009

4.	5.
8274108	5791350
3509270	246801
4680259	1384650
3584672	2794589
9876543	6532108
5279614	7999888

38. It is obvious that numbers can be added only when they refer to the same things. Five oranges and three books when "put together" are still 5 oranges and 3 books, and not 8 oranges or 8 books.

It is also obvious that digits can be added only when they refer to the same order of units. Nine hundreds and eight tens when put together are still 9 hundreds and 8 tens, and not 17 hundreds or 17 tens.

Care must be taken, therefore, in writing numbers to be added, that *all the units' digits shall fall in one column, all the tens' digits in the next column (to the left), and all the hundreds' digits in the next column, and so on.*

39. To add columns of digits with absolute accuracy and great rapidity is a real accomplishment, and the operation of addition should be continued until both these results are secured. The beginner, however, will need some **test** of the accuracy of his work. One test is to begin at the bottom of the right-hand column in adding, and write on a piece of waste-paper the entire sum of each column; then to begin at the top of the left-hand column and write also the entire sum of each column; finally, to add the sums obtained in the first addition, and the sums obtained in the second addition, and compare the results.

The study of an example will make the process understood.

Beginning at the top of the left-hand column in adding, and writing the entire sum of each column, we have:

```
28
 31
  23
   17
    28
     26
-------
3135006
```

```
 871254
 123456
 789098
 357912
 993286
-------
3135006
```

Beginning at the bottom of the right-hand column in adding, and writing the entire sum of each column, we have:

```
     26
    28
   17
  23
 31
28
-------
3135006
```

By comparing the results we find each sum to be 3,135,006, and so infer that the operation is correct.

Ex. 20.

Find the sums of:

1. 427, 342, 856, 728.
2. 483, 1000, 8000, 648, 3750, 9840.
3. 15, 603, 1145, 6342.
4. 41, 725, 60, 425, 7000, 4900, 398.
5. 39, 876, 5742, 3000, 478, 9873.
6. 327, 4960, 5000, 749, 3000, 7849.
7. 4284, 32, 679, 43, 5006, 7897.
8. 325, 6007, 983, 4050, 678, 9874.
9. 856, 9193, 8765, 4287, 6696, 9185, 979.
10. 7964, 5000, 303, 9784, 5673, 9004.
11. 9007, 34, 6876, 400, 9344, 7879.
12. 45,678, 96, 375, 4784, 9673, 11,980.
13. 7865, 3586, 4321, 8576.
14. $900,542 + 308,970 + 555,674 + 498,785$.
15. $456,789 + 304,590 + 600,792 + 480,893 + 514,763$.
16. $357,963 + 478,497 + 323,484 + 596,372 + 300,409$.
17. $706,963 + 78,405 + 907,342 + 503,476$.
18. A man bought a sleigh for $142, a carriage for $325, and a pair of horses for $476. What was the cost of all?
19. A man collected on Monday, $1290; on Tuesday, $340; on Wednesday, $1008. How much was collected in all?
20. A lady paid $912 for a piano, $342 for furniture, $187 for linen, $46 for silver. What did she pay for all?
21. A farmer had in one flock of sheep, 407; in another, 96; and in a third, 2584. How many had he in all?

22. A man owns four houses; the first is worth $47,050; the second, $9106; the third, $1492; the fourth, $512. What is the value of them all?

23. Five loads of flour weighed as follows: 3500 pounds, 4967 pounds, 3974 pounds, 7982 pounds, 7963 pounds. What was the weight of the whole?

24. A house was bought for $7895; repairs amounted to $1500; new fences, $97; repairs on stable, $463; furniture, $1285. What was the cost of the whole?

25. The population of six towns is: 1674, 9008, 3769, 4000, 7096, 3784. Find the whole population.

26. A house-lot cost $675; for building the house and furnishing materials the carpenters were paid $2245, the masons $540, the painters $320. What was expended on house and lot?

27. A merchant bought carpets to the amount of $4670; curtains, $300; paper-hangings, $1275; matting, $9765. What was the cost of the whole?

28. Find the sum of three hundred thousand six hundred fifty, seven thousand eight hundred thirty-two, eleven thousand five hundred sixty-seven, ten thousand fifty-six, four hundred seventy-two.

29. Find the sum of one hundred sixty-seven thousand, three hundred sixty-seven thousand, nine hundred six thousand, two hundred forty-seven thousand, ten thousand, seven hundred thousand, nine hundred seventy-six thousand, one hundred ninety-five thousand, ninety-seven thousand.

30. Find the sum of two hundred seven, three hundred sixty-two, nine hundred forty-five, two thousand three hundred forty-three, fifteen thousand six hundred twenty-two, forty-five thousand eight.

31. Add 3 thousand 4 hundred 92, one thousand four, 6 thousand 5 hundred seventy, 42 hundred eleven.

32. Add 386 million 591, 546 million 311 thousand 122, 796 thousand 351, 84 hundred 1, 9 thousand, 86 thousand 521, 3 hundred fifty-eight thousand 6 hundred, 8 million 888 thousand eight hundred eighty-eight, 1 hundred million.

33. Find the sum of six million sixty thousand six, seven million nine hundred fifty thousand ninety-nine, ten million nine thousand eight hundred seven, three hundred sixty-seven thousand forty-five.

34. Find the sum of 200 million 302 thousand, 200 thousand two hundred, 50 million 50 thousand 50, 25 million 860 thousand, 47 million 467 thousand, 202 million 6367.

35. What is the sum of eighteen thousand three hundred twenty, seventy-four thousand five hundred six, ten hundred seventeen thousand nine hundred twenty-one, fifty-three thousand seven hundred eleven, five hundred seventy-six thousand three hundred four, six hundred fifty thousand forty-four?

36. A man drew five loads of bricks; in the first load there were 4068; in the second, 1342; in the third, 3927; in the fourth, 1694; in the fifth, 2009. How many in all the loads?

37. What is the united population of the following cities: Utica, 28,804; Lowell, 40,928; Lynn, 28,236; Salem, 24,100; Erie, 19,500; Auburn, 17,225?

38. A fruit-grower sent to market the produce of six peach orchards; from the first, 7000 baskets; from the second, 6973; from the third, 1004; from the fourth, 3276; from the fifth, 1594; from the sixth, 3976. How many baskets in all?

39. Bought wheat for $7962, corn for $12,649, oats for $8763; sold the wheat at a profit of $780, the corn at a profit of $920, and the oats at a profit of $765. How much was received for the whole?

40. A man paid $7038 for 782 horses, $7776 for 972 horses, $9948 for 829 horses. How many horses did he buy, and what sum of money did he pay for all?

41. How many times does a clock strike from half-past 12 o'clock at night to half-past 12 o'clock at noon?

42. A gentleman is 19 years older than his sister, and she is 18 years older than her nephew, who is 15 years of age. How old is the gentleman, and how old is his sister?

43. Several persons contributed to the building of a college; A gave $7325, B gave $500 more than A, C gave $40 more than A and B together, and D gave as much as A, B, and C together. What did B, C, and D give, and what was the whole sum contributed?

44. Add one million nine hundred seventy-five thousand six hundred eighty-two, three hundred seventy-six, five million nine hundred seven, thirty-six, seven hundred sixty thousand nine, five thousand nine.

45. What number must be added to 6 that the right-hand figure in the sum shall be 0? 1? 2? 3? 4? 5?

46. A man divided his estate as follows: To his wife, $10,560; to his two sons, $7325 each; to his daughter, $485 more than each son's share. What was the whole estate?

47. A merchant has in his store flour worth $656, sugar worth $480, molasses worth $325. He owes for freight $125, for expressage $56, and for cartage $78. What is the whole value of his goods? What is the amount of his debts?

48. A farmer bought 78,580 sheep, and sold 3584; he then bought 58,960, and sold 7896. How many sheep did he buy, and how many did he sell?

49. A father gave to each of his three sons $6580, and to his daughter $920 more than to *two* sons. How much did he give to his children together?

50. If Julius Cæsar went to Gaul 58 years B.C., how many years have elapsed since that event up to the present date of the nineteenth century?

51. A man sold a horse for $986 and three Jersey cows for $680 each; he lost $125 on the horse and $220 on each cow. What was the cost of all?

52. A grocer bought 4 hogsheads of sugar for $397; in selling them he gained $37 on two together, and $12 apiece on the other two. What was received for all?

53. In an orchard there are 145 cherry trees, 534 apple trees, 127 pear trees, and 397 plum trees. How many trees in the orchard?

54. Add 56 million 6 thousand 3, 108 million 97 thousand 6 hundred 5, 996 million 690 thousand 4, 345 million 6 thousand 40, 987 million 40 thousand 3 hundred 25, 789 million 5 thousand, 7 hundred 85.

55. Bought 50,768 sheep, and then sold 2320; bought 47,962 sheep, and then sold 3648. How many sheep were bought? How many were sold?

CHAPTER III.

SUBTRACTION.

40. What number must be added to four to make seven? What, then, will be left if 4 is taken from 7?

What number must be added to seven to make ten? What, then, will be left if 7 is taken from 10?

Copy the following set of numbers, and find what number must be added to each one in the upper row to make the number below the line. Write the required numbers in the empty places above the lines:

7	6	12	4	2	9	0	5	6
—	—	—	—	—	—	—	—	—
17	14	20	7	12	10	5	7	12
13	19	24	7	13	8	9	4	10
—	—	—	—	—	—	—	—	—
15	25	28	12	20	10	11	16	21

When you have done this, you will see that, since 7 and 10 make 17, 7 taken from 17 leaves 10; since 6 and 8 make 14, 6 taken from 14 leaves 8; so with each set of numbers.

41. In the following set, under each number in the lower row, write the number that must be added to it to make the upper number:

9	12	7	12	15	10	6	9	7
3	4	2	8	6	5	0	5	4
—	—	—	—	—	—	—	—	—

11	18	17	5	10	9	16	8	3
5	2	16	1	3	2	5	2	0
—	—	—	—	—	—	—	—	—

To 3 we have to add 6 to make 9, so we write 6 under the 3. To 4 we must add 8 to make 12, so we write 8 under the 4.

Now in finding what number must be added to 3 to make 9, we have really found what number will be left if 3 is taken from 9. In finding what number must be added to 4 to make 12, we have really found what number will remain if 4 is taken from 12.

42. The operation of finding the number that remains, when a smaller number is taken from a larger, is called **subtraction.** The result is called the **remainder** or **difference.**

43. The number which is to be subtracted is called the **subtrahend;** and the number which is to be diminished (that is, the number from which the subtraction is made), is called the **minuend.**

44. A dash — is the sign of subtraction, and when placed between two numbers means that the first number is to be diminished by the second. It is called the **minus** sign.

The expression $4 - 1 = 3$ is read *four minus one equals three.*

45. Three dots $\therefore$ are often used for the word *therefore*.

The expression $6 + 2 = 8$, $\therefore 8 - 6 = 2$, is read *six plus two equals eight, therefore eight minus six equals two.*

Ex. 21. (*Oral.*)

1. What number with 5 makes 10?
   What number with 3 makes 10?
   What number with 2 makes 10?
   What number with 4 makes 10?

2. What number taken from 10 leaves 2?
   What number taken from 10 leaves 4?
   What number taken from 10 leaves 3?
   What number taken from 10 leaves 5?

3. 5 is one part of 12, what is the other?
   8 is one part of 12, what is the other?
   3 is one part of 12, what is the other?
   7 is one part of 12, what is the other?
   9 is one part of 12, what is the other?
   6 is one part of 12, what is the other?
   10 is one part of 12, what is the other?

4. What number taken from 12 leaves 11?
   What number taken from 12 leaves 9?
   What number taken from 12 leaves 5?
   What number taken from 12 leaves 8?
   What number taken from 12 leaves 2?
   What number taken from 12 leaves 6?
   What number taken from 12 leaves 7?
   What number taken from 12 leaves 1?

5.

$9 + 2 =$	$\therefore 11 - 2 =$	and $11 - 9 =$
$8 + 3 =$	$\therefore 11 - 3 =$	and $11 - 8 =$
$6 + 5 =$	$\therefore 11 - 5 =$	and $11 - 6 =$
$10 + 1 =$	$\therefore 11 - 1 =$	and $11 - 10 =$

6.	$8+5=$	$\therefore 13-5=$	and $13-8=$
	$6+7=$	$\therefore 13-7=$	and $13-6=$
	$9+4=$	$\therefore 13-4=$	and $13-9=$
7.	$6+8=$	$\therefore 14-6=$	and $14-8=$
	$5+9=$	$\therefore 14-9=$	and $14-5=$
	$7+7=$	$\therefore 14-7=$	
8.	$7+8=$	$\therefore 15-7=$	and $15-8=$
	$9+6=$	$\therefore 15-6=$	and $15-9=$
	$9+3=$	$\therefore 12-9=$	and $12-3=$
9.	$8+8=$	$\therefore 16-8=$	
	$7+9=$	$\therefore 16-7=$	and $16-9=$
	$9+8=$	$\therefore 17-9=$	and $17-8=$

10. Subtract by threes, from 100 to 1; from 102 to 2; by fours, from 101 to 1; from 102 to 2; from 103 to 3.

11. Subtract by fives, from 102 to 2; from 103 to 3; from 104 to 4; from 100 to 5.

12. Subtract by sixes, from 103 to 1; from 104 to 2; from 105 to 3; from 100 to 4; from 102 to 6.

13. Subtract by sevens, from 106 to 1; from 100 to 2; from 101 to 3; from 102 to 4; from 103 to 5; from 104 to 6; from 105 to 7.

14. Subtract by eighths, from 105 to 1; from 106 to 2; from 107 to 3; from 100 to 4; from 101 to 5; from 102 to 6; from 103 to 7; from 104 to 8.

15. Subtract by nines, from 100 to 1; from 101 to 2; from 102 to 3; from 103 to 4; from 104 to 5; from 105 to 6; from 106 to 7.

Ex. 22. (*Oral.*)

$5+4=$	$\therefore\ 9-5=$	$9-4=$
$9+3=$	$\therefore 12-9=$	$12-3=$
$6+5=$	$\therefore 11-6=$	$11-5=$
$7+6=$	$\therefore 13-7=$	$13-6=$
$9+6=$	$\therefore 15-6=$	$15-9=$
$7+9=$	$\therefore 16-9=$	$16-7=$

$14-8=$	$16-9=$	$18-6=$	$17-8=$	$25-9=$
$11-3=$	$33-8=$	$45-6=$	$76-8=$	$32-9=$
$16-7=$	$24-9=$	$37-8=$	$48-6=$	$53-9=$
$17-8=$	$35-8=$	$43-7=$	$50-4=$	$63-6=$
$12-4=$	$44-7=$	$24-8=$	$31-3=$	$26-9=$
$15-7=$	$68-9=$	$56-7=$	$43-5=$	$29-7=$
$13-6=$	$27-8=$	$34-9=$	$40-9=$	$50-7=$
$11-8=$	$13-8=$	$15-8=$	$13-9=$	$31-3=$
$27-9=$	$86-8=$	$85-9=$	$87-6=$	$84-5=$
$32-8=$	$73-5=$	$62-7=$	$26-9=$	$23-7=$
$25-4=$	$75-9=$	$73-7=$	$72-6=$	$83-8=$
$17-9=$	$31-8=$	$42-9=$	$50-3=$	$39-8=$
$42-3=$	$30-6=$	$38-9=$	$40-4=$	$93-7=$
$37-9=$	$58-9=$	$52-6=$	$63-8=$	$41-3=$
$24-7=$	$70-8=$	$21-9=$	$22-7=$	$38-9=$
$45-8=$	$42-3=$	$54-7=$	$71-8=$	$65-7=$
$19-8=$	$60-3=$	$65-9=$	$64-6=$	$17-9=$
$34-6=$	$95-6=$	$82-8=$	$79-9=$	$76-8=$
$28-9=$	$72-7=$	$90-9=$	$65-6=$	$81-7=$
$54-5=$	$77-8=$	$85-7=$	$69-9=$	$71-4=$

From 876 take 631.

Write units under units, tens under tens, and so on. Then 1 unit from 6 units leaves 5 units, and we write 5 under the units' column; 3 tens from 7 tens leave 4 tens, and we write 4 under the tens' column; 6 hundreds from 8 hundreds leave 2 hundreds, and we write 2 under the hundreds' column. The remainder, therefore, is 2 hundreds 4 tens 5 units; that is, 245.

Operation.

Minuend,	876
Subtrahend,	631
Remainder,	245

46. The minuend is the sum of the subtrahend and the remainder. Hence, to test the accuracy of the work, add the subtrahend and remainder together, and if the work is correct, their sum will be equal to the minuend.

47. It is obvious that one number can be subtracted from another only when both numbers refer to the same things. Thus, we can subtract 3 oranges from 5 oranges, but we cannot subtract 3 apples from 5 oranges.

Ex. 23.

Find the results of:

1. 59 — 23.	**13.** 89 — 41.	**25.** 786 — 45.
2. 54 — 23.	**14.** 67 — 23.	**26.** 674 — 52.
3. 67 — 14.	**15.** 58 — 17.	**27.** 569 — 38.
4. 65 — 32.	**16.** 75 — 34.	**28.** 857 — 43.
5. 78 — 25.	**17.** 96 — 53.	**29.** 294 — 82.
6. 75 — 41.	**18.** 87 — 42.	**30.** 348 — 37.
7. 85 — 33.	**19.** 69 — 37.	**31.** 489 — 76.
8. 78 — 25.	**20.** 78 — 26.	**32.** 768 — 47.
9. 96 — 42.	**21.** 64 — 43.	**33.** 976 — 53.
10. 97 — 54.	**22.** 98 — 35.	**34.** 897 — 75.
11. 87 — 54.	**23.** 89 — 53.	**35.** 588 — 64.
12. 86 — 31.	**24.** 77 — 46.	**36.** 467 — 45.

37. 874 — 632.	**42.** 6982 — 5431.	**47.** 725,419 — 613,208.
38. 792 — 261.	**43.** 7629 — 4518.	**48.** 965,420 — 342,100.
39. 798 — 627.	**44.** 7824 — 6821.	**49.** 854,267 — 723,150.
40. 764 — 532.	**45.** 8542 — 6131.	**50.** 549,830 — 438,820.
41. 862 — 741.	**46.** 8792 — 6281.	**51.** 628,300 — 517,200.

48. If the number of units of any order in the minuend is less than the number of units of the corresponding order in the subtrahend, one of the next higher order of units in the minuend must be added to the units of the order we are considering. The process will be understood by an example.

From 783 take 469.

Operation.

Minuend,	783
Subtrahend,	469
Remainder,	314

Since we cannot take 9 units from 3 units, we add 1 of the 8 *tens* to the 3 *units*, making 13 *units;* then 9 units from 13 units leave 4 units. Now as we have added 1 of the 8 *tens* to the 3 *units* of the minuend, we have only 7 tens remaining, and 6 tens from 7 tens leave 1 ten; 4 hundreds from 7 hundreds leave 3 hundreds. The remainder, therefore, is 3 hundreds 1 ten 4 units; that is, 314.

From 359 take 186.

Operation.

Minuend,	359
Subtrahend,	186
Remainder,	173

Here 6 units from 9 units leave 3 units. Since we cannot take 8 tens from 5 tens we add 1 of the 3 *hundreds* to the 5 *tens*, making 15 *tens*; then 8 tens from 15 tens leave 7 tens. Now as we have added 1 of the 3 *hundreds* to the 5 *tens* of the minuend, we have only 2 hundreds remaining; and 1 hundred from 2 hundreds leaves 1 hundred. The remainder, therefore, is 1 hundred 7 tens 3 units; that is, 173.

Ex. 24.

1. 867 — 325.	13. 90 — 35.	25. 70 — 28.
2. 985 — 312.	14. 40 — 13.	26. 50 — 13.
3. 746 — 213.	15. 70 — 26.	27. 80 — 37.
4. 384 — 132.	16. 50 — 24.	28. 60 — 48.
5. 479 — 235.	17. 80 — 32.	29. 90 — 25.
6. 679 — 215.	18. 60 — 33.	30. 50 — 27.
7. 857 — 324.	19. 60 — 47.	31. 80 — 43.
8. 956 — 532.	20. 70 — 45.	32. 70 — 36.
9. 795 — 362.	21. 70 — 52.	33. 90 — 32.
10. 687 — 321.	22. 80 — 36.	34. 60 — 27.
11. 978 — 333.	23. 90 — 28.	35. 80 — 49.
12. 835 — 214.	24. 90 — 27.	36. 70 — 36.

Ex. 25.

1. 52 — 26.	13. 63 — 29.	25. 680 — 247.
2. 73 — 38.	14. 74 — 37.	26. 570 — 236.
3. 81 — 49.	15. 92 — 68.	27. 860 — 218.
4. 94 — 57.	16. 81 — 56.	28. 690 — 254.
5. 72 — 48.	17. 75 — 38.	29. 750 — 419.
6. 91 — 64.	18. 96 — 48.	30. 830 — 214.
7. 75 — 48.	19. 85 — 57.	31. 690 — 275.
8. 92 — 48.	20. 93 — 75.	32. 750 — 326.
9. 83 — 26.	21. 54 — 18.	33. 860 — 247.
10. 95 — 47.	22. 81 — 27.	34. 970 — 358.
11. 86 — 57.	23. 75 — 29.	35. 580 — 149.
12. 95 — 66.	24. 94 — 58.	36. 870 — 146.

Ex. 26.

1. 407 — 84.	7. 462 — 38.	13. 608 — 247.
2. 308 — 75.	8. 374 — 57.	14. 706 — 253.
3. 609 — 58.	9. 281 — 65.	15. 805 — 364.
4. 205 — 81.	10. 592 — 83.	16. 904 — 472.
5. 506 — 63.	11. 476 — 68.	17. 809 — 581.
6. 807 — 42.	12. 852 — 39.	18. 705 — 694.

19. 508 — 294.
20. 609 — 385.
21. 707 — 246.
22. 806 — 324.
23. 405 — 132.
24. 709 — 328.
25. 781 — 246.
26. 892 — 387.
27. 643 — 418.
28. 954 — 216.
29. 763 — 419.
30. 655 — 247.
31. 461 — 239.
32. 572 — 238.
33. 693 — 447.
34. 754 — 536.
35. 835 — 226.
36. 973 — 237.

Ex. 27.

1. 612 — 78.
2. 523 — 64.
3. 845 — 87.
4. 417 — 58.
5. 731 — 94.
6. 324 — 65.
7. 942 — 74.
8. 635 — 89.
9. 522 — 56.
10. 417 — 68.
11. 325 — 86.
12. 712 — 94.
13. 732 — 458.
14. 816 — 237.
15. 624 — 158.
16. 936 — 489.
17. 567 — 298.
18. 715 — 348.
19. 623 — 417.
20. 861 — 375.
21. 453 — 286.
22. 817 — 329.
23. 643 — 457.
24. 415 — 186.
25. 531 — 352.
26. 642 — 263.
27. 763 — 174.
28. 824 — 296.
29. 915 — 468.
30. 812 — 357.
31. 514 — 136.
32. 972 — 489.
33. 624 — 248.
34. 512 — 136.
35. 713 — 364.
36. 817 — 259.

Ex. 28.

1. 500 — 78.
2. 600 — 83.
3. 700 — 92.
4. 800 — 64.
5. 600 — 57.
6. 400 — 76.
7. 802 — 68.
8. 304 — 95.
9. 506 — 87.
10. 403 — 75.
11. 902 — 94.
12. 504 — 69.
13. 600 — 235.
14. 800 — 217.
15. 900 — 386.
16. 700 — 427.
17. 400 — 128.
18. 800 — 372.
19. 600 — 345.
20. 700 — 562.
21. 800 — 427.
22. 900 — 368.
23. 500 — 321.
24. 600 — 487.
25. 902 — 146.
26. 805 — 347.
27. 704 — 215.
28. 607 — 238.
29. 503 — 267.
30. 906 — 387.
31. 904 — 328.
32. 802 — 467.
33. 705 — 258.
34. 603 — 318.
35. 701 — 427.
36. 705 — 348.

Ex. 29.

1. 7689 — 2345.	9. 9580 — 5136.	17. 8300 — 2746.
2. 6837 — 4216.	10. 7480 — 2367.	18. 7400 — 2843.
3. 9876 — 1234.	11. 9560 — 1423.	19. 8020 — 3647.
4. 8697 — 3274.	12. 8670 — 4324.	20. 7050 — 6873.
5. 7586 — 2145.	13. 8700 — 3218.	21. 6040 — 2895.
6. 6789 — 4321.	14. 9600 — 2745.	22. 8030 — 2746.
7. 8470 — 2138.	15. 9600 — 4347.	23. 7050 — 4873.
8. 6790 — 3245.	16. 7200 — 3647.	24. 6020 — 2748.

Ex. 30.

1. 6005 — 2347.	9. 8021 — 3472.	17. 9000 — 3725.
2. 8002 — 2636.	10. 8064 — 2397.	18. 9000 — 2745.
3. 8003 — 2746.	11. 9012 — 3684.	19. 6324 — 2538.
4. 6005 — 2748.	12. 7054 — 2768.	20. 6245 — 3789.
5. 9004 — 2615.	13. 7000 — 2546.	21. 4517 — 1638.
6. 6003 — 2846.	14. 7000 — 3748.	22. 7253 — 4867.
7. 7035 — 2648.	15. 8000 — 5318.	23. 9215 — 4757.
8. 7023 — 2896.	16. 8000 — 3526.	24. 7214 — 4869.

Ex. 31.

1. 56,739 — 24,316.	13. 59,001 — 16,739.
2. 68,507 — 47,623.	14. 89,076 — 569.
3. 47,865 — 12,341.	15. 60,020 — 24,156.
4. 72,006 — 48,315.	16. 57,490 — 598.
5. 65,043 — 17,872.	17. 70,000 — 25,487.
6. 81,000 — 25,143.	18. 70,000 — 4,139.
7. 90,000 — 30,906.	19. 60,300 — 36,428.
8. 90,503 — 47,628.	20. 70,302 — 5,648.
9. 41,009 — 31,214.	21. 80,040 — 23,619.
10. 43,020 — 36,748.	22. 63,008 — 47,236.
11. 26,735 — 9,856.	23. 50,004 — 47,825.
12. 75,986 — 43,264.	24. 80,047 — 26,578.

Ex. 32.

1. 431,250 – 153,697.
2. 920,503 – 476,829.
3. 523,146 – 286,759.
4. 647,352 – 268,574.
5. 502,304 – 186,475.
6. 625,030 – 274,384.
7. 720,301 – 368,596.
8. 842,003 – 459,687.
9. 715,324 – 369,857.
10. 900,500 – 465,783.
11. 512,435 – 126,867.
12. 600,000 – 285,436.
13. 723,514 – 536,945.
14. 801,050 – 469,872.

Ex. 33.

1. What number must be added to 7428 to make 8047?
2. What number must be taken from 3015 to leave 2405?
3. If the minuend is 78,206, and the subtrahend 35,264, what is the remainder?
4. A man borrowed at one time $463, at another time $578, at another $3648; he then paid $3122. How much did he still owe?
5. A flour merchant bought 783 barrels of Minnesota flour, 349 barrels of St. Louis flour, 906 barrels of Ohio flour; he sold 1203 barrels. How many barrels had he left?
6. Mr. Brown's yearly income is $5067; his family expenses amount to $3625; his charities to $468. How much has he left to invest?
7. The population of the New England States in 1860 was as follows: Maine, 628,279; Vermont, 315,098; New Hampshire, 326,073; Massachusetts, 1,231,060; Connecticut, 460,147; Rhode Island, 174,620. In 1870 the population of these six states amounted to 3,487,924. What was the increase?

8. A house cost: for masonry, $3460; painting, $1325; carpenters' work, $4683. If $5367 are paid, what remains due?

9. The sum of two numbers is 890,375, and one of them is 309,007. What is the other?

10. A is worth $98,760, B is worth $4586 less than A, and C is worth the difference between A and B. What are B and C each worth?

11. The sum of four numbers is 67,896; the first is 45,009; the second, 3748; the third, 18,909. Required the fourth.

12. In 1860 California had 379,994 inhabitants; Illinois had 1,711,951. In 1870 California had 560,247, and Illinois had 2,539,891. What was the gain in population of each state for the ten years?

13. What number increased by $1850 + 13{,}789$ will be 28,984?

14. What number subtracted from nine hundred eighty-seven thousand three hundred fifty-nine will leave three hundred thousand two hundred eight?

15. A cotton planter raised 9675 pounds of cotton; he sent to New York 2008 pounds, to Memphis 3906 pounds, to New Orleans 1962 pounds. How many pounds had he left?

16. There were 322 apples on a tree, of which 198 were gathered, and 87 were blown off by the wind. How many were left on the tree?

17. There are 60 minutes in an hour; how many minutes between 4 minutes after 10 o'clock and 3 minutes before 11 o'clock? Between 9 minutes after 1 o'clock and 3 minutes before 2 o'clock?

18. A man wishes to buy a farm worth $15,960, and stock to the amount of $8709. To make these purchases, how much money must he add to $13,708?

19. Eight hundred seventy-six thousand four hundred twenty-five added to a certain number makes eleven million seven hundred nine thousand three hundred four. What is the number?

20. Two men, A and B, start together from the same place and in the *same* direction. A walks the first day twenty-nine miles, and rides the second day seventy-six miles; B rides the first day sixty-seven miles, and walks the second day nineteen miles. How many miles are they apart at the end of the second day? How many miles would they have been apart if they had set off in *opposite* directions?

21. A merchant deposited in a bank $9540, and then drew a check for $3780; he then deposited $500. How much had he in the bank after his last deposit?

22. What is the difference between ninety-eight thousand sixty-five plus eight thousand nine and thirty-eight thousand six minus nine thousand seven?

23. A man went to market with $10.25. He paid for steak $2; for sugar, $1; for coffee, $1; for fruit, $2; for flour, $2. How much money had he left?

24. A horse cost $340 and was sold for $563. How much was gained?

25. A lady bought a horse for $320.70 and a carriage for $138.45; the horse was sold for $450.70, the carriage for $119.45. What was the loss on the carriage, and what was the gain on the horse?

26. A cow that cost $152 was sold at an advance of $19.25. What was she sold for?

27. A man bought a village lot for $1265. He paid $25 for taxes, and sold it for $1106. How much did he lose?

28. A horse, harness, and saddle were bought for $375. Paid for repairs on the harness $1, and on the saddle $2. Sold the horse and harness for $293, the saddle for $18. Was there gain or loss, and how much?

29. A man owing $7862.50 has paid $5678. How much is still due?

30. From a $50 bank-note a bill of $38.50 was paid. What change was given back?

31. A man left his property to his wife and two children. To his wife $3794; to his daughter $1478 less than to his wife; to his son $325 less than to his daughter. What did the daughter and son receive, and what was the whole sum divided?

32. The length of the Missouri river from its source to the Mississippi is three thousand ninety-six miles, and from its source to the Gulf of Mexico four thousand five hundred six miles. How many miles is it from the junction of the two rivers to the Gulf of Mexico?

33. The sum of 4 numbers is eight hundred ninety-seven thousand six. The first is eighty-five thousand three hundred seven, the second is twenty-five thousand nine hundred eighty-seven more than the first, the third is twelve thousand four hundred sixty-two less than the second. What is the fourth?

34. A gentleman received from his father $65,784. He paid for a house, $18,620; for furniture, $6,978; for a carriage and a pair of horses, $3000. What had he left?

35. A's estate is thirty-six thousand five hundred eight minus nine thousand eight hundred seventy-two dollars. B's estate is forty-three thousand five hundred twenty-four dollars more than A's. What is the value of B's estate?

36. A merchant invested ninety-six thousand dollars in a ship: the first year he made three thousand seven hundred ninety-five dollars; the second year he made four hundred fifty dollars; the third year he lost four thousand two hundred forty-five. Did he gain or lose by the investment, and how much?

37. In 1860 St. Louis had a population of one hundred sixty-thousand seven hundred seventy-three; Cincinnati, one hundred sixty-one thousand forty-four. In 1870 St. Louis had three hundred ten thousand eight hundred sixty-four inhabitants; Cincinnati, two hundred sixteen thousand two hundred thirty-nine. What was the increase of each? What was the excess of the increase of one above that of the other?

38. Mr. Smith paid fifteen thousand nine hundred eighty-seven dollars for his farm, implements, and stock. The stock cost three thousand six hundred dollars, the implements one thousand four hundred eighty-six dollars. Required the value of the farm.

39. How many hours from eight o'clock in the morning until five o'clock in the afternoon?

40. Four men, A, B, C, D, bought a factory for $78,960. A was to pay $12,607; B, $3,000 more than A; C, $8,000 more than B. What had D to pay?

CHAPTER IV.

MULTIPLICATION.

49. If the cost of 6 tons of coal at $7 a ton is required, the amount can be found by writing $7 six times in a column convenient for adding, as in the margin, and finding the sum of the column.

$$\begin{array}{r} \$7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ \hline \$42 \end{array}$$

50. If the cost of a whole cargo of coal was required, this operation would be long and tedious, and therefore a shorter process has been devised, called **Multiplication.**

By this process $7 is written only once, 6 is written beneath the $7 to show the *number of times* $7 must be taken in order to obtain the required amount, and this amount is found by saying 6 times $7 are $42. Thus:

$$\begin{array}{r} \$7 \\ 6 \\ \hline \$42 \end{array}$$

51. In this operation $7 is called the **multiplicand,** 6 the **multiplier,** and $42 the **product.** The multiplier 6 is the sum of six 1's, and the product 42 is the sum of six 7's. Hence, it will be seen that:

Multiplication is an operation by which, when two numbers are given, called **multiplicand** and **multiplier,** a third number is found called **product,** which is formed from the multiplicand as the multiplier is formed from unity.

Ex. 34. (*Oral.*)

1. Of what number are 2 and 4 the factors? 3 and 3? 5 and 3? 2 and 5? 3 and 6?

2. What are the factors of 14? of 9? of 8? of 18? of 6? of 21? of 10?

3. 4 is one factor of 8; what is the other?
   3 is one factor of 12; what is the other?
   9 is one factor of 18; what is the other?

4. 3 times what number make 15?
   6 times what number make 18?
   5 times what number make 25?
   4 times what number make 28?
   3 times what number make 21?
   7 times what number make 14?

5. 8 times what number make 24?
   6 times what number make 24?
   6 times what number make 12? 42? 30? 18?

6. 7 times what number make 21? 63? 35? 49? 56? 14?

7. 8 times what number make 32? 64? 16? 40? 24? 56?

8. 9 times what number make 27? 72? 45? 63? 36? 18?

9.	$6 \times 2 =$	**10.**	$7 \times 3 =$	**11.**	$8 \times 4 =$	**12.**	$0 \times 5 =$
	$8 \times 2 =$		$9 \times 3 =$		$1 \times 4 =$		$3 \times 5 =$
	$3 \times 2 =$		$0 \times 3 =$		$0 \times 4 =$		$2 \times 5 =$
	$9 \times 2 =$		$4 \times 3 =$		$3 \times 4 =$		$7 \times 5 =$
	$7 \times 2 =$		$3 \times 8 =$		$9 \times 4 =$		$1 \times 5 =$
	$1 \times 2 =$		$3 \times 3 =$		$7 \times 4 =$		$8 \times 5 =$
	$0 \times 2 =$		$3 \times 0 =$		$4 \times 4 =$		$6 \times 5 =$
	$5 \times 2 =$		$3 \times 5 =$		$6 \times 4 =$		$4 \times 5 =$
	$4 \times 2 =$		$3 \times 1 =$		$4 \times 4 =$		$9 \times 5 =$

13.	$1 \times 6 =$	**14.**	$3 \times 7 =$	**15.**	$4 \times 8 =$	**16.**	$1 \times 9 =$
	$9 \times 6 =$		$0 \times 7 =$		$0 \times 8 =$		$7 \times 9 =$
	$8 \times 6 =$		$1 \times 7 =$		$3 \times 8 =$		$0 \times 9 =$
	$3 \times 6 =$		$2 \times 7 =$		$9 \times 8 =$		$3 \times 9 =$
	$5 \times 6 =$		$4 \times 7 =$		$7 \times 8 =$		$8 \times 9 =$
	$0 \times 6 =$		$8 \times 7 =$		$1 \times 8 =$		$6 \times 9 =$
	$7 \times 6 =$		$5 \times 7 =$		$5 \times 8 =$		$9 \times 9 =$
	$2 \times 6 =$		$9 \times 7 =$		$8 \times 8 =$		$5 \times 9 =$
	$4 \times 6 =$		$6 \times 7 =$		$8 \times 6 =$		$9 \times 9 =$
17.	$4 \times 6 =$	**18.**	$3 \times 2 =$	**19.**	$5 \times 2 =$	**20.**	$7 \times 4 =$
	$7 \times 3 =$		$7 \times 9 =$		$8 \times 2 =$		$8 \times 8 =$
	$9 \times 2 =$		$8 \times 3 =$		$6 \times 4 =$		$0 \times 2 =$
	$5 \times 3 =$		$4 \times 5 =$		$7 \times 3 =$		$1 \times 9 =$
	$7 \times 4 =$		$9 \times 6 =$		$0 \times 9 =$		$6 \times 5 =$
	$8 \times 2 =$		$7 \times 4 =$		$7 \times 6 =$		$7 \times 7 =$
	$5 \times 7 =$		$8 \times 9 =$		$8 \times 5 =$		$9 \times 9 =$
	$9 \times 3 =$		$6 \times 5 =$		$9 \times 5 =$		$4 \times 8 =$
	$4 \times 9 =$		$7 \times 8 =$		$3 \times 6 =$		$7 \times 2 =$

56. When the multiplicand consists of two or more digits, and the multiplier is a single digit, it is necessary to multiply each digit of the multiplicand by the multiplier. Thus, the product of 6×4587 is the sum of six numbers, each the same as the multiplicand.

$$\begin{array}{r} 4587 \\ 4587 \\ 4587 \\ 4587 \\ 4587 \\ \underline{4587} \end{array} \quad \text{or} \quad \begin{array}{r} 4587 \\ \underline{6} \\ 27522 \end{array}$$

The sum of the six 7's is 6 times 7 = 42, and we write the 2 units in the column of units, and reserve the 4 tens to be added to the product of the tens; then 6 times 8 tens = 48 tens, which, with the 4 tens, make 52 tens, or 5 hundreds and 2 tens, and we write the 2 tens in the column of tens; then 6 times 5 hundreds = 30 hundreds, which, with the 5 hundreds, make 35 hundreds, or 3 thousands and 5 hundreds, and we write the 5 hundreds in the column of hundreds: then 6 times 4 thousands = 24 thousands, which, with the 3 thousands, make 27 thousands, and we write 27 to the left of the 5 hundreds.

57. When the multiplier is 10, 100, 1000, etc., the product is obtained by simply annexing as many zeros to the multiplicand as are found in the multiplier. Thus:

$$10 \times 4587 = 45{,}870.$$

Likewise, when the multiplier is any one of the nine significant* digits followed by zeros, the product is obtained by multiplying the multiplicand by the significant digit and annexing to the result as many zeros as are found in the multiplier. Thus, if the multiplicand is 4587, and the multiplier is 600, we multiply by 6 and obtain 27,522, and annex to this result 2 zeros, and have for the required product 2,752,200:

$$\begin{array}{r} 4587 \\ 600 \\ \hline 2{,}752{,}200 \end{array}$$

Ex. 35.

Find the products of:

1. 4×80.	**13.** 6×32.	**25.** 3×97.	**37.** 9×96.
2. 8×40.	**14.** 2×62.	**26.** 8×57.	**38.** 6×59.
3. 9×70.	**15.** 7×47.	**27.** 8×75.	**39.** 4×83.
4. 7×60.	**16.** 3×53.	**28.** 9×74.	**40.** 7×84.
5. 5×60.	**17.** 8×54.	**29.** 9×28.	**41.** 5×94.
6. 9×80.	**18.** 4×87.	**30.** 2×86.	**42.** 8×96.
7. 6×90.	**19.** 9×63.	**31.** 2×67.	**43.** 8×86.
8. 9×40.	**20.** 5×96.	**32.** 3×95.	**44.** 9×78.
9. 7×40.	**21.** 5×78.	**33.** 7×85.	**45.** 7×53.
10. 5×90.	**22.** 6×58.	**34.** 4×79.	**46.** 8×83.
11. 8×50.	**23.** 4×86.	**35.** 8×74.	**47.** 9×68.
12. 5×70.	**24.** 7×89.	**36.** 5×68.	**48.** 7×94.

* The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are called *significant* digits.

Ex. 36.

Find the products of:

1. 7×800.	13. 6×703.	25. 5×974.	37. 8×948.
2. 4×200.	14. 9×507.	26. 4×789.	38. 9×827.
3. 9×700.	15. 5×809.	27. 4×947.	39. 7×825.
4. 5×300.	16. 7×604.	28. 5×987.	40. 8×493.
5. 8×600.	17. 4×906.	29. 6×896.	41. 9×672.
6. 7×400.	18. 6×803.	30. 6×456.	42. 7×756.
7. 6×750.	19. 2×986.	31. 7×627.	43. 8×359.
8. 4×340.	20. 2×593.	32. 7×645.	44. 6×387.
9. 7×960.	21. 3×593.	33. 5×865.	45. 9×865.
10. 6×580.	22. 3×486.	34. 8×329.	46. 5×739.
11. 8×680.	23. 4×867.	35. 6×496.	47. 9×648.
12. 8×630.	24. 3×837.	36. 9×584.	48. 4×867.

Ex. 37.

Find the products of:

1. 9×6000.	13. 8×6070.	25. 2×6007.
2. 4×8000.	14. 4×9080.	26. 9×7008.
3. 7×8000.	15. 6×5080.	27. 3×8005.
4. 7×9000.	16. 7×4070.	28. 8×4007.
5. 8×6000.	17. 3×7040.	29. 4×6009.
6. 6×7000.	18. 9×3050.	30. 7×5006.
7. 6×7300.	19. 9×6320.	31. 7×8026.
8. 6×7400.	20. 7×3980.	32. 6×7054.
9. 7×8500.	21. 6×8570.	33. 5×9045.
10. 6×8600.	22. 5×7390.	34. 4×6072.
11. 5×3900.	23. 6×8570.	35. 9×6038.
12. 7×7500.	24. 8×6780.	36. 5×5076.

Ex. 38.

Find the products of:

1. 7 × 7204.	**13.** 2 × 4716.	**25.** 6 × 3725.
2. 3 × 6305.	**14.** 3 × 3825.	**26.** 7 × 5273.
3. 8 × 9308.	**15.** 4 × 6918.	**27.** 8 × 6531.
4. 6 × 4706.	**16.** 5 × 5724.	**28.** 9 × 1365.
5. 4 × 6407.	**17.** 6 × 6375.	**29.** 2 × 8417.
6. 9 × 3809.	**18.** 7 × 8413.	**30.** 3 × 7148.
7. 7 × 3628.	**19.** 8 × 5823.	**31.** 4 × 6528.
8. 8 × 6984.	**20.** 9 × 3285.	**32.** 5 × 8256.
9. 8 × 5746.	**21.** 2 × 7619.	**33.** 6 × 3748.
10. 4 × 4968.	**22.** 3 × 9167.	**34.** 7 × 4873.
11. 9 × 9786.	**23.** 4 × 4682.	**35.** 8 × 5329.
12. 7 × 3715.	**24.** 5 × 2864.	**36.** 9 × 9235.

Ex. 39.

Multiply by 2; by 3; and so on to 9:

1. 2739.	**4.** 7658.	**7.** 7463.	**10.** 6483.
2. 4519.	**5.** 5396.	**8.** 8367.	**11.** 3526.
3. 8526.	**6.** 5783.	**9.** 8562.	**12.** 5417.

Multiply by 20; by 30; and so on to 90:

13. 5732. **14.** 6749. **15.** 8345. **16.** 7952.

Multiply by 200; by 300; and so on to 900:

17. 6738. **18.** 3579. **19.** 5742. **20.** 5793.

Multiply by 2000; by 3000; and so on to 9000:

21. 4827. **22.** 9357. **23.** 6519. **24.** 7953.

58. Suppose the product of 649×4587 is required. The multiplier 649 is $600+40+9$, and the product is found by multiplying by 9, then by 40, and then by 600, and adding the partial products. Thus,

$$
\begin{array}{rrl}
 & 4587 & \\
 & \underline{649} & \\
9 \text{ times the multiplicand} = & 41283 & \left.\right\} \\
40 \text{ times the multiplicand} = & 183480 & \text{Partial} \\
\underline{600} \text{ times the multiplicand} = & \underline{2752200} & \text{products.} \\
649 \text{ times the multiplicand} = & 2976963 &
\end{array}
$$

59. The zeros at the right of the partial products do not affect the result of the addition, and may be omitted if care is taken to put the right-hand digit of each partial product directly under the multiplier used. Thus,

$$
\begin{array}{r}
4587 \\
\underline{649} \\
41283 \\
18348 \\
\underline{27522} \\
2976963
\end{array}
$$

60. If the multiplier contains zeros, the products that correspond to them will be zero, and need not be written.

Find the product of 2007×4587.

$$
\begin{array}{r}
4587 \\
\underline{2007} \\
32109 \\
\underline{9174} \\
9206109
\end{array}
\qquad
\textit{Proof:}
\left\{
\begin{array}{r}
2007 \\
\underline{4587} \\
14049 \\
16056 \\
10035 \\
\underline{8028} \\
9206109
\end{array}
\right.
$$

61. To test the accuracy of the work in multiplication, interchange the multiplicand and the multiplier. If the numerical result is the same in both cases, as in the last example, the work may be assumed to be correct.

Ex. 40.

Find the products of:

1.	27 × 8436.	13.	83 × 8495.
2.	26 × 7358.	14.	86 × 5283.
3.	36 × 3579.	15.	91 × 5246.
4.	37 × 5684.	16.	93 × 6475.
5.	45 × 5823.	17.	26 × 8167.
6.	43 × 4263.	18.	29 × 7384.
7.	53 × 4271.	19.	38 × 7496.
8.	54 × 7538.	20.	34 × 4976.
9.	64 × 9057.	21.	47 × 4982.
10.	65 × 8154.	22.	46 × 8217.
11.	78 × 6381.	23.	56 × 6284.
12.	74 × 9472.	24.	57 × 9582.

Ex. 41.

Find the products of:

1.	364 × 6492.	13.	843 × 6527.
2.	327 × 4756.	14.	935 × 5729.
3.	283 × 5718.	15.	297 × 7186.
4.	465 × 3862.	16.	487 × 8526.
5.	592 × 4718.	17.	752 × 3849.
6.	583 × 5926.	18.	594 × 6392.
7.	647 × 8529.	19.	265 × 6973.
8.	637 × 6548.	20.	378 × 7495.
9.	741 × 9438.	21.	374 × 8247.
10.	758 × 4857.	22.	648 × 9238.
11.	824 × 3741.	23.	864 × 9753.
12.	826 × 3297.	24.	798 × 5937.

Ex. 42.

1. What will 29 acres of land cost, at $475 an acre?
2. What will 89 passenger-cars cost, at $3785 each?
3. An orchard contains 25 rows of 24 apple trees each; if these trees average 16 bushels of apples apiece, how many bushels will the orchard yield?
4. A gentleman's investments yield him $75 per day. What income will he receive in two years, allowing 365 days for a year?
5. A square mile contains 640 acres How many acres in a county containing 26 townships of 36 square miles each?
6. A cotton factory has 240 looms; if each loom makes 39 yards of cloth daily, how much cloth will the factory make in 60 days?
7. A merchant living 25 miles out from New York City goes to his business and returns every day. How many miles does he travel in 6 weeks?
8. The cost of building a certain road was, on an average, $1789 to the mile. What was the cost of 327 miles of this road?
9. A line of telegraph is 585 miles in length; the wire necessary for one mile weighs 378 pounds. What will be the weight of the wire for the whole line?
10. There are 4 fields, each containing 585 hills of potatoes, and every hill averages 12 potatoes. How many potatoes in the 4 fields?
11. If a saw-mill turns out 5708 feet of boards in a day, how many feet will it turn out in 49 weeks of 6 days each?

12. If ninety-five men can do a piece of work in three thousand four hundred seventy-nine hours, in how many hours will one man, laboring at the same rate, perform the work?

13. 640 acres make one square mile. How many acres in 3481 square miles?

14. If a railroad train runs 38 miles in an hour, how many miles will it run in 84 trips of 3 hours each?

15. Two towns, two hundred ninety-four miles apart, are to be connected by a railroad, at a cost of twenty-four thousand six hundred forty-five dollars per mile. What will be the cost of the road?

16. A house requires 7865 shingles on each side of the roof. How many shingles would be necessary for three hundred forty-eight houses of the same size?

17. If 125 tons of steel rails are required for one mile of railroad, how many tons will be necessary for 389 miles?

18. A mile contains 5280 feet. How many feet in 542 miles?

19. At the rate of 1275 words in an hour, how many words will be sent over a telegraph line in a week, allowing six days for the week, and eighteen working-hours each day?

20. The garrison of a fort consume 785 pounds of bread per day. How many pounds will they consume in three years of 365 days each?

In all the examples containing dollars and cents, the pupil will remember that two places on the right are used for cents, and that dollars and cents are *always separated by the decimal point.*

(1) Required the cost of fifteen yards of cloth, at $1.25 per yard.

(2) Required the cost of nineteen yards of linen, at 60 cents per yard.

OPERATION.	OPERATION.
$1.25	$0.60
15	19
625	540
125	60
$18.75, cost of cloth.	$11.40, cost of linen.

Find the amount of the following bills:

21. Mrs. Anne Brown bought of P. Marsh 6 pairs of shoes, at $3.25; 8 yards of silk, at $2.48; 3 pairs of gloves, at $1.73; 9 collars, at $0.25.

OPERATION.

6 pairs of shoes,	@ $3.25,	cost	$19.50
8 yards of silk,	@ $2.48,	cost	$19.84
3 pairs of gloves,	@ $1.73,	cost	$ 5.19
9 collars,	@ $0.25,	cost	$ 2.25
		Ans.	$46.78

22. Mr. Henry Carey bought of John Rouse 52 pounds of butter, at $0.45; 16 yards of cassimere, at $2.25; 23 pairs of boots, at $3.88; 19 lamps, at $2.37; 28 pounds of sugar, at $0.16.

23. Mrs. Jane Wilcox bought of G. Ferrey 12 rolls of paper, at $0.85; 32 yards of carpet, at $2.32; 15 curtain fixtures, at $0.75; 14 yards of drugget, at $0.87; 18 yards of Nottingham lace, at $1.45; 14 yards of oil-cloth, at $1.85.

24. Mr. H. Humphrey bought of D. Thomson 6 bureaus, at $8.75; 3 easy-chairs, at $15.32; 12 dining-room chairs, at $4.67; 15 spring mattresses, at $3.75; 2 extension tables, at $17.85; 4 mirrors, at $9.79.

Find the balance due on the following accounts:

25. Mr. James Green bought of C. Jenkins 125 pounds of sugar, at $0.12; 17 barrels of flour, at $12.42; 48 pounds of lard, at $0.15; 482 pounds of meal, at $0.08.

And sold to him 75 bushels of apples, at $0.85; 5 tons of hay, at $10.50; 50 bushels of potatoes, at $0.75; 50 cabbages, at $0.10.

26. Mr. Albert Wood bought of G. May 378 yards of muslin, at $0.12; 45 pairs of gloves, at $1.50; 127 yards of gingham, at $0.18; 5 parasols, at $3.25.

And sold to him 3 reams of paper, at $2.75; 6 dictionaries, at $7.87; 7 writing-cases, at $4.38; 8 gold pens, at $2.50.

27. James White bought of J. G. Moore, 16 yards of cloth, at $3.75; 135 yards of sheeting, at $0.28; 25 yards of table linen, at $2.75; 13 dozen towels, at $4.35.

And sold to him 17 lambs, at $6.25; 1 yearling, $15; 1 yoke of oxen, $150.

28. Mr. G. P. Brown bought of Haight & Halsey, 15 tons railroad iron, at $9.25; 782 pounds steel, at $0.35; 17 dozen bolts, at $1.16; 5 gross screws, at $1.67.

And sold to them 5 thousand shingles, at $8.28; 189 pounds white paint, at $0.12; 125 gallons linseed oil, at $0.35; 40 gallons white varnish, at $1.25.

29. A drover bought 325 horses, at an average of $178 each; 143 cows, at $68 a head; 247 oxen, at $79 a head. What did he pay for the whole?

30. The state of Massachusetts has fourteen counties. If each county has twenty-five towns, each town two hundred and seventy-five houses, and each house nine inhabitants, what is the population of the State?

31. If John Brown starts for Washington, from a place 2796 miles distant, and travels 87 miles per day for 25 days, how far will he then be from Washington?

32. A butcher bought four thousand three hundred forty-five pounds of beef, and sold three thousand one hundred twenty-four pounds; he then bought three times as many pounds as he had left. How many pounds did he buy in all?

33. Multiply the difference between sixty-five thousand two and forty-eight thousand nine hundred sixty-three by seven hundred sixty-eight.

34. Three boys sold chestnuts to a merchant: George, 20 quarts; John, 19 quarts; Henry as many quarts as George and John together. Five dollars were paid for the chestnuts. If the merchant retailed them at 12 cents per quart, what did he make?

35. A family uses for necessary purposes 24 gallons of water daily, and wastes 6 gallons. How many gallons will be required to supply the family for two years of 365 days each?

36. A western speculator has three tracts of land: the first contains 462 acres, the second twice that number of acres, and the third 25 times the sum of the first and second. How many acres in the third?

37. A farmer has 2678 sheep; he sells 347 to his neighbor, and 1792 to a drover; he then buys 12 times the number left. How many does he buy?

38. A passenger-train and a freight-train are running in the same direction, one at the rate of 15 miles per hour, the other at the rate of 36 miles per hour. How far apart will they be in one hour? How far apart in 16 hours? How far apart in 13 hours if they travel in *opposite* directions?

CHAPTER V.

DIVISION.

62. To divide \$42 by 6 is to find *the number of dollars* that must be taken 6 times to make \$42. Again, to divide \$42 by \$6 is to find *the number of times* that it is necessary to take \$6 to make \$42. In either case, the *product* and *one factor* are given and *the other factor* is required. Hence,

63. Division is an operation by which when **the product** and **one factor** are given **the other factor** is found.

64. The number to be divided is called the **dividend,** the number by which the dividend is to be divided is called the **divisor,** and the result is called the **quotient.**

65. Division is indicated by *the sign of division* $\div$, or by writing the dividend over the divisor with a line between them. Thus, each of the expressions $42 \div 6 = 7$, and $\frac{42}{6} = 7$, means and is read "forty-two **divided by** six equals seven."

Ex. 43. (*Oral.*)

$2 \times 8 =$	$\therefore 16 \div 2 =$	$2 \times 6 =$	$\therefore 12 \div 6 =$
	$16 \div 8 =$		$12 \div 2 =$
$2 \times 2 =$	$\therefore\ 4 \div 2 =$	$2 \times 3 =$	$\therefore\ 6 \div 3 =$
			$6 \div 2 =$
$2 \times 5 =$	$\therefore 10 \div 2 =$	$2 \times 7 =$	$\therefore 14 \div 7 =$
	$10 \div 5 =$		$14 \div 2 =$

$2 \times 9 =$	$\therefore 18 \div 2 =$ $18 \div 9 =$	$5 \times 9 =$	$\therefore 45 \div 5 =$ $45 \div 9 =$
$3 \times 4 =$	$\therefore 12 \div 4 =$ $12 \div 3 =$	$5 \times 8 =$	$\therefore 40 \div 8 =$ $40 \div 5 =$
$3 \times 3 =$	$\therefore 9 \div 3 =$	$5 \times 3 =$	$\therefore 15 \div 3 =$ $15 \div 5 =$
$3 \times 6 =$	$\therefore 18 \div 6 =$ $18 \div 3 =$	$5 \times 6 =$	$\therefore 30 \div 6 =$ $30 \div 5 =$
$3 \times 9 =$	$\therefore 27 \div 9 =$ $27 \div 3 =$	$5 \times 4 =$	$\therefore 20 \div 4 =$ $20 \div 5 =$
$3 \times 7 =$	$\therefore 21 \div 3 =$ $21 \div 7 =$	$5 \times 7 =$	$\therefore 35 \div 7 =$ $35 \div 5 =$
$3 \times 8 =$	$\therefore 24 \div 8 =$ $24 \div 3 =$	$6 \times 9 =$	$\therefore 54 \div 6 =$ $54 \div 9 =$
$3 \times 5 =$	$\therefore 15 \div 5 =$ $15 \div 3 =$	$6 \times 3 =$	$\therefore 18 \div 6 =$ $18 \div 3 =$
$4 \times 5 =$	$\therefore 20 \div 4 =$ $20 \div 5 =$	$6 \times 6 =$	$\therefore 36 \div 6 =$
$4 \times 3 =$	$\therefore 12 \div 4 =$ $12 \div 3 =$	$6 \times 7 =$	$\therefore 42 \div 6 =$ $42 \div 7 =$
$4 \times 6 =$	$\therefore 24 \div 6 =$ $24 \div 4 =$	$6 \times 8 =$	$\therefore 48 \div 8 =$ $48 \div 6 =$
$4 \times 9 =$	$\therefore 36 \div 9 =$ $36 \div 4 =$	$6 \times 5 =$	$\therefore 30 \div 6 =$ $30 \div 5 =$
$4 \times 7 =$	$\therefore 28 \div 7 =$ $28 \div 4 =$	$6 \times 4 =$	$\therefore 24 \div 6 =$ $24 \div 4 =$
$4 \times 8 =$	$\therefore 32 \div 4 =$ $32 \div 8 =$	$7 \times 3 =$	$\therefore 21 \div 3 =$ $21 \div 7 =$
$4 \times 4 =$	$\therefore 16 \div 4 =$	$7 \times 9 =$	$\therefore 63 \div 9 =$ $63 \div 7 =$
$5 \times 5 =$	$\therefore 25 \div 5 =$	$7 \times 7 =$	$\therefore 49 \div 7 =$

$7 \times 4 =$	$\therefore 28 \div 4 =$	$8 \times 9 =$	$\therefore 72 \div 9 =$
	$28 \div 7 =$		$72 \div 8 =$
$7 \times 8 =$	$\therefore 56 \div 7 =$	$8 \times 4 =$	$\therefore 32 \div 4 =$
	$56 \div 8 =$		$32 \div 8 =$
$7 \times 5 =$	$\therefore 35 \div 7 =$	$9 \times 3 =$	$\therefore 27 \div 9 =$
	$35 \div 5 =$		$27 \div 3 =$
$7 \times 6 =$	$\therefore 42 \div 6 =$	$9 \times 5 =$	$\therefore 45 \div 9 =$
	$42 \div 7 =$		$45 \div 5 =$
$8 \times 8 =$	$\therefore 64 \div 8 =$	$9 \times 9 =$	$\therefore 81 \div 9 =$
$8 \times 3 =$	$\therefore 24 \div 8 =$	$9 \times 6 =$	$\therefore 54 \div 9 =$
	$24 \div 3 =$		$54 \div 6 =$
$8 \times 7 =$	$\therefore 56 \div 7 =$	$9 \times 8 =$	$\therefore 72 \div 8 =$
	$56 \div 8 =$		$72 \div 9 =$
$8 \times 5 =$	$\therefore 40 \div 5 =$	$9 \times 7 =$	$\therefore 63 \div 9 =$
	$40 \div 8 =$		$63 \div 7 =$

66. In the following exercises, the divisor for each line of dividends is written at the left. The quotients should be named without a moment's hesitation.

Ex. 44. (*Oral.*)

1.	6)	54	18	42	12	6	24	36	60	30	48
2.	4)	28	16	20	32	40	36	4	24	12	8
3.	8)	32	48	64	16	40	24	56	8	72	0
4.	9)	0	27	72	81	18	63	54	9	45	36
5.	7)	21	0	14	7	28	42	35	56	63	49
6.	5)	30	45	20	5	40	50	25	15	35	10
7.	3)	9	12	3	30	27	18	15	21	6	24

Ex. 45. (*Oral.*)

Give the quotients and remainders in the following examples:

1.	2) 11	7	9	15	13	14	18	19	16	17
2.	3) 7	11	10	13	16	14	17	15	5	19
3.	4) 19	7	13	17	25	15	22	31	33	29
4.	9) 71	83	15	25	34	17	19	62	26	44
5.	7) 15	19	27	38	40	54	48	60	17	39
6.	8) 23	14	31	17	9	25	37	68	71	28
7.	6) 20	15	19	27	32	10	13	45	57	40
8.	5) 14	9	13	21	43	12	49	32	29	38
9.	7) 11	18	26	37	34	53	47	59	16	33
10.	5) 13	11	22	16	42	32	48	31	49	37
11.	6) 21	16	31	19	11	26	39	46	56	41
12.	8) 33	34	39	18	27	41	69	70	75	63
13.	9) 73	16	84	29	35	43	51	64	80	70
14.	5) 37	41	34	27	36	23	28	33	44	48
15.	6) 37	44	17	10	51	58	25	34	59	50
16.	8) 26	39	30	42	53	20	36	43	51	57
17.	9) 21	37	23	41	47	11	55	50	60	65
18.	7) 13	23	61	46	55	69	25	58	62	18
19.	8) 55	67	78	44	61	50	74	65	52	77

SHORT DIVISION.

67. When the divisor is so small that the work can be performed mentally, the process is called **Short Division,** and will be understood from the following examples:

(1) Divide 697,425 by 3.

The divisor is written at the left of the dividend, as in the margin.

```
3)697425
  232475
```

Wording. 3 in 6, **2**; in 9, **3**; in 7, **2**; in 14, **4**; in 22, **7**; in 15, **5**.

Here the divisor is contained in 6 twice, in 9 three times, and in 7 twice with remainder 1; this 1 is equal to 10 of the next lower order, and with the 4, the next order of the dividend, makes 14. Then 14 is divided by 3; the quotient is 4 with remainder 2; this 2 is equal to 20 of the next lower order, and with the 2 makes 22. Then 22 is divided by 3; the quotient is 7 with remainder 1. Then 15 is divided by 3, and the quotient is 5.

(2) Divide 4,236,158 by 7.

7)4236158
605165 *with remainder* 3.

In this example, 7 is not contained in 3, so 0 is the second figure of the quotient: then the next figure 6 of the dividend is joined to the 3, making 36, and the division is continued. When the division is finished, there is a remainder 3.

(3) Divide 54,123 by 9.

9)54123
6013 *with remainder* 6.

Each quotient figure is of the same **order of units** as the right-hand figure of that part of the dividend used in obtaining it. Thus, 54 in this example are 54 thousands, and the first figure of the quotient is 6 thousands.

(4) Divide $23,087 by 5.

5)$23087
$4617 *with* $2 *remaining.*

In this example, we are required to divide 23087 dollars into **five equal parts**, and find the *number of dollars* in each part. The answer is 4617 *dollars*, with 2 dollars over. The complete quotient may be written $\$4617\frac{2}{5}$.

(5) Divide \$23,087 by \$5.

$$\$5)\underline{\$23087}$$
$$4617 \textit{ with } \$2 \textit{ remaining.}$$

In this example, we are required to find the *number of times* we can take away \$5 from \$23,087, and the answer is 4617 *times*, with \$2 left over. The complete quotient may be written $4617\frac{2}{5}$; and the meaning is, that we can take \$5 away 4617 times from \$23,087, and the next time have \$2 to take away.

68. The last two examples illustrate the different meanings of division. When the divisor corresponds to the multiplier in multiplication the quotient corresponds to the multiplicand, and denotes *the same kind of units as the dividend;* when the divisor corresponds to the multiplicand the quotient corresponds to the multiplier, and denotes the *number of times* the divisor must be taken to obtain a quantity equal to the dividend.

69. A number, when divided by 10, will have a quotient consisting of the same series of figures, the last one being cut off for the remainder. Thus, $35764 \div 10 = 3576$ *with remainder* 4. In this case, the value of each figure in the result is diminished ten-fold, the *tens* becoming *units*, the hundreds becoming *tens*, and so on. A number, when divided by 100, 1000, etc., will have the same series of figures in the quotient, the last *two*, *three*, etc., figures being cut off for the remainder. Hence,

When a divisor ends in one or more zeros, cut off the zeros and an equal number of figures from the right of the dividend, perform the division with the numbers left,

and for the total remainder annex the figures cut off from the dividend to the remainder from the division.

Divide 5,786,342 by 200.

$$\begin{array}{l} 2\cancel{0}\cancel{0})\underline{57863\cancel{4}\cancel{2}} \\ \qquad\quad 28931 \textit{ with remainder } 142. \end{array}$$

In this example, we cut off the two zeros at the right of the divisor and two figures at the right of the dividend; then we divide, putting the first figure of the quotient under the figure 8, which is the right-hand figure of the first partial dividend when the entire divisor 200 is used.

70. The product of the divisor and quotient increased by the remainder is equal to the dividend. Hence,

To test the accuracy of the work of division, find the product of the divisor and quotient, and to this product add the remainder; this result will be equal to the dividend if the work is correct.

Thus, in the last example,

$$200 \times 28{,}931 = 5{,}786{,}200,$$

and $$5{,}786{,}200 + 142 = 5{,}786{,}342 \text{ (the dividend).}$$

Ex. 46.

Find the quotients of:

1. $48 \div 2$.	**10.** $75 \div 5$.	**19.** $91 \div 8$.	**28.** $815 \div 5$.
2. $72 \div 3$.	**11.** $98 \div 7$.	**20.** $94 \div 9$.	**29.** $714 \div 6$.
3. $56 \div 4$.	**12.** $92 \div 4$.	**21.** $94 \div 5$.	**30.** $826 \div 7$.
4. $85 \div 5$.	**13.** $57 \div 2$.	**22.** $87 \div 4$.	**31.** $952 \div 8$.
5. $96 \div 6$.	**14.** $83 \div 3$.	**23.** $95 \div 6$.	**32.** $972 \div 9$.
6. $84 \div 7$.	**15.** $75 \div 4$.	**24.** $77 \div 3$.	**33.** $912 \div 8$.
7. $96 \div 8$.	**16.** $48 \div 5$.	**25.** $734 \div 2$.	**34.** $492 \div 4$.
8. $99 \div 9$.	**17.** $77 \div 6$.	**26.** $768 \div 3$.	**35.** $675 \div 5$.
9. $90 \div 6$.	**18.** $82 \div 7$.	**27.** $956 \div 4$.	**36.** $918 \div 6$.

37. 513 ÷ 2.	**53.** 9354 ÷ 6.	**69.** 4017 ÷ 7.
38. 719 ÷ 3.	**54.** 8176 ÷ 7.	**70.** 7139 ÷ 8.
39. 623 ÷ 4.	**55.** 9456 ÷ 8.	**71.** 9415 ÷ 6.
40. 749 ÷ 5.	**56.** 8568 ÷ 9.	**72.** 8793 ÷ 5.
41. 875 ÷ 6.	**57.** 3712 ÷ 8.	**73.** 3794 ÷ 2.
42. 643 ÷ 7.	**58.** 2226 ÷ 7.	**74.** 7929 ÷ 3.
43. 927 ÷ 8.	**59.** 2550 ÷ 6.	**75.** 6728 ÷ 4.
44. 705 ÷ 9.	**60.** 2895 ÷ 5.	**76.** 6380 ÷ 5.
45. 591 ÷ 8.	**61.** 5391 ÷ 2.	**77.** 8322 ÷ 6.
46. 853 ÷ 7.	**62.** 7418 ÷ 3.	**78.** 9219 ÷ 7.
47. 735 ÷ 6.	**63.** 5327 ÷ 4.	**79.** 7395 ÷ 2.
48. 923 ÷ 5.	**64.** 8236 ÷ 5.	**80.** 7684 ÷ 3.
49. 7594 ÷ 2.	**65.** 7129 ÷ 6.	**81.** 7315 ÷ 4.
50. 7458 ÷ 3.	**66.** 8513 ÷ 7.	**82.** 8369 ÷ 5.
51. 9656 ÷ 4.	**67.** 9237 ÷ 8.	**83.** 5869 ÷ 6.
52. 7985 ÷ 5.	**68.** 5682 ÷ 9.	**84.** 4239 ÷ 7.

Divide by 2; by 3; and so on to 9:

85. 5794. **86.** 4572. **87.** 9785. **88.** 7163.

Divide by 20; by 30; and so on to 90.

89. 8239. **90.** 5197. **91.** 3274. **92.** 5834.

Divide by 200; by 300; and so on to 900:

93. 4571. **94.** 5768. **95.** 9563. **96.** 9876.

Ex. 47.

1. There were 72 children in a Sunday-school, and they walked two and two to church. How many rows would they make? How many rows would there have been if they had walked three and three?

2. A boy had 97 filberts. He kept 34 for himself, and divided the rest equally among his 9 class-mates. How many did he give to each?

3. How many times must we take the number 7 to make 819? How many times the number 9?

4. Divide a paper of 264 pins equally into 8 papers.

5. 2691 poles were used in a certain hop-yard, and 3 were required for each plant. How many plants were there?

6. A blacksmith uses 9 nails in putting on one shoe, and in one day he used 432 nails. How many hoofs did he shoe?

7. A forest of 1995 trees is to be thinned by cutting down 1 tree in 7. How many will be left?

8. A regiment consists of 1200 men and 60 officers. How many men are there to each officer?

9. When beef is $7 per hundred-weight, how many hundred-weight can be bought for $9,700,327?

10. How many tons of coal, at $9, can be bought for $3,596,301?

11. A wagon travels 58,068 feet. How many times will a wheel 12 feet in circumference turn in going that distance?

12. A square yard contains 9 square feet. How many square yards in 3,917,502 square feet?

13. Aaron Reed left $325,645 for his wife and four children. How much had each, if the property was divided equally among them?

14. A grocer sells brown sugar at $9 per hundred-weight. If he receives $976,482, how many hundred-weight does he sell?

15. John Brown paid $375,008 for a tract of wild land, at $8 per acre. How many acres did he buy?

16. How many tons of coal, at $7 per ton, can be purchased for $3,785,908?

17. A merchant received $397,640 in selling a quantity of flour, at $8 per barrel. How many barrels did he sell?

18. What must be paid for 12 yards of cloth, if 5 yards cost $25?

SOLUTION. If 5 yards cost $25, to find the cost of 1 yard $25 must be divided by 5; $\$25 \div 5 = \$5$, cost of 1 yard. 12 yards will cost $12 \times \$5 = \$60$. *Ans.*

19. A drover paid $20 for 5 sheep. What will be the cost of 125 sheep?

20. Three cows cost $156. What must be paid for 27 cows?

21. If 7 tons of hay cost $105, what will be the cost of 63 tons?

22. If 9 barrels of flour are worth $63, how many barrels of apples, at $3 a barrel, will pay for 72 barrels of flour?

23. If 7 cords of birch wood are worth $28, how many cords of birch wood will pay for 6 barrels of sugar worth $16 a barrel?

24. If 12 men do a piece of work in 12 hours, how many hours would it take 8 men to do the same work?

LONG DIVISION.

71. The process of Long Division is the same as that of Short Division, except that the work is written in full, and the quotient is written *over* the dividend.

Divide 41,668 by 78.

The beginner will find it convenient to form a table of products of the divisor by the numbers 1, 2, 3, ..., as follows:

$1 \times 78 = 78$	$4 \times 78 = 312$	$7 \times 78 = 546$
$2 \times 78 = 156$	$5 \times 78 = 390$	$8 \times 78 = 624$
$3 \times 78 = 234$	$6 \times 78 = 468$	$9 \times 78 = 702$

The third product is found by adding the first and second products, the fourth by adding the first and third, and so on.

As 78 is more than 41, it is necessary to take *three* figures of the dividend for the first partial dividend. Of the products in the table that do not exceed 419 the greatest is 390, that is, 5×78. Hence the first quotient figure is 5, and is written over the 9 in the dividend; then 390 is subtracted from 419. To the remainder 29, the next figure 9 of the dividend is annexed. Of the products that do not exceed 299, the greatest is 234, that is, 3×78. Hence 3 is the next figure of the quotient, and the next remainder is 65, to which the 8 of the dividend is annexed. Of the products that do not exceed 658, the greatest is 624, that is, 8×78. Hence the next figure of the quotient is 8, and the remainder 34.

OPERATION.

```
     538
78)41998
   390
    299
    234
     658
     624
      34 remainder.
```

After a little practice the operation of division can be performed without the aid of a table of products. Each quotient figure is estimated by taking for a trial divisor the left-hand figure of the divisor (or the left-hand figure in-

creased by 1, when the next figure is greater than 5), and by taking for a trial dividend one or two figures only of each partial dividend. When the trial divisor is increased by 1, the trial dividend should be increased by 1.

Divide 2,791,163 by 394.

The first partial dividend is 2791. As 9, the second figure of the divisor, is greater than 5, we take 4 for a trial divisor. As we have increased the trial divisor, we increase the trial dividend by 1, making it 28. 4 is contained in 28 7 times. We write the 7 over the 1, and multiply the divisor 394 by 7. We subtract the product 2758 from 2791 and have for a remainder 33, to which we annex the 1 of the dividend. As 331 is less than 394, the next quotient figure is 0. To 331 we annex the next figure 6 of the dividend. 4 is contained in 34 8 times. We therefore write 8 for the next quotient figure, and find the product of 8×394 to be 3152. The remainder obtained by subtracting 3152 is 164, to which the 3 of the dividend is annexed. 4 is contained 4 times in 17. The product of 4×394 is 1576, and this subtracted from 1643 leaves 67 for the final remainder.

Operation.

```
        7084
394)2791163
    2758
      3316
      3152
       1643
       1576
         67 remainder.
```

Note. If the product of the divisor by the quotient figure is greater than the partial dividend, the quotient figure is too large, and must be diminished; and, if the difference between the partial dividend and the product of the divisor by the quotient figure is greater than the divisor, the quotient figure is too small and must be increased.

Ex. 48.

Find the quotients of:

1. $4386 \div 21$.	5. $9357 \div 61$.	9. $6985 \div 22$.
2. $5271 \div 31$.	6. $5263 \div 71$.	10. $9876 \div 32$.
3. $8056 \div 41$.	7. $3046 \div 82$.	11. $2378 \div 42$.
4. $7158 \div 51$.	8. $7219 \div 92$.	12. $4068 \div 52$.

13.	8359 ÷ 63.	**21.**	6,543 ÷ 68.	**29.**	79,853 ÷ 63.
14.	4573 ÷ 73.	**22.**	8,319 ÷ 78.	**30.**	82,569 ÷ 73.
15.	7358 ÷ 84.	**23.**	5,432 ÷ 89.	**31.**	94,365 ÷ 84.
16.	3985 ÷ 94.	**24.**	9,753 ÷ 99.	**32.**	98,765 ÷ 94.
17.	6973 ÷ 25.	**25.**	41,268 ÷ 21.	**33.**	82,639 ÷ 25.
18.	7413 ÷ 36.	**26.**	74,306 ÷ 31.	**34.**	64,372 ÷ 35.
19.	8765 ÷ 47.	**27.**	89,415 ÷ 42.	**35.**	59,036 ÷ 46.
20.	7654 ÷ 57.	**28.**	67,834 ÷ 52.	**36.**	42,837 ÷ 56.

Ex. 49.

Find the quotients of:

1.	84,317 ÷ 67.	**13.**	437,650 ÷ 23.	**25.**	437,650 ÷ 53.
2.	72,659 ÷ 77.	**14.**	657,320 ÷ 35.	**26.**	657,320 ÷ 65.
3.	64,980 ÷ 88.	**15.**	327,045 ÷ 47.	**27.**	327,045 ÷ 77.
4.	52,196 ÷ 98.	**16.**	632,008 ÷ 59.	**28.**	632,008 ÷ 89.
5.	47,028 ÷ 29.	**17.**	437,650 ÷ 33.	**29.**	437,650 ÷ 63.
6.	74,369 ÷ 39.	**18.**	657,320 ÷ 45.	**30.**	657,320 ÷ 75.
7.	54,371 ÷ 14.	**19.**	327,045 ÷ 57.	**31.**	327,045 ÷ 87.
8.	68,594 ÷ 15.	**20.**	632,008 ÷ 69.	**32.**	632,008 ÷ 99.
9.	73,109 ÷ 16.	**21.**	437,650 ÷ 43.	**33.**	437,650 ÷ 73.
10.	82,563 ÷ 17.	**22.**	657,320 ÷ 55.	**34.**	657,320 ÷ 85.
11.	94,069 ÷ 18.	**23.**	327,045 ÷ 67.	**35.**	327,045 ÷ 97.
12.	47,938 ÷ 19.	**24.**	632,008 ÷ 79.	**36.**	632,008 ÷ 29.

Ex. 50.

Find the quotients of:

1.	50,576 ÷ 101.	**7.**	76,593 ÷ 415.	**13.**	96,432 ÷ 781.
2.	50,576 ÷ 102.	**8.**	76,593 ÷ 516.	**14.**	96,432 ÷ 592.
3.	50,576 ÷ 203.	**9.**	76,593 ÷ 621.	**15.**	96,432 ÷ 864.
4.	50,576 ÷ 205.	**10.**	76,593 ÷ 732.	**16.**	96,432 ÷ 972.
5.	50,576 ÷ 302.	**11.**	76,593 ÷ 843.	**17.**	96,432 ÷ 492.
6.	50,576 ÷ 106.	**12.**	76,593 ÷ 954.	**18.**	96,432 ÷ 993.

Ex. 51.

Find the quotients of:

1. $861{,}345 \div 4001$.
2. $861{,}345 \div 2048$.
3. $861{,}345 \div 3507$.
4. $861{,}345 \div 6409$.
5. $861{,}345 \div 8157$.
6. $861{,}345 \div 3965$.
7. $730{,}604 \div 8403$.
8. $972{,}817 \div 7184$.
9. $854{,}235 \div 8794$.
10. $730{,}604 \div 5748$.
11. $972{,}817 \div 4981$.
12. $730{,}604 \div 1984$.

Ex. 52.

1. How many stoves can be bought for $1120, if 3 stoves cost $105?
2. If 12 carriages are valued at $1728, how many can be bought at the same rate for $54,000?
3. A horse dealer bought 5 horses for $625. What would he pay for 485 horses, at the same rate?
4. How many barrels of sugar can be bought for $8352 when $108 are paid for three barrels?
5. A merchant bought 297 barrels of flour for $1887, and sold it at a gain of $786. What was the selling-price per barrel?
6. George Clifford paid $10,250 for oxen when 4 could be bought for $328. How many did he buy?
7. Mark Stone paid $430 for 332 fruit-trees; his brother, buying at the same time, spent $860. How many fruit-trees did the brother buy?
8. Eight shares of bank-stock are worth $784. How many shares can be bought for $22,050?

9. A farmer bought 19 sheep for $114. He sold 9 of them for $32. For how much a head must he sell the rest in order to gain $38?

10. John Jones, with $1752, went to buy sheep; his expenses were $125; he bought 283 sheep at a uniform rate, and had $212 left. What did he pay for each?

11. A fruit grower received $1755 for 195 barrels of cranberries. What was the price per barrel?

12. In one square foot there are 144 square inches. How many square feet in 1,375,920 square inches?

13. A public library has a yearly circulation of 56,966 books. How many books are taken daily, if the library is open 313 days a year?

14. One mile contains 320 rods. How many miles in 348,160 rods?

15. Dividend 514,478, divisor 327, remainder 107. What is the quotient?

16. A railroad 478 miles in length cost $3,500,872. What was the average cost per mile?

17. How many house-lots, at $321 for each, can be bought for $772,326?

18. A company of 547 men took equal shares in a mine valued at $705,083. How much money did each man invest?

19. If 325 workmen are paid $583,700, what sum does each receive?

20. At $89 per acre, how many acres of land can be purchased for $713,513?

21. Divide one million three hundred seventy-five thousand eight hundred nine by two hundred eighty-seven.

22. A ship averaging 215 miles per day has to sail 3678 miles. How many days will be required for the trip?

23. A New Orleans merchant sends to New York 376,705 gallons of molasses. How many casks will there be if each cask contains 235 gallons?

24. The capital of a bank, amounting to $518,077, is divided among 679 stockholders. What is the average amount held by each?

25. If 34,823 tons of coal are required for 97 steamships, what is the average number of tons for each?

26. A carpet-factory running 45 looms makes 17,820 yards of carpet in a fortnight. What number of yards is woven daily by one loom?

27. How much greater is the quotient of one hundred seventy-nine thousand three hundred sixty-four divided by six than the difference between forty thousand and twenty-eight thousand two hundred?

28. In one cubic foot there are 1728 cubic inches. How many cubic feet are there in a pile of wood containing 3,507,840 cubic inches?

29. Henry Sturgis has $250 at the beginning of a college term of 16 weeks. He spends $40 for books, $15 for fuel and lights, $25 for travelling expenses, and has $74 left at the end of the term. What did he pay per week for board?

30. A speculator paid $428 for 4 horses. At that rate how many horses can he buy for $317,897?

31. A man having an income of $3874 a year (52 weeks) spent $1826 for family expenses, gave $488 for charitable purposes, and saved the rest. How much did he save per week?

32. A planter sends to market 428,197 pounds of cotton in three lots. In the first lot the bales weigh 232 pounds; in the second lot 425 pounds; in the third lot 256 pounds; the number of bales in each lot is the same. What is the whole number of bales?

33. How many pieces of carpet, at $75, $96, and $132 per piece, the quantity of each kind being the same, can be bought for $998,385?

34. A has 425 horses valued at $58,650; B has 382 acres of land worth $48,514. What is the difference in value between one of A's horses and an acre of B's land?

---

35. What is that number to which if 8 be added, and the sum divided by 3, the quotient will be 12?

NOTE. In all such questions the pupil is to work from the end of the example to the beginning, and *reverse* the operations.

36. If 17 be taken from a certain number and the remainder be divided by 10, the quotient will be 5. Required the number.

37. What number is that to which if 84 be added and the sum be divided by 72, the quotient will be 312?

38. By what number must 200 be multiplied that the product increased by 250 may be 30,250?

39. Dividend 13,642, remainder 2, quotient 110. Required the divisor.

40. Dividend 325,682, divisor 284, remainder 218. Required the quotient.

41. Divisor 235, quotient 423, remainder 34. Required the dividend.

NOTE. When dollars and cents are to be expressed, it must not be forgotten that two places on the right are to be used for cents, and that a point is to separate cents from dollars.

If four dollars and fifty cents are paid for nine pounds of tea, what is the price of one pound?

$$9\underline{)\$4.50}$$
$$\$0.50 = \text{price of one pound.}$$

Here 9 is not contained in 4, so that 0 is written in the quotient under the 4; and the decimal point is written in the quotient directly under the decimal point in the dividend.

42. If 8 yards of muslin can be bought for $2.00, what must be paid for 28 yards?

43. If 16 melons cost 96 cents, how many dollars and cents must be paid for 3585 melons?

44. A lady bought 19 yards of silk for $66.50, and 48 yards of carpet for $108. How much more was paid for a yard of silk than for a yard of carpet?

45. Martin Green paid $72.75 for 97 acres of land. What would be the cost of 437 acres, at the same rate?

46. Sixty-five yards of cloth cost $325. What will be lost per yard by selling the cloth at the rate of 10 yards for $45.00? and what the loss on the whole?

47. A grain merchant bought 684 bushels of grain for $752.40. At what rate must he sell it to gain 15 cents per bushel?

48. Hiram Jenkins bought 25 cows for \$1262.50, and sold them at \$62.25 each. What did he gain on each? What would have been his whole gain if he had sold them at \$75 each?

49. If corn be bought at the rate of \$3.55 for 5 bushels, and is sold at the rate of \$7.84 for 8 bushels, how much money will be made in buying and selling 600 bushels?

50. A merchant, at a loss of 25 cents on a yard, sold cloth for which he paid \$6.00 per piece of 12 yards. What did he lose in selling 12 pieces of this cloth? To equal the loss, how many pairs of shoes must he sell at a gain of \$2 a pair?

51. If 25 horses are worth \$1862.50, and one horse is equal in value to 25 sheep, what is one sheep worth?

52. If three dozen pairs of gloves which cost \$52 were sold for \$58.48, what was made on each pair?

53. When flour is worth \$11.76 per barrel of 196 pounds, how many pounds of flour must be given for 5 dozen eggs at 18 cents per dozen?

54. A coal dealer makes \$6 profit on a car load of 24 tons. What will he make on 1200 tons?

55. A man bought 290 acres of land for \$9860, and sold a part of it for \$10,000, at \$40 an acre. How many acres did he have left, and how much did he gain on every acre sold?

CHAPTER VI.

DECIMALS.

72. Numbers which denote **whole units** are called **Integral** numbers; but it is often necessary to express **parts** of a unit.

If a unit is divided into **two equal parts,** each part is called **one-half,** and is expressed by $\frac{1}{2}$. If a unit is divided into **three equal parts,** each part is called **one-third,** and is expressed by $\frac{1}{3}$; two of the parts are called **two-thirds,** and are expressed by $\frac{2}{3}$. Again, if a unit is divided into **four equal parts,** each part is called **one-fourth,** and is expressed by $\frac{1}{4}$; into **five equal parts,** each part is called **one-fifth,** and is expressed by $\frac{1}{5}$; into **six equal parts,** each part is called **one-sixth,** and is expressed by $\frac{1}{6}$; into **seven equal parts,** each part is called **one-seventh,** and is expressed by $\frac{1}{7}$; into **eight equal parts,** each part is called **one-eighth,** and is expressed by $\frac{1}{8}$; into **nine equal parts,** each part is called **one-ninth,** and is expressed by $\frac{1}{9}$; into **ten equal parts,** each part is called **one-tenth,** and is expressed by $\frac{1}{10}$.

If AB (see page opposite) represent a unit of length, each division of the line next below AB represents one-half of a unit; and each division of the second line below AB represents one-third of a unit; and so on.

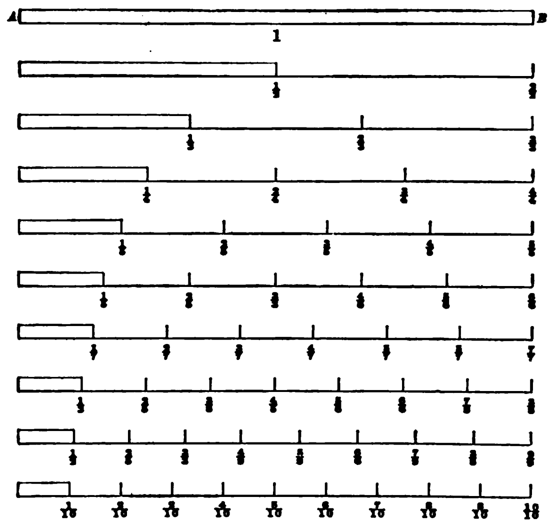
How many halves of a unit make a whole unit?

How many fourths make a half? how many make a whole unit?

How many sixths make a third? how many make a half? how many make a whole unit?

How many eighths make a half? a fourth? a whole unit?

How many tenths make a fifth? a half? a whole unit?

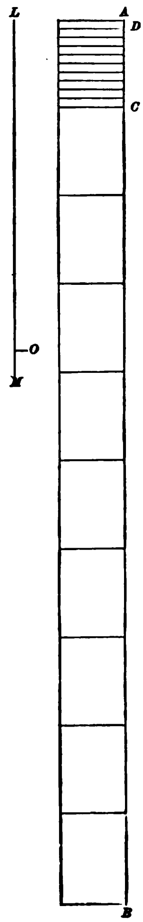


73. When a unit is divided into **ten equal parts,** and we wish to express in figures one or more of these parts, we do not usually write them $\frac{1}{10}$, $\frac{2}{10}$, etc., but we write 1, 2, 3, etc., and separate the number which denotes *parts* of a unit from the number which denotes *whole* units by a decimal point. Thus, two units and three-tenths of a unit are written, 2.3.

If each *tenth* of a unit is divided into ten equal parts, that is, the entire unit into a **hundred equal parts,** each part is called a **hundredth** of the unit; and if each hundredth is divided into ten equal parts, that is, the entire unit into a **thousand equal parts,** each part is called a **thousandth** of the unit; and so on.

These tenth-parts are called **Decimal** parts, from the Latin word *decem*, which means *ten;* and these parts are commonly called **Decimal Fractions.**

Let AB, for example, represent the unit of length by which a certain distance is to be measured. Suppose the given distance to contain AB 137 times, and a remainder LM to be left, which is less than AB. Take AC, a tenth of AB, and suppose AC is contained in LM 4 times, with a remainder OM less than AC. Again, suppose AD, a tenth of AC (that is, a hundredth of AB), to be contained in OM 3 times, with a remainder less than AD. And again, suppose a tenth of AD (that is, a thousandth of AB), to be contained in this last remainder 9 times. Then the whole distance expressed in lengths of AB will be 137.439.



The series of figures 137.439 means 1 hundred + 3 tens + 7 units + 4 tenths + 3 hundredths + 9 thousandths; as 1 hundred = 10 tens = 100 units, and 3 tens = 30 units, the integral value is 137 units; so, 4 tenths = 40 hundredths = 400 thousandths, and 3 hundredths = 30 thousandths; the decimal value therefore is 439 thousandths.

If the unit is the yard-stick, the whole is read "one hundred thirty-seven and four hundred thirty-nine thousandths yards"; if the unit is the meter-stick, the whole is read "137 and 439 thousandths meters."

NOTE. The pupil will get the clearest notions of decimals by taking a meter-stick (which is divided in tenths, hundredths, and thousandths) and measuring given lengths; such as, the length of the side of the room, of the platform, of the window-sill, etc., etc., and writing down the result in each case. Whenever the length measured is less than a meter, he should write down 0, and after it the decimal point, then the actual measure. Thus, if the length is found to be 8 tenths 2 hundredths and 7 thousandths, it is expressed by 0.827, and read "eight hundred twenty-seven thousandths of a meter."

74. It will be seen that 1 tenth = 10 hundredths, 1 hundredth = 10 thousandths; and, conversely, 10 thousandths = 1 hundredth, 10 hundredths = 1 tenth, 10 tenths = 1 unit; so that in *decimal* numbers, as in *integral* numbers, 10 in any place is equal to 1 in the next place to the left, and 1 in any place is equal to 10 in the next place to the right.

Hence figures in the *first* decimal place denote *tenths*, in the *second* place *hundredths*, in the *third* place *thousandths*, in the *fourth* place *ten-thousandths*, in the *fifth* place *hundred-thousandths*, in the *sixth* place *millionths*, and so on.

75. In reading decimals, read precisely as if the decimal were an integral number, and add the name of the lowest decimal place. It is best to pronounce the word "and" at the decimal point, and omit it in all other places. Thus, 100.023 is read one hundred *and* twenty-three thousandths. Ambiguity in reading, from having zeros at the end of a decimal, is avoided by a pause; thus, 0.300 is read three hundred . . . thousandths, while 0.00003 is read three . . . hundred-thousandths.

76. Read the following numbers:

0.3; 0.7; 0.65; 0.99; 37.5; 26.9; 425.312; 617.624; 94.57; 83.28; 0.9; 0.96; 57.09; 3.207; 2.03; 3.045; 40.7; 0.055; 0.074; 0.0215; 7.3945; 0.14875; 0.00005; 2.000375; 100.015625; 3.7525; 2.1136257.

77. Express in the decimal notation:

Seven tenths; nine tenths; eleven hundredths; eight hundredths; one hundred thirty-four thousandths; twenty-five thousandths; two hundred and thirty-four thousandths; nineteen *and* forty-one hundred-thousandths; twenty-five *and* sixteen ten-thousandths;

thirteen *and* two hundred one hundred-thousandths; six hundred fifty-eight thousand three hundred forty-two millionths; eighty-six *and* eight hundred three thousand three hundred four millionths; three *and* twenty-nine hundredths; fifteen *and* six hundred seventy-one thousandths; fifty-three ten-thousandths; twenty-two *and* sixty-seven hundredths; fourteen *and* two thousand three hundred fifty-one ten-thousandths; two *and* two hundred nineteen thousandths; three *and* one hundred fifty-seven thousandths.

78. Zeros occurring at the end of a decimal do not affect its value. Thus, 3.50700 means 3 units + 5 tenths + 0 hundredths + 7 thousandths + 0 ten-thousandths + 0 hundred-thousandths, and is, therefore, 3 and 507 thousandths, the same as 3.507.

79. The arrangement and method of working employed in decimals is precisely like that employed in integral numbers, the decimal point being the only new consideration.

Addition of Decimals.

Add 17.5163, 236.3, 1.7162, 0.00132.

Operation.

$$\begin{array}{r@{.}l} 17&5163 \\ 236&3 \\ 1&7162 \\ 0&00132 \\ \hline 255&53382 \end{array}$$

Write the numbers in columns, units under units, tens under tens, tenths under tenths, and so on, so that the decimal points will fall in a vertical line, and add as in integral numbers.

Ex. 53.

Find the value of:

1. 2.514 + 3.7 + 9.6304 + 0.24876.
2. 1.916 + 6.3 + 0.4782 + 9.35634.
3. 0.415 + 8.0 + 6.3746 + 8.29426.
4. 7.516 + 9.6 + 1.9238 + 7.21442.
5. 7.03 + 7.2456 + 0.483 + 9.23579 + 8.3.
6. 2.576 + 3.4203 + 1.5 + 6.27948 + 0.362357.
7. 3.29 + 15.671 + 0.0053 + 22.67.
8. 14.2351 + 651 + 2.219 + 3.157.
9. 213.7 + 2.913 + 14.769 + 0.007871.
10. 1.4178 + 0.2 + 2.356709 + 1.14 + 2.0.
11. 4.96 + 3.2728 + 0.7 + 3.54219 + 4.7.
12. 1.198 + 3.5 + 7.635487 + 4.23 + 1.5724.
13. 4.372 + 9.5 + 7.369248 + 1.72 + 3.2948.
14. 0.4293 + 0.7 + 6.954326 + 3.14 + 7.005.
15. 3.87 + 2.6493 + 0.8 + 2.63495 + 9.3.
16. 6.9 + 5.71 + 0.0431 + 329.2 + 4.4.
17. 3.571 + 0.008 + 12.51 + 649 + 3.051.
18. 15.753 + 2.069 + 17.6143 + 3.2107.
19. 1.1 + 20.02 + 13 + 2.845 + 1.0001.
20. 31.826 + 3.471 + 0.004 + 45 + 0.6.
21. 82.537 + 2000 + 1.354 + 0.006 + 13.
22. 64.27 + 1.1 + 23 + 17.12 + 8.8.
23. 72.5 + 140 + 340.03 + 21.5715 + 4.00087.
24. 0.96 + 7.3004 + 8010 + 0.00093 + 124650.

Subtraction of Decimals.

80. Subtract 37.286 from 41.1325; and 1.00523 from 9.3.

Operation.	Operation.
41.1325	9.30000
37.286	1.00523
3.8465	8.29477

Write the subtrahend under the minuend, so that the decimal points may fall in a vertical line. If the number of decimal places in the subtrahend exceed the number in the minuend, zeros may be annexed to the minuend, as such zeros have no effect on its value.

Ex. 54.

1. 0.58 – 0.39.
2. 0.67 – 0.59.
3. 3.927 – 1.836.
4. 4.825 – 1.763.
5. 4.325 – 1.672.
6. 6.283 – 3.576.
7. 9.025 – 6.387.
8. 6.275 – 3.829.
9. 7.57 – 6.385.
10. 9.26 – 2.375.
11. 8.4 – 3.228.
12. 9.5 – 2.732.
13. 14.3846 – 4.8003.
14. 3.4370 – 0.3045.
15. 0.3290 – 0.0089.
16. 136.0200 – 1.5423.
17. 1.9990 – 0.063.
18. 13.5298 – 10.0060.
19. 2.1808 – 0.0009.
20. 1.9870 – 1.0873.
21. 48.9370 – 30.3000.
22. 0.9990 – 0.9009.
23. 15.1409 – 3.8579.
24. 5.9009 – 0.0909.
25. 1.3993 – 0.9090.
26. 10.1010 – 0.0999.
27. 3.5 – 0.075.
28. 517 – 0.0076.
29. 1.325 – 0.4736.
30. 192.3 – 17.294.
31. 175.8 – 1.0024.
32. 186.257 – 13.794.
33. 0.715 – 0.70451.
34. 1111.116 – 22.22222.
35. 71.0047 – 9.0008167.
36. 9161.0098 – 7149.16716.

Multiplication of Decimals.

81. A change in position of the decimal point of a number will affect the local value of each figure of that number. Thus, if in place of 79.213 we write 792.13, we increase the value of each figure ten-fold, the 7 tens become 7 hundreds, the 9 units become 9 tens, the 2 tenths become 2 units, the 1 hundredth becomes 1 tenth, and the 3 thousandths become 3 hundredths, and, as the value of every figure is increased ten-fold, the entire number is increased ten-fold. If the decimal point is moved one place to the left, the local value of each figure is diminished ten-fold, and consequently the value of the entire number is diminished ten-fold. Hence,

To multiply a decimal by 10, 100, 1000, etc., we have only to move the decimal point in the multiplicand as many places to the *right*, annexing zeros if necessary, as there are zeros in the multiplier.

To divide a decimal by 10, 100, 1000, etc., we have only to move the decimal point in the dividend as many places to the *left*, prefixing zeros if necessary, as there are zeros in the divisor.

Thus, $100 \times 36.123 = 3612.3$, and $1000 \times 36.1 = 36100$; $36.123 \div 10 = 3.6123$, and $36.123 \div 1000 = 0.036123$.

82. To multiply a number by 0.1, 0.01, 0.001, etc., we have, by the definition of multiplication, to divide the multiplicand by 10, 100, 1000, etc.; that is, to remove the decimal point one place, two places, etc., to the left.

To divide by 0.1, 0.01, 0.001, etc., we have only to move the decimal point in the dividend one place, two places, etc., to the right.

Thus, $0.1 \times 86.32 = 8.632$, and $0.01 \times 1.236 = 0.01236$; $86.32 \div 0.1 = 863.2$, and $1.236 \div 0.01 = 123.6$.

Multiply 123.826 by 3.

OPERATION.

$$\begin{array}{r} 123.826 \\ 3 \\ \hline 371.478 \end{array}$$

Here 3×6 thousandths = 18 thousandths, or 1 hundredth and 8 thousandths; the 8 therefore is written in the thousandths' column; then, 3×2 hundredths = 6 hundredths, which, with the 1 hundredth, make 7 hundredths, and the 7 is written in the hundredths' column; then, 3×8 tenths = 24 tenths, or 2 units and 4 tenths, and the 4 is written in the tenths' column; then, 3×3 units = 9 units, which, with the 2 units, make 11 units, and so on.

Multiply 123.826×0.3.

The multiplier $0.3 = 3 \times 0.1$. We may therefore multiply first by 3, and the resulting product by 0.1. But multiplying by 0.1 simply moves the decimal point in the product one place to the left. Hence, the product will have three decimal places for the decimal in the multiplicand, and one more place for the decimal in the multiplier; thus,

OPERATION.

$$\begin{array}{r} 123.826 \\ 0.3 \\ \hline 37.1478 \end{array}$$

Multiply 123.826 by 0.32.

OPERATION.

$$\begin{array}{r} 123.826 \\ 0.32 \\ \hline 247652 \\ 371478 \\ \hline 39.62432 \end{array}$$

The multiplier $0.32 = 32 \times 0.01$; and we therefore multiply first by 32, and the resulting product by 0.01. But multiplying by 0.01 simply moves the decimal point in the product two places to the left. Hence, the product has three decimal places for the decimal in the multiplicand, and two more places for the decimal in the multiplier.

In the multiplication of decimals, therefore, point off in the product as many decimal places as there are in the multiplicand and multiplier taken together.

83. By putting the units' figure of the multiplier under the right-hand figure of the multiplicand, and writing the right-hand figure of each partial product directly under the multiplying digit that produces it, the decimal points will all fall in a vertical column, and the necessity of counting the number of decimal places in the multiplicand and multiplier to determine the number in the product will be avoided; thus,

Multiply 1.394 by 52.76; and 0.0363 by 0.0502.

```
OPERATION.        OPERATION.

  1.394           0.0363
    5276             00502
  --------        ---------
    8364               726
   9758             1815
  2 788           ---------
 69 70            0.00182226
 --------
 73.54744
```

Multiply 51.068 by 112.3; and 1.0401 by 21.21.

```
OPERATION.        OPERATION.

  51.068           1.0401
     1123              2121
  --------         --------
  15 3204            10401
 102 136            20802
 510 68            1 0401
5106 8            20 802
---------         ---------
5734.9364         22.060521
```

Ex. 55.

Find the products of:

1. 5×0.3.
2. 8×0.27.
3. 12×0.375.
4. 15×0.256.
5. 9×0.7.
6. 6×0.75.
7. 16×0.284.
8. 11×0.386.
9. 10×0.65.
10. 100×0.721.
11. 1000×3.736.
12. 1000×0.074.
13. 10×0.99.
14. 100×0.615.
15. 1000×2.409.
16. 1000×0.055.
17. 0.5×37.
18. 0.9×99.
19. 0.25×428.
20. 0.36×7384.
21. 0.9×26.
22. 0.7×67.
23. 0.48×237.
24. 0.18×3692.
25. 0.312×425.
26. 0.716×388.
27. 0.725×96.
28. 0.085×88.
29. 0.624×617.
30. 0.358×776.
31. 0.145×48.
32. 0.017×44.
33. 57×9.4.
34. 26×3.8.
35. 3×972.3.
36. 65×87.2.
37. 2.8×83.
38. 3.2×64.
39. 7.8×369.
40. 3.7×815.
41. 1.44×9.6.
42. 2.88×4.8.
43. 3.21×72.5.
44. 2.16×40.7.
45. 3.26×4.37.
46. 2.03×3.207.
47. 2.472×9.525.
48. 3.264×3.045.
49. 0.7×0.5.
50. 0.9×0.57.
51. 0.45×0.57.
52. 0.72×0.324.
53. 0.6×0.9.
54. 0.8×0.96.
55. 0.72×0.72.
56. 0.36×0.648.
57. 416×0.416.
58. 57×0.015.
59. 693×0.83.
60. 4.625×7.14.
61. 99.9×4.09.
62. 753×0.672.
63. 928×8.302.
64. 56.704×0.413.
65. 2.052×0.0037.
66. 0.00948×29.
67. 372×0.468.
68. 9.43×0.054.
69. 786×3.62.
70. 0.632×85.
71. 2.406×0.008.
72. 6824×3.7.
73. 42.53×0.685.
74. 0.832×59.
75. 763.24×4.078.

Division of Decimals.

84. In Division, if the dividend and divisor are both multiplied or both divided by the same number, the quotient is not changed. Thus, $18 \div 6 = 3$, and (when both dividend and divisor are multiplied by 2) $36 \div 12 = 3$. Again (when both dividend and divisor are divided by 2), $9 \div 3 = 3$.

If, therefore, the divisor contains decimal places, we may remove the decimal point from the divisor, provided we carry the decimal point in the dividend as many places to the right as there are decimal places in the divisor.

Divide 78.528 by 0.8.

Here the decimal point is removed from the divisor, and the decimal point in the dividend is carried one place to the right; that is, both dividend and divisor are multiplied by 10.

Operation.

$$\begin{array}{r} 8\,\underline{)\,785.28} \\ 98.16 \end{array}$$

When the divisor is a whole number, each quotient figure is of the **same order of units** as the right-hand figure of the partial dividend used in obtaining it. Hence, *the decimal point is put in the quotient as soon as the decimal point in the dividend is reached.*

Divide 28.3696 by 1.49.

Operation.

$$\begin{array}{r} 19.04 \\ 149\overline{)2836.96} \\ \underline{149} \\ 1346 \\ \underline{1341} \\ 596 \\ \underline{596} \end{array}$$

Here the decimal point is removed from the divisor, and is moved two places to the right in the dividend; in other words, both dividend and divisor are **multiplied by 100.**

If the divisor is not contained in the dividend without a remainder, ciphers may be mentally annexed to the dividend, and the division continued.

Divide 0.39842 by 3.7164 to four decimal places.

OPERATION.

```
            0.1072
37164)3984.2
      37164
       267800
       260148
         76520
         74328
          2192
```

If the divisor is a whole number, and ends in zeros, we may cut off the zeros from the divisor, and move the decimal point in the dividend as many places *to the left* as there are zeros cut off.

Divide 42.08 by 8000.

OPERATION.

```
8)0.04208
  0.00526
```

Here the three zeros are cut off from the divisor, and the decimal point in the dividend is moved three places to the left. In other words, both divisor and dividend are divided by 1000.

Ex. 56.

Find the quotients of:

1. $34.24 \div 4.28$.
2. $24.56 \div 6.14$.
3. $52.90 \div 5.75$.
4. $37.576 \div 6.832$.
5. $281.232 \div 7.812$.
6. $97.524 \div 5.16$.
7. $738.0930 \div 0.023$.
8. $5.18466 \div 1.02$.
9. $0.018 \div 9.6$.
10. $34.96818 \div 0.381$.

11. $0.003125 \div 25$.

12. $859.95 \div 136.5$.

13. $5.468 \div 0.08$.

14. $0.04922 \div 0.0023$.

15. $0.00044408 \div 0.0112$.

16. $0.20412 \div 0.0084$.

17. $0.07504 \div 23.45$.

18. $0.00025 \div 2.5$.

19. $0.03217 \div 1250$.

20. $171.99 \div 27.3$.

21. $0.012305 \div 1.07$.

22. $15.625 \div 2.5$.

23. $5.418 \div 2.58$.

24. $0.59064 \div 0.0276$.

25. $0.73807 \div 0.023$.

26. $15.4546 \div 0.019$.

27. $6.7288 \div 64.7$.

28. $72.36 \div 144$.

29. $0.01124 \div 11.24$.

30. $15.625 \div 5$.

31. $8.192 \div 0.00128$.

32. $0.00512 \div 2.048$.

33. $0.00972 \div 0.0004$.

34. $0.07504 \div 23.45$.

35. $15.21 \div 11.7$.

36. $6 \div 0.008$.

37. $4.8 \div 0.00016$.

38. $1562.5 \div 0.00025$.

39. $64 \div 0.016$.

40. $5.76 \div 0.0048$.

41. $3.012 \div 0.0006$.

42. $91844152.5 \div 1.1575$.

43. $7 \div 0.0035$.

44. $0.39237 \div 0.319$.

45. $0.3230864 \div 0.5072$.

46. $3.1 \div 0.0025$.

47. $63.8406 \div 0.18345$.

48. $181.3 \div 0.00037$.

49. $12.5 \div 2.56$.

50. $284.7432 \div 0.00004$.

51. $130.4 \div 0.0004$.

52. $113.4 \div 0.0108$.

53. $68.97516 \div 0.9246$.

54. $0.022185 \div 0.0306$.

55. $0.276766 \div 0.371$.

56. $286 \div 0.013$.

57. $0.10724 \div 0.003125$.

58. $0.03 \div 0.001$.

59. $105 \div 43.75$.

60. $8.468 \div 0.0292$.

Ex. 57.

1. To enclose a certain farm 225 rods of fence are needed. What will be the cost of the fence at the rate of $0.125 per foot, 16.5 ft. being a rod?

2. A section of land costs $49,878; what must be paid for 0.375 of a section?

3. When 0.7 of a ton of coal is worth $6.30, what will be the cost of 12.5 tons?

4. Coal being worth $7.00 per ton, what part of a ton can be bought for $2.59?

5. If a man can build 0.45 of a rod of wall in one hour, how many rods will 4 men build in 3.8 days, working 7.5 hours per day?

6. Twelve dozen penknives cost $90. If they are sold at $0.75 each, what will be the gain on each?

7. Divide $125.15 by $25.03.

8. Twelve yards of velvet cost $150. At that rate, what must be paid for 18 yards?

9. What will be the cost of 9.75 cords of white oak wood at the rate of $10 a cord?

10. Twenty-five hundredths of a farm cost $5000; what will nine-tenths of it cost?

11. A merchant bought 575 pounds of sugar for $51.75; he sold four-tenths of it at $0.11 per pound, and the remainder at $0.125. What was his gain?

12. A railroad train has 201 miles to run. If it averages 26.8 miles per hour, how many hours will be required?

13. An errand boy receives $2.75 per week. In how many weeks will he earn a pair of boots worth $3.25, a coat worth $4.75, a hat worth $1.50, and three handkerchiefs worth $0.50 each?

14. How many cords of pine wood at $3.375 a cord must be given for 12 yards of broadcloth at $2.25 a yard?

15. A and B start from the same place and travel in opposite directions. If A travels 6.25, B 7.75 miles per hour, how many miles apart will they be in one hour? When they are 70 miles apart, in how many hours will they meet if they turn and travel toward each other?

16. A farmer's wife bought 19 yards of table linen at $0.875, 15 yards of muslin at $0.47, and paid in butter at $0.25 per pound. How many pounds did she sell?

17. The milk from a herd of 75 Jersey cows, sold at 6 cents a quart, amounted in one summer to $2025. How many quarts were sold, and what was the average quantity from each cow?

18. A merchant sold three pieces of India matting, each containing 45.5 yards, at $0.375 per yard. How many bushels of corn at $0.75 a bushel should he receive in payment?

19. Henry Hodge earns $12 per week. He pays $4.25 for board, $0.625 for car-fare, $0.375 for library fees, and $4.875 for other expenses. In how many weeks will he save $97.50?

20. A grocer bought one hundred and fifty-six boxes of oranges at $5.625 each; he sold them for $916.875, and invested his gain in tea at $1.125 per pound. How many pounds did he buy?

21. George Mason built a house for $5000. He employed for a certain number of days 3 carpenters at $2.75 per day, 2 painters at $2.25, 1 mason at $3.50, and spent for lumber $1500, for stone $250, for hair, lime, and sand $250, for paints, oil, and paper $250, for locks, hinges, nails, etc., $50.50, for plumbing $262. How many days were the workmen employed?

22. How much broadcloth valued at $3 a yard must be given for 19 bushels of wheat at $1.875 per bushel?

Ex. 58.

To find the cost of goods sold by the hundred.

Point off two places for decimals at the right of the number denoting the *quantity*, and multiply the *price* of a hundred by this number.

To find the cost of goods sold by the thousand.

Point off three places for decimals at the right of the number denoting the *quantity*, and multiply the *price* of a thousand by this number.

In measuring lumber, M is often used for thousand and C for hundred.

1. What will 9875 feet of boards cost at $9 per M?

2. At $3.25 per C, what must be paid for 3784 feet of Georgia pine?

3. For the roof of a building 8000 tiles are to be used. What will they cost at $9.875 per M?

4. Required the cost of 98,762 laths at $0.35 per C.

5. An architect estimates that 1,500,784 bricks will be needed for a schoolhouse. What will they cost at $7.75 a thousand?

6. What must be paid for 4879 paving stones at \$9.375 a hundred?

7. A lumber dealer sold for \$5.75 per M shingles for which he paid \$4.50 per M. What did he gain on 3500 shingles?

8. If the freight from New York to Boston is \$0.12 per hundred pounds, what must be paid on five boxes of goods weighing respectively 348.25, 227.25, 429.25, 396.125, 419.125 pounds?

9. If the freight from New Orleans to St. Louis by steamboat is \$0.25 a hundred weight, what will be paid on one bushel of corn, there being 56 pounds in a bushel?

10. A farmer contracted for boards for fencing at the rate of \$12.375 per M. His bill for lumber amounted to \$61.875. How many thousand feet did he buy?

To find the cost of goods by the ton.

Find the retail price of 7846 pounds of coal at \$8.75 a ton.

OPERATION.

$$\begin{array}{r}
\$8.75 \\
\underline{7.846} \\
5250 \\
3500 \\
7000 \\
\underline{6125} \\
2\overline{)68.65250} \\
\$34.32625
\end{array}$$

Point off from the right of the number denoting the *quantity* three decimal places, multiply the *price* of a ton by this number, and divide this result by 2. The reason for this operation follows from the fact that *two* thousand pounds make a ton.

11. A merchant bought 3 loads of hay weighing severally 3684.25, 2976.06, 4764.28 pounds; each load included the weight of the wagon, 825 pounds. What was the hay worth at $12.75 per ton?

12. What must be paid for 9785 pounds of plaster at $6.75 per ton?

13. If 25,000 pounds of plaster cost $131.25, what is that per ton?

14. A dealer in New York retails coal at $7.75 per ton. If a ton costs $3.25 at the mine and $0.75 for freight, what will he make on 8758 pounds of coal?

15. At $10.50 per ton, what is the cost of 25,000 pounds of plaster?

16. When freight on wheat from Chicago to New York is $0.12 per bushel of 60 pounds, what is that per hundred?

17. What is the freight from St. Paul to Albany on 489 bushels of wheat at $0.25 per hundred pounds?

18. How many pounds of plaster at $10.50 per ton can be bought for $131.25?

19. What will be paid for shipping 1500 tons of wheat from Cleveland to Buffalo, freight on the lakes being 5 cents a bushel?

20. What is the retail price of coal per ton when 17,520 pounds are sold for $74.46?

21. How much more will it cost to send 12,000 bushels of wheat from Buffalo to New York by the Erie Canal, at 5 cents a bushel, than to send the wheat by the New York Central Railway, at 7 cents a hundredweight?

22. When nine eggs weigh a pound, at what price should they be sold per dozen to be worth 18 cents per pound?

23. What must a carpenter pay for the following bill of lumber? 6500 shingles at \$4.75 per M; 15,964 feet of boards at \$39.25 per M; 4849 feet of planks at \$45.32 per M; 19,496 laths at \$0.35 per C.

24. A merchant can ship his coal from Scranton to New York by water for 10 cents per hundred weight, or pay freight on the railroad at the rate of \$2.00 per ton. Which will be the cheaper way?

25. A Boston grocer bought 150 pounds of butter in Syracuse at 35 cents per pound. How must he sell it to gain 10 cents per pound, and pay \$1.50 a hundred weight for freight and packing?

26. The proprietor of a Minnesota flour-mill had 2 pounds toll from every bushel of wheat ground. Selling the wheat taken for toll in one day at \$3.00 per hundred, he received \$366. How many bushels were ground in that one day?

27. Two merchants send from St. Louis to Cincinnati 12,660 pounds of wheat each; one by steamboat, paying 12 cents per bushel, the other by railroad, paying 18 cents per hundred. Which conveyance is the cheaper?

28. A Western farmer's corn crop appraised at \$1.08 per hundred pounds is worth \$831.60. If he sells it at \$0.75 per bushel, how much more than the appraised value will he receive? (A bushel weighs 56 pounds.)

Ex. 59.

1. Find the price of 30 Parian statuettes at $8.875 each.

2. In February, 1884, the number of days during which rain fell in New England was 22, and the amount which fell was 4.57 inches. Find the daily average for the 22 days.

3. How many acres are in a park containing 0.08 of 115.1875 acres?

4. If 31.75 rods of fence are made for $10.90, what is the cost of a rod?

5. On a certain day in February, 1884, the thermometer at the highest was 51.1°, and at the lowest 29.4°. Find the difference.

6. Of 100 parts of matter in beans, sugar and gum form 61.10, other vegetable matter forms 31.55, and moisture 5 parts. Of how many parts does the remainder, which is mineral matter, consist?

7. If 0.1571 of the weight of superphosphate is organic matter, find the weight in tons of organic matter in 80 tons of superphosphate.

8. In January, 1884, the barometer at the highest was 30.543 inches, and at the lowest 28.843 inches. Find the difference.

9. A cubic inch of pure water weighs 252.458 grains. Find the weight in grains of a cylindrical inch, which is 0.7854 of a cubic inch.

10. Divide $31.40 among 6 men and 11 youths, giving a youth 0.525 of a man's share. What is each man's share?

11. The polar and equatorial diameters of the earth are respectively 41,707,620 and 41,847,426 feet. Express each in miles to three places of decimals, there being 5280 feet in a mile.

12. A zinc bar which at 32° Fahrenheit measures 1 inch, at 212° measures 1.003 inch. Find the length of a bar of the same metal at 212°, which at 32° measures 2.25 inches.

13. The yard measure made by Bird in 1758 was 36.00023 inches long. How many times would this measure be contained in a mile, there being 63,360 inches in a mile?

14. In 1825 the Stirling jug was measured in Edinburgh, and found to contain 104.2034 cubic inches. Reduce this to the decimal of a liquid gallon (231 cubic inches).

15. The paving of a street has cost $16,473, of which $1173 were paid for the work. If the paving stones were bought at the rate of $9 per hundred, and it takes 36 of them for every square yard of surface, find the surface of the street.

16. A farmer sowed an acre of land with 44 quarts of wheat. The return was 70 sheaves. If 100 sheaves produce 705 quarts of wheat, what is the product from a quart of wheat sown?

17. A building is lighted by 12 gas-jets, each burning 6 cubic feet per hour. If the gas is burnt five hours a day for 6 months (from October to March inclusive), and 3 hours a day for the rest of the year, what is the annual expense of lighting the building, gas being worth $3 per thousand cubic feet?

18. A man bought 56 dozen eggs at $1.70 a hundred, and sold them at $0.25 a dozen. How much did he gain?

19. The salt water which is obtained from the bottom of a mine of rock salt contains 0.09 of its weight of pure salt. What weight of salt water is it necessary to evaporate in order to obtain 4734 pounds of salt?

20. The weight of ashes from the burning of oak wood is 0.03 of the weight of the wood, and the weight of carbonate of potash contained in the ashes is 0.065 of the weight of the ashes. Find the weight of carbonate of potash which may be obtained from burning 1170 pounds of wood.

21. The weight of sugar from the sugar beet is nearly 0.06 of the weight of the beet. An acre produces 30,000 pounds of beets, which are sold at the rate of $2 a thousand pounds. How many acres of land is it necessary to sow to furnish beets to a sugar factory which produces 150,000 pounds a year, and what will be the value of the crop obtained?

22. If a workman has taken every day for the last 12 years two glasses of beer at 5 cents a glass, how much could he have saved if he had not indulged this habit, reckoning 365 days each year?

23. A woman has three children. She pays for each $15 a year for having their clothes made, $1.50 a month for mending, and $0.35 a week for washing. How much could she save in a year if she knew how to wash, make clothes, and mend?

24. A farmer sells for $448.40 his crop of grass at the rate of $15.25 a ton. If his farm produces 2 tons to an acre, how many acres of grass has he?

Ex. 60.

1. Fifteen men and eight boys are [illegible]. Their weekly pay is $342. If a [illegible] man's pay, what are the da[illegible] [illegible] also of a boy?

85. When the multiplier is an integral number, the product is called a **multiple** of the multiplicand; and, in division, when the quotient is an integral number, the divisor is called a **measure** of the dividend. Thus, $8 \times 7 = 56$; the number 56 is a multiple of 7. Again, $56 \div 7 = 8$; the number 7 is a measure of 56.

86. A number which cannot be divided by any other number except unity without remainder is called a **prime number.**

Thus, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc., are prime numbers.

87. Other numbers are each the product of a **fixed set** of prime numbers, and are called **composite numbers.**

88. Numbers which can be divided by 2 without remainder are called **even numbers**; and all other numbers are called **odd numbers.** Even numbers end in 2, 4, 6, 8, or 0; odd numbers end in 1, 3, 5, 7, or 9.

89. By way of distinction, when a number is used without reference to any designated unit, it is called an **abstract number**; and, when used with reference to a specified unit, it is called a **concrete number.**

Thus, 5, 7, 8 are abstract numbers, and 5 horses, 7 chairs, 8 dollars are called, by way of distinction, concrete numbers.

18. A man bought 56 dozen eggs at $1.70 a hundred, and sold them at $0.25 a dozen. How much did he gain?

19. The salt water which is obtained from the bottom of a mine of rock salt contains 0.09 of its weight of pure salt. What weight of salt water is it necessary to evaporate in order to obtain 4734 pounds of salt?

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21. The weight of sugar from the sugar beet is nearly 0.06 of the weight of the beet. An acre produces 30,000 pounds of beets, which are sold at the rate of $2 a thousand pounds. How many acres of land is it necessary to sow to furnish beets to a sugar factory which produces 150,000 pounds a year, and what will be the value of the crop obtained?

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24. A farmer sells for $448.40 his crop of grass at the rate of $15.25 a ton. If his farm produces 2 tons to an acre, how many acres of grass has he?

Ex. 60.

1. Fifteen men and eight boys are employed in a factory. Their weekly pay is \$ 342. If a boy's pay is half a man's pay, what are the daily wages of a man, and also of a boy?

---

85. When the multiplier is an integral number, the product is called a **multiple** of the multiplicand; and, in division, when the quotient is an integral number, the divisor is called a **measure** of the dividend. Thus, $8 \times 7 = 56$; the number 56 is a multiple of 7. Again, $56 \div 7 = 8$; the number 7 is a measure of 56.

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Thus, 5, 7, 8 are abstract numbers, and 5 horses, 7 chairs, 8 dollars are called, by way of distinction, concrete numbers.

7. A quart contains 1600 beans of average size, and a field is planted with 22 rows of 800 hills each, with 6 beans in a hill. The increase is tenfold. What is the value of the crop at $3 a bushel? (There are 32 quarts in a bushel.)

8. For making 25 gallons of ordinary beer 60 pounds of

evaporate in order to obtain 4734 pounds of salt? If

20. The weight of ashes from the burning of oak wood is 0.03 of the weight of the wood, and the weight of carbonate of potash contained in the ashes is 0.065 of the weight of the ashes. Find the weight of carbonate of potash which may be obtained from burning 1170 pounds of wood.

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24. A farmer sells for $448.40 his crop of grass at the rate of $15.25 a ton. If his farm produces 2 tons to an acre, how many acres of grass has he?

CHAPTER VII.

MULTIPLES AND MEASURES.

85. When the multiplier is an integral number, the product is called a **multiple** of the multiplicand; and, in division, when the quotient is an integral number, the divisor is called a **measure** of the dividend. Thus, $8 \times 7 = 56$; the number 56 is a multiple of 7. Again, $56 \div 7 = 8$; the number 7 is a measure of 56.

86. A number which cannot be divided by any other number except unity without remainder is called a **prime number.**

Thus, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc., are prime numbers.

87. Other numbers are each the product of a **fixed set** of prime numbers, and are called **composite numbers.**

88. Numbers which can be divided by 2 without remainder are called **even numbers**; and all other numbers are called **odd numbers.** Even numbers end in 2, 4, 6, 8, or 0; odd numbers end in 1, 3, 5, 7, or 9.

89. By way of distinction, when a number is used without reference to any designated unit, it is called an **abstract number**; and, when used with reference to a specified unit, it is called a **concrete number.**

Thus, 5, 7, 8 are abstract numbers, and 5 horses, 7 chairs, 8 dollars are called, by way of distinction, concrete numbers.

90. To factor a composite number is to separate the number into its factors.

Find the prime factors of 144.

$$\begin{array}{r|r} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

That is, $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$.

91. To avoid the necessity of writing long rows of equal factors, a small figure called the **exponent** is written at the right of a number to show how many times the number is taken as a factor.

Thus, $2 \times 2 \times 2 \times 2 \times 3 \times 3$ is written $2^4 \times 3^2$.

The expression 2^4 is called the fourth **power** of 2, and 3^2 is called the second power of 3.

92. It is evident from § 90 that the method of separating a composite number into its prime factors is,

Divide the given number by any prime number that is contained in it without remainder; then the quotient by any prime number that is contained in it without remainder; and so on until the quotient is itself a prime number. The several divisors and the last quotient are the prime factors.

If no prime factor is found before the quotient becomes equal to or less than the divisor, the number is prime.

93. The following tests are useful for determining without actual division if a number contains certain factors:

1. A number is divisible by 2 if its *last*, or *right hand, digit is even.*

2. A number is divisible by 4 (2^2) if the number denoted by the *last two digits* is divisible by 4.

3. A number is divisible by 8 (2^3) if the number denoted by the *last three digits* is divisible by 8.

4. A number is divisible by 3 if the *sum of its digits* is divisible by 3.

5. A number is divisible by 9 (3^2) if the *sum of its digits* is divisible by 9.

6. A number is divisible by 5 if its *last digit* is either 5 or 0.

7. A number is divisible by 25 (5^2) if the number denoted by the *last two digits* is divisible by 25.

8. A number is divisible by 125 (5^3) if the number denoted by the *last three digits* is divisible by 125.

9. A number is divisible by 6 if its *last digit is even*, and the *sum of its digits* is divisible by 3.

10. A number is divisible by 11 if the *difference between the sum of the digits in the even places and the sum of the digits in the odd places* is either 0 or a multiple of 11.

Ex. 61.

Find the prime factors of:

1. 32; 48; 56; 60; 75; 63; 92; 44; 88; 72; 84; 85.
2. 51; 69; 68; 87; 54; 98; 74; 90; 86; 70; 42; 62.
3. 112; 140; 132; 216; 162; 176; 252; 240; 360; 384.
4. 484; 476; 512; 525; 560; 572; 632; 648; 696; 720.
5. 748; 775; 824; 876; 888; 948; 960; 925; 117; 119.

94. The number 1.56 may be put in the form of $156 \times .01$, and thus separated into $2^2 \times 3 \times 13 \times .01$.

Ex. 62.

Find the prime factors of:

1. 1.05; 12.5; 14.3; 1.65; 19.2; 2.42; 62.4; 27.5.
2. 34.3; 5.39; 62.1; 118.8; 1.331; 1.452; 1.584; 92.4.

Greatest Common Measure.

95. The measures of 12 are 1, 2, 3, 4, 6, 12, and the measures of 18 are 1, 2, 3, 6, 9, 18. These two numbers have the measures 1, 2, 3, 6 in common, and of these measures 6 is the greatest.

The measures that two or more numbers have in common are called their *common measures*, and the greatest of these is called their **Greatest Common Measure**, which, for the sake of brevity, is denoted by the letters G. C. M.

If two or more numbers have no common measure they are said to be *prime to each other*. Thus, 27 and 125 are prime to each other.

96. The prime factors of 12 are 2^2, 3.

The prime factors of 18 are 2, 3^2.

The prime factors common to 12 and 18 are 2, 3. The G. C. M. of 12 and 18, namely 6, is 2×3.

That is, the G. C. M. of two or more numbers is,

The product of the prime factors common to the numbers, each prime factor having the least exponent that it has in any one of the numbers.

Hence, to find the G. C. M. of two or more numbers,

Separate the numbers into their prime factors.

Select the lowest power of each factor that is common to the given numbers, and find the product of these powers.

Find the G. C. M. of 84, 105, 63.

$$\begin{array}{r|r} 2 & 84 \\ \hline 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array} \qquad \begin{array}{r|r} 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

$84 = 2^2 \times 3 \times 7$. $\quad 105 = 3 \times 5 \times 7$. $\quad 63 = 3^2 \times 7$.

Hence, G. C. M. $= 3 \times 7$ or 21.

97. Common factors of two or more numbers may be taken out of the numbers simultaneously, as follows:

$$\begin{array}{r|rrr} 3 & 84 & 105 & 63 \\ \hline 7 & 28 & 35 & 21 \\ \hline & 4 & 5 & 3 \end{array}$$

The number 3 is seen to be a factor of all the numbers, and 7 of the resulting quotients 28, 35, 21. The quotients 4, 5, and 3 have no common factor. Therefore, 3 and 7 are the only common factors, and the G.C.M. is 3×7, or 21.

Ex. 63.

Find the G.C.M. of:

1. 48, 128.
2. 36, 90.
3. 64, 256.
4. 24, 105.
5. 125, 600.
6. 56, 138.
7. 63, 108.
8. 40, 600.
9. 65, 91.
10. 39, 273.
11. 56, 126.
12. 232, 493.
13. 365, 511.
14. 148, 592.
15. 216, 360.
16. 279, 403.
17. 294, 378.
18. 210, 294.
19. 182, 196.
20. 225, 375.
21. 195, 299.
22. 288, 360.
23. 133, 152.
24. 23, 111.
25. 352, 384.
26. 123, 579.
27. 960, 1536.
28. 336, 884.
29. 352, 364.
30. 1344, 1536.
31. 21, 35, 56.
32. 42, 133, 56.
33. 32, 48, 128.
34. 27, 36, 108.
35. 96, 48, 60, 108.
36. 33, 297, 198.
37. 56, 63, 315.
38. 75, 225, 500.
39. 232, 290, 493.
40. 365, 511, 803.

98. When it is required to find the G.C.M. of two or more numbers that cannot readily be separated into factors, the method to be employed is as follows:

Find the G.C.M. of 63 and 217.

OPERATION.

$$
\begin{array}{l}
63)217(3 \\
\quad\;\; \underline{189} \\
\quad\quad 28)63(2 \\
\quad\quad\quad\;\; \underline{56} \\
\quad\quad\quad\quad 7)28(4 \\
\quad\quad\quad\quad\quad\; \underline{\underline{28}}
\end{array}
$$

Therefore, the G.C.M. is 7.

Hence, by this method,

Divide the greater number by the less, and then the divisor by the remainder ~~left,~~ and so on till there is no remainder. The last divisor will be the G. C. M. required.

To find the G.C.M. of several numbers, find the G.C.M. of two of the numbers, then of that result and a third number, and so on. The last G.C.M. is the one required.

Ex. 64.

Find the G.C.M. of:

1. 342, 665.
2. 841, 899.
3. 961, 1178.
4. 1243, 1469.
5. 1001, 1287.
6. 1131, 2639.
7. 9889, 986.
8. 1792, 1832.
9. 1847, 1792.
10. 1850, 1517.
11. 3927, 5049.
12. 1287, 1551.
13. 1537, 1802.
14. 3056, 3629.
15. 2108, 3813.
16. 4844, 5536.
17. 696, 1305.
18. 232, 3219.
19. 949, 1387.
20. 1081, 1311.
21. 4067, 2573.
22. 216, 105, 405.
23. 112, 192, 128.
24. 168, 132, 352.
25. 198, 495, 209, 660.
26. 146, 730, 365, 219.
27. 924, 378, 612, 246.

LEAST COMMON MULTIPLE.

99. The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, etc.

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, etc.

The multiples common to 3 and 5 are 15, 30, etc., and of these 15 is the least.

100. The multiples that two or more numbers have in common are called their *common multiples*, and the least of these is called their **Least Common Multiple,** which is denoted by the letters L. C. M.

Find the L. C. M. of 7, 8, 9, 21.

The L. C. M. of 7, 8, 9, 21, must contain the factor 7, or it would not be a multiple of 7. It must also contain 2^3 to be a multiple of 8, and 3^2 to be a multiple of 9. It must contain the factors 3 and 7 to be a multiple of 21. That is, the L. C. M. of 7, 8, 9, 21, is the product of the factors 7, 2^3, 3^2; therefore, it is $7 \times 8 \times 9 = 504$. Hence,

To find the L. C. M. of two or more numbers,

Separate each number into its prime factors.

Select from these the **highest power** *of each factor, and find the product of these powers.*

Find the L. C. M. of 16, 21, 24, 30, 32.

$$16 = 2^4,$$
$$21 = 3 \times 7,$$
$$24 = 2^3 \times 3,$$
$$30 = 2 \times 3 \times 5,$$
$$32 = 2^5.$$

Hence, the L. C. M. $= 2^5 \times 3 \times 5 \times 7 = 3360$.

The L.C.M. of 16, 21, 24, 30, 32, may be found as follows:

$$
\begin{array}{r|rrrrr}
2 & \cancel{16} & 21 & 24 & 30 & 32 \\ \hline
2 & & 21 & 12 & 15 & 16 \\ \hline
2 & & 21 & 6 & 15 & 8 \\ \hline
3 & & 21 & \cancel{3} & 15 & 4 \\ \hline
 & & 7 & & 5 & 4
\end{array}
$$

Hence, the L.C.M. $= 2^3 \times 3 \times 7 \times 5 \times 4 = 3360$.

Since 16 is a measure of 32, it is elided, for any multiple of 32 is also a multiple of 16. The even numbers are divided by 2; the quotients and the odd numbers are written below the horizontal line. This operation is repeated so long as 2 is a measure of more than one number. In the fourth line 3, a measure of 15, is elided. The division by 3 leaves in the fifth line the numbers 7, 5, 4, which are prime to each other.

Therefore, the factors contained in the numbers are 2, 2, 2, 3, and 7, 5, 4.

Hence, the L. C. M. $= 2 \times 2 \times 2 \times 3 \times 7 \times 5 \times 4 = 3360$.

When two or more numbers are *prime* to each other, their L.C.M. is their product. Thus, the L.C.M. of 3, 5, 7, is $3 \times 5 \times 7$.

Ex. 65.

Find the L.C.M. of:

1. 3, 9, 27, 54.

2. 6, 9, 24, 40.

3. 144, 12, 18, 96.

4. 3, 8, 12, 22.

5. 16, 30, 48, 15.

6. 12, 24, 63, 84.

7. 9, 27, 33, 54.

8. 12, 20, 36, 54.

9. 22, 44, 88, 108.

10. 15, 30, 45, 60.

11. 8, 16, 24, 32.

12. 13, 15, 26, 39.

13. 7, 17, 51, 119.

14. 8, 6, 28, 32.

15. 4, 21, 42, 63.

16. 3, 6, 18, 22.

17. 5, 15, 24, 30.

18. 7, 2, 3, 5.

19. 13, 5, 2, 26.

20. 5, 10, 20, 100.

21. 19, 38, 2, 76.

22. 3, 9, 27, 81.

23. 6, 18, 22, 99.

24. 18, 26, 117, 312.

25. 13, 26, 39, 65.

26. 9, 36, 3, 45.

27. 16, 24, 13, 7.

28. 5, 9, 14, 96, 128.

29. 32, 36, 49, 56, 42.

30. 20, 24, 25, 27, 45.

31. 28, 30, 32, 36, 42.

32. 35, 40, 42, 49, 28.

33. 14, 18, 21, 32, 28.

34. 24, 27, 32, 36, 56.

35. 21, 24, 27, 28, 35.

36. 28, 32, 56, 72, 96.

101. If the given numbers are large, and contain no prime factors that can readily be detected, the common factors may be obtained by the process of finding the G.C.M. under like circumstances.

Find the L.C.M. of 1189 and 2117.

```
1189)2117(1
     9118
     ----
      928)1189(1
           928
           ---
           261)928(3
               783
               ---
               145)261(1
                   145
                   ---
                   116)145(1
                       116
                       ---
                        29)116(4
                           116
                           ---
```

Hence, the G.C.M. $= 29$.

Therefore, $1189 = 29 \times 41$; $2117 = 29 \times 73$.

Therefore, the L.C.M. $= 29 \times 41 \times 73 = 1189 \times 73$.

From this process it will be seen that:

The L. C. M. of two numbers may be found by dividing one of the numbers by their G. C. M., and multiplying the quotient by the other number.

Ex. 66.

Find the L. C. M. of:

1. 510 and 595.
2. 217 and 643.
3. 506 and 308.
4. 296 and 407.
5. 645 and 275.
6. 468 and 923.
7. 187 and 255.
8. 1261 and 663.
9. 255 and 357.
10. 432 and 840.
11. 949 and 2920.
12. 1247 and 1769.

Ex. 67.

1. A farmer owns 132 acres of wood-land, and 99 acres of pasture; he wishes to divide them into equal lots of the largest possible size. How many lots will there be, and what number of acres in each one?

2. A merchant has 75 yards of one kind of silk, 225 of a second, and 200 of a third; if he cut them into dress patterns of equal size, what is the largest number of yards which each pattern can contain?

3. Simeon Jones has 260 bushels of rye, 384 of oats, and 416 of wheat. He sends his grain to market in bags of equal size. What is the greatest number of bushels which each bag can hold, provided there is no mixture of the different kinds of grain?

4. What width of carpet will fit three rooms, the first 15 feet wide, the second 21 feet, and the third 33 feet?

5. A milkman has four different measures, holding 2, 3, 5, and 6 quarts, respectively. What is the smallest vessel that can be emptied by each one of them?

6. Find the length of the greatest line that exactly measures the sides of an enclosure 216 yards long and 111 broad.

7. Find the contents of the smallest vessel that may be filled by using a 4-quart, a 5-quart, or a 6-quart measure.

8. Two apprentices carry 1147 and 961 ivory balls, respectively, from the workshop to the showroom. The balls are carried in baskets of equal size, which are filled and emptied several times. How many balls in a basketful?

9. Find the shortest distance that each of three lines 8 feet, 9 feet, and 12 feet long will exactly measure.

10. A rides at the rate of 10 miles an hour, B at the rate of 6 miles an hour, and C walks at the rate of 3 miles an hour. Find the shortest distance they may all go in an integral number of hours.

11. Two frigates, with different numbers of guns, fire the same number of *rounds;* the one fires 608, and the other 1102 shots. How many guns has each?

12. Two opposition coaches, which have run full during the season for the same number of days, have carried 4807 and 3971 passengers respectively. How many days has the season lasted, and how many more passengers does the larger coach contain than the smaller?

13. A gardener has four square lots: a side of the first is 348 feet, the second 408 feet, the third 528 feet, the fourth 684 feet. If he uses boards of the same length for fencing the four lots, how long must the boards be?

14. A man has 195 cigars which he wishes to put into packages of equal size. What is the smallest number greater than one must be put in each package?

15. Four boys start from the same place at the same time and in the same direction to walk around a field. The first can walk around it in 10 minutes, the second in 12 minutes, the third in 15 minutes, and the fourth in 20 minutes. How many times must they each walk round the field before they will all be together at the starting place?

Cancellation.

102. Cancellation is the operation of striking out equal factors from the dividend and the divisor.

Find the quotient of $\frac{12 \times 3}{3 \times 2}$.

Operation.

$$\frac{\overset{6}{\cancel{12}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}}} = \frac{6 \times 1}{1 \times 1} = 6.$$

The 3 in the divisor cancels the 3 in the dividend. Then the factor 2 of the divisor is cancelled, and 2 is cancelled from 12 in the dividend. The resulting dividend is 6×1, and the divisor 1×1; and therefore the quotient is 6.

The process of cancellation is used to shorten arithmetical work.

Ex. 68.

1. $\frac{9 \times 18 \times 24}{12 \times 3 \times 6}$.
2. $\frac{15 \times 36 \times 27}{9 \times 12 \times 5}$.
3. $\frac{12 \times 6 \times 25}{5 \times 12 \times 3}$.
4. $\frac{19 \times 27 \times 30}{9 \times 15 \times 3}$.
5. $\frac{45 \times 16 \times 9}{8 \times 15 \times 3}$.
6. $\frac{32 \times 49 \times 6}{3 \times 16 \times 7}$.
7. $\frac{1728}{12 \times 12 \times 12}$.
8. $\frac{25 \times 6 \times 28}{6 \times 4 \times 35}$.
9. $\frac{6 \times 54 \times 7 \times 24}{7 \times 8 \times 9 \times 12}$.
10. $\frac{64 \times 105 \times 12}{21 \times 12 \times 8 \times 8}$.
11. $\frac{39 \times 18 \times 9}{13 \times 27 \times 2}$.
12. $\frac{50 \times 9 \times 84}{12 \times 3 \times 75}$.
13. $\frac{144 \times 81}{27 \times 96}$.
14. $\frac{625}{75 \times 3}$.
15. $\frac{75 \times 3 \times 96}{12 \times 15 \times 9}$.
16. $\frac{87 \times 15 \times 9}{5 \times 18 \times 29}$.
17. $\frac{84 \times 91 \times 8}{32 \times 60 \times 13}$.
18. $\frac{169 \times 196}{26 \times 14 \times 56}$.
19. $\frac{60 \times 9 \times 90}{42 \times 27 \times 15}$.
20. $\frac{13 \times 19 \times 17 \times 20}{17 \times 260 \times 4 \times 361}$.

PRACTICAL APPLICATIONS.

A man carried to a store 49 bushels of potatoes, which he sold at 35 cents a bushel, and took his pay in sugar at 7 cents a pound. How many pounds of sugar did he receive?

SOLUTION. 1 bushel of potatoes is sold for 35 cents.
49 bushels are sold for 49×35 cents.
1 pound of sugar is bought for 7 cents.
∴ Number of pounds bought for 49×35 cents is $\frac{49 \times 35}{7}$.

$$\frac{\overset{7}{\cancel{49}} \times 35}{\cancel{7}} = 245.$$

Ans. 245 pounds.

Ex. 69.

1. How many yards of cloth, at $3 a yard, can be bought for 12 tons of hay, at $15 per ton.

2. How many pairs of boots, at $4 a pair, can be bought for 40 pounds of butter, at 40 cents per pound?

3. How many jars of lard of 36 pounds each, at 8 cents per pound, must be given for 16 pieces of cloth containing 24 yards each, at 48 cents a yard?

4. How many coats, at $4 each, can be bought for 32 yards of broadcloth, at $2.50 a yard?

5. A milkman having 30 cows which daily give 9 quarts each, sells the milk at 5 cents per quart. How many pieces of cloth containing 40 yards each, at 12 cents per yard, ought he to receive for the milk of 6 days?

6. A market gardener sold 16 lots of celery, 120 bunches in each, at 28 cents per bunch; how many 240-pound barrels of sugar, at 8 cents a pound, will the celery pay for?

7. John Peters sold 9 firkins of butter weighing 78 pounds each, at 25 cents per pound; how many pieces of matting having 45 yards in a piece, at 56 cents per yard, should he receive?

8. A workman has received for 15 days' work of 7 hours each, 21 dollars. How much would he receive for 19 days' work of 5 hours each?

9. Thirty workmen have made in 9 days 215 yards of wall. At the same rate, how much would 36 workmen make in 15 days?

10. A telegraph operator transmits 80 words, averaging 4 letters each, in the space of 5 minutes. At the same rate, how many minutes will be required to send a dispatch of 130 words, averaging 5 letters each?

CHAPTER VIII.

COMMON FRACTIONS.

103. What is the name of one of the parts when a unit is divided into:

1. Two equal parts?
2. Three equal parts?
3. Four equal parts?
4. Five equal parts?
5. Six equal parts?
6. Eight equal parts?
7. Ten equal parts?
8. Twelve equal parts?
9. Sixteen equal parts?
10. Twenty equal parts?

A unit contains how many:

1. Halves?
2. Thirds?
3. Fourths?
4. Fifths?
5. Sixths?
6. Eighths?
7. Tenths?
8. Twelfths?
9. Sevenths?
10. Ninths?
11. Elevenths?
12. Thirteenths?
13. Twentieths?
14. Twenty-fourths?
15. Thirtieths?
16. Thirty-seconds?

When a unit is divided into twelve equal parts, what is the name of:

1. One part?
2. Three parts?
3. Five parts?
4. Two parts?
5. Four parts?
6. Six parts?
7. Eight parts?
8. Nine parts?
9. Twelve parts?

Express in figures:

1. Three-sevenths.
2. Five-ninths.
3. Seven-eighths.
4. Five-twelfths.
5. Seven-sixteenths.
6. Five-eighteenths.
7. Four-elevenths.
8. Nine-twentieths.

Read: $\frac{7}{10}$, $\frac{6}{13}$, $\frac{5}{12}$, $\frac{4}{21}$, $\frac{11}{16}$, $\frac{73}{100}$, $\frac{23}{24}$, $\frac{11}{32}$, $\frac{13}{55}$, $\frac{7}{23}$, $\frac{9}{28}$.

104. The expression $\frac{7}{9}$ means:

I. Seven of the parts when a unit has been divided into nine equal parts.

II. One-ninth of seven units; for, if *seven* units be divided into nine equal parts, one of these parts will be *seven* times as great as one of the parts obtained by dividing one unit into nine equal parts.

III. The quotient of seven divided by nine.

105. In the fraction $\frac{7}{9}$, the lower figure shows the number of equal parts into which the whole has been divided, and is therefore a **divisor**; but, since it shows the number of parts into which the whole has been divided, it shows the **name** of each part, and is therefore called the **denominator.**

The upper figure shows the **number** of these parts taken, and is therefore called the **numerator.**

The figure, then, above the line denotes **number,** the figure below the line **name.**

106. The numerator and denominator are called the **terms** of a fraction.

107. A **proper fraction** is one of which the numerator is less than the denominator; as $\frac{7}{9}$.

108. An **improper fraction** is one of which the numerator equals or exceeds the denominator; as $\frac{9}{9}$, $\frac{17}{4}$.

When the numerator is greater than the denominator, more than one unit must be regarded as divided into equal parts; thus, $\frac{9}{4}$

means that three units have been divided each into four equal parts, and that all the parts of two units and one part of the third unit are taken.

109. A **mixed number** is an expression consisting of a whole number and a fraction; as $4\frac{3}{7}$, 5.35. These expressions are read four *and* three-sevenths, five *and* thirty-five hundredths.

Every mixed number means that some entire units are taken, and the fraction of another unit.

Select the proper fractions, the improper fractions, and mixed numbers from the following expressions:

$\frac{8}{9}$, $\frac{21}{2}$, $4\frac{1}{5}$, $\frac{3}{8}$, $9\frac{4}{7}$, $\frac{125}{89}$, $\frac{7}{3}$, $\frac{16}{17}$, $\frac{17}{7}$, $6\frac{3}{7}$, $\frac{15}{22}$, $\frac{487}{385}$, $\frac{4}{5}$, $\frac{9}{3}$, $5\frac{1}{4}$, $\frac{9}{10}$, $8\frac{4}{9}$, $19\frac{4}{12}$, $\frac{17}{9}$, $6\frac{3}{4}$, $\frac{25}{7}$, $18\frac{2}{3}$, $\frac{5}{4}$.

110. An improper fraction represents a quantity which can also be represented by a whole number or else by a mixed number. Thus, $\frac{19}{7} = 2\frac{5}{7}$.

For, if we suppose several units to be divided each into seven equal parts, and we take 19 of these parts, 14 (that is, 2×7) will make 2 units, and the five remaining parts will be five-sevenths of another unit.

111. To reduce an improper fraction to a whole or mixed number,

Divide the numerator by the denominator.

The quotient will be the whole number, and the remainder, if any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.

Ex. 70. (*Oral.*)

Reduce to whole or mixed numbers:

1. $\frac{15}{3}$.	7. $\frac{19}{7}$.	13. $\frac{9}{2}$.	19. $\frac{13}{4}$.
2. $\frac{17}{9}$.	8. $\frac{29}{6}$.	14. $\frac{25}{5}$.	20. $\frac{87}{10}$.
3. $\frac{18}{8}$.	9. $\frac{56}{9}$.	15. $\frac{14}{12}$.	21. $\frac{40}{3}$.
4. $\frac{26}{12}$.	10. $\frac{13}{4}$.	16. $\frac{28}{7}$.	22. $\frac{45}{11}$.
5. $\frac{38}{5}$.	11. $\frac{14}{8}$.	17. $\frac{30}{2}$.	23. $\frac{23}{6}$.
6. $\frac{48}{12}$.	12. $\frac{12}{9}$.	18. $\frac{35}{6}$.	24. $\frac{70}{11}$.

Ex. 71.

Reduce to whole or mixed numbers:

1. $\frac{97}{16}$.	9. $\frac{397}{78}$.	17. $\frac{575}{79}$.	25. $\frac{18506}{72}$.
2. $\frac{319}{24}$.	10. $\frac{563}{45}$.	18. $\frac{587}{38}$.	26. $\frac{39679}{83}$.
3. $\frac{415}{17}$.	11. $\frac{584}{92}$.	19. $\frac{1234}{91}$.	27. $\frac{14271}{250}$.
4. $\frac{683}{25}$.	12. $\frac{796}{97}$.	20. $\frac{3597}{429}$.	28. $\frac{3615}{411}$.
5. $\frac{497}{89}$.	13. $\frac{847}{109}$.	21. $\frac{12322}{111}$.	29. $\frac{1384}{87}$.
6. $\frac{498}{47}$.	14. $\frac{693}{420}$.	22. $\frac{5142}{117}$.	30. $\frac{3145}{43}$.
7. $\frac{576}{53}$.	15. $\frac{318}{95}$.	23. $\frac{360}{13}$.	31. $\frac{3782}{73}$.
8. $\frac{300}{87}$.	16. $\frac{417}{89}$.	24. $\frac{9172}{63}$.	32. $\frac{580}{37}$.

112. A whole number or a mixed number represents a quantity which can also be represented by an improper fraction. Thus, $\$3\frac{3}{4} = \$\frac{15}{4}$.

For each dollar contains 4 *fourths;* therefore 3 dollars contain 3×4 *fourths* or 12 *fourths;* which, together with the 3 *fourths*, make 15 *fourths*. Hence,

113. To reduce a mixed number to an improper fraction,

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; under this sum write the denominator.

114. A whole number may be expressed as a fraction with any given denominator. Thus, $9 = \frac{63}{7}$.

For, as 1 unit contains 7 *sevenths*, 9 units contain 9×7 *sevenths*, or 63 *sevenths*.

A whole number may be written in the form of a fraction with 1 for a denominator. Thus, $9 = \frac{9}{1}$.

Ex. 72. (*Oral.*)

Reduce to improper fractions:

1. $4\frac{1}{5}$.	7. $12\frac{1}{3}$.	13. $11\frac{3}{7}$.	19. $25\frac{1}{2}$.
2. $7\frac{2}{3}$.	8. $10\frac{5}{8}$.	14. $13\frac{1}{10}$.	20. $30\frac{2}{5}$.
3. $8\frac{1}{2}$.	9. $7\frac{4}{8}$.	15. $7\frac{3}{13}$.	21. $17\frac{3}{10}$.
4. $9\frac{3}{7}$.	10. $3\frac{9}{10}$.	16. $9\frac{6}{12}$.	22. $40\frac{1}{4}$.
5. $5\frac{4}{9}$.	11. $8\frac{4}{11}$.	17. $20\frac{2}{12}$.	23. $50\frac{7}{8}$.
6. $6\frac{3}{7}$.	12. $3\frac{8}{12}$.	18. $15\frac{1}{3}$.	24. $80\frac{2}{3}$.

25. Change 12 to thirds; 8 to fourths; 7 to fifths; 9 to halves; 12 to ninths; 13 to sixths; 11 to sevenths; 14 to eighths.

Ex. 73.

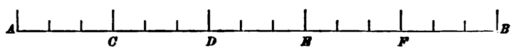
Change to improper fractions:

1. $15\frac{1}{23}$.	6. $45\frac{3}{12}$.	11. $5\frac{27}{160}$.	16. $155\frac{29}{37}$.
2. $36\frac{11}{12}$.	7. $56\frac{7}{15}$.	12. $46\frac{5}{126}$.	17. $17\frac{29}{84}$.
3. $5\frac{61}{79}$.	8. $77\frac{8}{17}$.	13. $2\frac{7}{80}$.	18. $167\frac{37}{56}$.
4. $3\frac{18}{55}$.	9. $183\frac{5}{21}$.	14. $9\frac{32}{79}$.	19. $29\frac{18}{19}$.
5. $12\frac{15}{61}$.	10. $72\frac{31}{48}$.	15. $10\frac{8}{17}$.	20. $29\frac{39}{68}$.

21. Change 25 to 94ths; 218 to 23ds; 375 to 87ths.

Reduction of Fractions to Lower Terms.

115. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction is not altered.



Thus, if the line AB be divided into 5 equal parts at the points C, D, E, and F, then AF is $\frac{4}{5}$ of AB.

Now, if each of the parts be sub-divided into 3 equal parts, AB will contain 15 of these sub-divisions, and AF 12 of these sub-divisions. Therefore AF is $\frac{12}{15}$ of AB.

Since AF is $\frac{4}{5}$ of AB and also $\frac{12}{15}$ of AB, it follows that $\frac{12}{15} = \frac{4}{5}$. But $\frac{4}{5}$ is obtained from $\frac{12}{15}$ by dividing both numerator and denominator by 3. Therefore,

To reduce a fraction to lower terms,

Divide the numerator and denominator by any common factor.

A fraction is expressed in its lowest terms when both the numerator and denominator are divided by the greatest common divisor.

Reduce $\frac{336}{784}$ to its lowest terms.

$$\frac{336}{784} = \frac{84}{196} = \frac{21}{49} = \frac{3}{7}.$$

The common factors cancelled are 4, 4, and 7.

Reduce $\frac{259}{333}$ to its lowest terms.

Since no common factor can readily be detected, we find the G.C.M.

```
259)333(1
    259
     74)259(3
        222
         37)74(2
            74
```

Divide 259 and 333 each by 37, their G.C.M. Then $\frac{259}{333} = \frac{7}{9}$.

Ex. 74. (*Oral.*)

Reduce to lowest terms by inspection :

1. $\frac{9}{12}$.	8. $\frac{11}{22}$.	15. $\frac{25}{30}$.	22. $\frac{14}{30}$.
2. $\frac{15}{20}$.	9. $\frac{10}{20}$.	16. $\frac{21}{28}$.	23. $\frac{25}{50}$.
3. $\frac{10}{12}$.	10. $\frac{15}{18}$.	17. $\frac{18}{24}$.	24. $\frac{30}{42}$.
4. $\frac{14}{16}$.	11. $\frac{7}{14}$.	18. $\frac{14}{21}$.	25. $\frac{27}{45}$.
5. $\frac{12}{15}$.	12. $\frac{9}{15}$.	19. $\frac{9}{21}$.	26. $\frac{24}{56}$.
6. $\frac{16}{24}$.	13. $\frac{18}{27}$.	20. $\frac{21}{27}$.	27. $\frac{32}{48}$.
7. $\frac{12}{18}$.	14. $\frac{16}{20}$.	21. $\frac{13}{26}$.	28. $\frac{36}{63}$.

Ex. 75.

Reduce to lowest terms by the method of inspection or by the method of finding the G.C.M.

1. $\frac{125}{500}$.	13. $\frac{78}{416}$.	25. $\frac{330}{660}$.	37. $\frac{492}{744}$.
2. $\frac{144}{192}$.	14. $\frac{308}{506}$.	26. $\frac{128}{256}$.	38. $\frac{55}{121}$.
3. $\frac{96}{264}$.	15. $\frac{135}{255}$.	27. $\frac{256}{1280}$.	39. $\frac{744}{906}$.
4. $\frac{120}{180}$.	16. $\frac{147}{189}$.	28. $\frac{252}{288}$.	40. $\frac{505}{707}$.
5. $\frac{36}{252}$.	17. $\frac{136}{168}$.	29. $\frac{365}{511}$.	41. $\frac{420}{864}$.
6. $\frac{154}{220}$.	18. $\frac{180}{234}$.	30. $\frac{775}{1800}$.	42. $\frac{882}{1512}$.
7. $\frac{48}{128}$.	19. $\frac{336}{576}$.	31. $\frac{144}{960}$.	43. $\frac{630}{882}$.
8. $\frac{130}{234}$.	20. $\frac{47}{83}$.	32. $\frac{198}{297}$.	44. $\frac{344}{2268}$.
9. $\frac{28}{154}$.	21. $\frac{63}{217}$.	33. $\frac{315}{745}$.	45. $\frac{168}{224}$.
10. $\frac{28}{343}$.	22. $\frac{196}{324}$.	34. $\frac{225}{450}$.	46. $\frac{216}{648}$.
11. $\frac{48}{224}$.	23. $\frac{384}{416}$.	35. $\frac{154}{220}$.	47. $\frac{1242}{2323}$.
12. $\frac{108}{252}$.	24. $\frac{209}{253}$.	36. $\frac{119}{121}$.	48. $\frac{2268}{3444}$.

NOTE. In the answers to *all* examples, fractions should be left in their lowest terms.

MULTIPLICATION OF FRACTIONS.

116. 7×3 *horses* $= 21$ *horses*.

7×3 *fifths* $= 21$ *fifths*.

If three like quantities are taken 7 times, the result will be 7 times 3 of the *same quantities*.

$\frac{3}{5} \times 15$ means $\frac{3}{5}$ of 15, which equals 9. Hence,

117. To find the product of a whole number and a fraction,

Find the product of the numerator and whole number, and divide the result by the denominator.

A factor common to the whole number and the denominator of the fraction may be cancelled. For, cancelling a factor common to the whole number and the denominator of the fraction *before* the multiplication, is evidently equivalent to dividing the numerator and denominator of the resulting fraction by that factor *after* the multiplication. Which may be done by § 115.

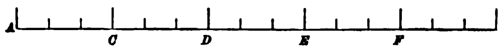
Ex. 76. (*Oral.*)

Find the products of:

1. $18 \times \frac{1}{3}$.	**10.** $14 \times \frac{1}{2}$.	**19.** $8 \times \frac{11}{20}$.	**28.** $18 \times \frac{3}{20}$.
2. $25 \times \frac{1}{5}$.	**11.** $20 \times \frac{3}{5}$.	**20.** $16 \times \frac{9}{20}$.	**29.** $25 \times \frac{3}{10}$.
3. $27 \times \frac{8}{9}$.	**12.** $16 \times \frac{1}{4}$.	**21.** $36 \times \frac{5}{12}$.	**30.** $\frac{4}{13} \times 26$.
4. $10 \times \frac{1}{5}$.	**13.** $7 \times \frac{11}{21}$.	**22.** $24 \times \frac{5}{8}$.	**31.** $\frac{1}{6} \times 9$.
5. $24 \times \frac{2}{3}$.	**14.** $16 \times \frac{3}{4}$.	**23.** $32 \times \frac{3}{16}$.	**32.** $\frac{7}{15} \times 12$.
6. $12 \times \frac{1}{4}$.	**15.** $\frac{17}{20} \times 10$.	**24.** $12 \times \frac{5}{16}$.	**33.** $22 \times \frac{5}{33}$.
7. $21 \times \frac{1}{7}$.	**16.** $\frac{11}{20} \times 12$.	**25.** $27 \times \frac{2}{9}$.	**34.** $\frac{5}{21} \times 28$.
8. $30 \times \frac{5}{6}$.	**17.** $27 \times \frac{7}{9}$.	**26.** $18 \times \frac{5}{9}$.	**35.** $\frac{3}{14} \times 21$.
9. $16 \times \frac{1}{8}$.	**18.** $\frac{13}{20} \times 15$.	**27.** $12 \times \frac{7}{9}$.	**36.** $\frac{4}{15} \times 35$.

118. To multiply a fraction by a fraction.

Multiply $\frac{4}{5}$ by $\frac{2}{3}$.



$\frac{4}{5}$ multiplied by $\frac{2}{3}$ means $\frac{2}{3}$ of $\frac{4}{5}$.

If the line AB be divided into 5 equal parts at the points C, D, E, and F, AF will be $\frac{4}{5}$ of AB. Now, if each part be sub-divided into three equal parts, there will evidently be 15 such parts in the whole line, and each part will be $\frac{1}{15}$ of the line.

That is, $\frac{1}{3}$ of $\frac{1}{5}$ is $\frac{1}{15}$ of the whole.

$\frac{1}{3}$ of $\frac{4}{5}$ will be $\frac{1}{15}+\frac{1}{15}+\frac{1}{15}+\frac{1}{15}$, or $\frac{4}{15}$ of the whole. And $\frac{2}{3}$ of $\frac{4}{5}$ will be twice $\frac{4}{15}$; that is, $\frac{8}{15}$ of the whole. Therefore,

To multiply a fraction by a fraction,

Find the product of the numerators for the required numerator, and of the denominators for the required denominator.

Mixed numbers must first be reduced to improper fractions.

Any factor common to a numerator and denominator should be cancelled before the multiplication.

(1) $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

(2) $\frac{13}{30} \times 2\frac{8}{13} \times \frac{10}{17}$.

Reducing the mixed number to an improper fraction, we have,

$$\frac{13}{30} \times \frac{34}{13} \times \frac{10}{17}.$$

By cancellation,

$$\frac{\cancel{13}}{\underset{3}{\cancel{30}}} \times \frac{\overset{2}{\cancel{34}}}{\cancel{13}} \times \frac{\overset{1}{\cancel{10}}}{\cancel{17}} = \frac{2}{3}.$$

Ex. 77. (*Oral.*)

Find the products of:

1. $\frac{1}{3} \times \frac{3}{5}$.
2. $\frac{1}{2} \times \frac{8}{9}$.
3. $\frac{1}{3} \times \frac{9}{8}$.
4. $\frac{1}{2} \times \frac{3}{8}$.
5. $\frac{1}{3} \times \frac{3}{7}$.
6. $\frac{5}{12} \times \frac{7}{9}$.
7. $\frac{6}{7} \times \frac{7}{4}$.
8. $\frac{4}{25} \times \frac{5}{6}$.
9. $\frac{7}{30} \times 10$.
10. $\frac{7}{30} \times \frac{10}{3}$.
11. $\frac{1}{5} \times \frac{10}{3}$.
12. $\frac{17}{20} \times \frac{5}{4}$.

Ex. 78.

Find the products of:

1. $\frac{1}{5} \times \frac{15}{19}$.
2. $\frac{2}{3} \times 4\frac{1}{2}$.
3. $2\frac{1}{5} \times \frac{3}{11} \times \frac{2}{7}$.
4. $4\frac{1}{8} \times 3\frac{7}{11} \times \frac{2}{5}$.
5. $\frac{5}{9}$ of $\frac{12}{25}$ of $1\frac{1}{5}$.
6. $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$.
7. $\frac{68}{95} \times \frac{2}{5} \times \frac{4}{5}$.
8. $3\frac{2}{3} \times 5\frac{1}{9} \times \frac{27}{46}$.
9. $\frac{12}{23} \times 9\frac{1}{5} \times \frac{25}{6}$.
10. $2\frac{1}{9} \times \frac{3}{7} \times \frac{11}{38}$.
11. $2\frac{2}{9}$ of $\frac{9}{2}$ of $\frac{3}{25}$.
12. $2\frac{1}{2}$ of $\frac{7}{4}$ of $\frac{13}{21}$.
13. $\frac{3}{4} \times 7\frac{1}{2} \times \frac{3}{25}$.
14. $\frac{4}{3} \times 7\frac{1}{2} \times \frac{3}{25}$.
15. $2\frac{1}{3}$ of $3\frac{1}{9}$ of $\frac{30}{7}$.
16. $\frac{7}{18}$ of $\frac{3}{14}$ of $7\frac{1}{5}$.
17. $3\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{10}{11}$.
18. $3\frac{1}{3}$ of $\frac{9}{40}$ of $\frac{9}{10}$.
19. $\frac{6}{19}$ of $2\frac{1}{9}$ of 21.
20. $\frac{7}{4}$ of $1\frac{1}{14}$ of $3\frac{1}{5}$.
21. $8\frac{2}{3} \times \frac{2}{3}$ of $\frac{7}{8}$.
22. $16 \times \frac{4}{7}$ of $\frac{8}{13}$.
23. $3 \times 7\frac{1}{2} \times \frac{11}{15} \times 3\frac{8}{11}$.
24. $\frac{3}{10}$ of $\frac{6}{7} \times \frac{8}{9} \times \frac{10}{11} \times 77$.
25. $\frac{16}{25} \times \frac{4}{3} \times \frac{75}{94} \times \frac{1}{4}$.
26. $\frac{3}{13} \times \frac{20}{81} \times \frac{13}{11} \times \frac{11}{12}$.
27. $3\frac{2}{3} \times 4\frac{7}{8} \times 15$.
28. $\frac{9}{10}$ of $7 \times \frac{11}{15}$ of $87\frac{3}{11}$.
29. $11\frac{3}{7} \times 16\frac{4}{11}$ of $\frac{7}{80}$ of $\frac{1}{90}$.
30. $\frac{1}{3}$ of $\frac{9}{8}$ of $2\frac{2}{3} \times 15\frac{2}{7}$.
31. $3\frac{1}{3}$ of $\frac{7}{15} \times 2\frac{1}{7}$ of $\frac{5}{9}$.
32. $3\frac{2}{3}$ of $\frac{51}{70}$ of $6\frac{2}{3}$ of $\frac{21}{25}$.
33. $\frac{1}{5} \times 2\frac{1}{9} \times 4\frac{1}{2} \times \frac{5}{19}$.
34. $\frac{2}{5} \times 2\frac{1}{7} \times 7\frac{1}{2} \times 4\frac{1}{5} \times \frac{8}{9}$.

35. $2\frac{1}{2} \times \frac{7}{9} \times \frac{4}{5} \times 3\frac{3}{8} \times 5\frac{1}{3}$.

36. $\frac{1}{7} \times \frac{2}{9} \times 7\frac{1}{5} \times 5\frac{1}{4} \times 6\frac{1}{3}$.

37. $5\frac{1}{9} \times \frac{20}{23} \times 7\frac{1}{5} \times \frac{15}{29} \times 7\frac{1}{4}$.

38. $\frac{3}{13}$ of $\frac{60}{81}$ of $\frac{78}{55}$ of $\frac{11}{12}$.

39. $3\frac{2}{5}$ of $\frac{2}{13}$ of $\frac{1}{5}$ of 10.

40. $\frac{5}{18} \times 2\frac{1}{3} \times 3\frac{3}{5} \times 2\frac{1}{4} \times \frac{8}{45}$.

119. When the product of a mixed number and a whole number is required, it is generally best to find the product of the whole number and the fractional part of the mixed number, then the product of the whole number and the integral part of the mixed number, and combine the results. Thus,

The product of 9 times $7\frac{1}{8}$ is found as follows:

OPERATION.

$$\begin{array}{r} 7\frac{1}{8} \\ 9 \\ \hline 64\frac{1}{8} \end{array}$$

Here 9 times $\frac{1}{8}$ equals $1\frac{1}{8}$, the $\frac{1}{8}$ is written, and the 1 is carried to the product of 9×7, making 64.

Ex. 79.

Find the products of:

1. $3 \times 4\frac{5}{7}$.	**11.** $5\frac{5}{8} \times 9$.	**21.** $20 \times 5\frac{1}{5}$.
2. $5 \times 7\frac{2}{9}$.	**12.** $2\frac{3}{10} \times 15$.	**22.** $4\frac{3}{4} \times 17$.
3. $21 \times 18\frac{3}{7}$.	**13.** $2\frac{11}{12} \times 20$.	**23.** $5\frac{2}{3} \times 18$.
4. $22 \times 29\frac{3}{13}$.	**14.** $5\frac{3}{5} \times 12$.	**24.** $6\frac{7}{8} \times 15$.
5. $25 \times 12\frac{3}{4}$.	**15.** $7\frac{1}{3} \times 8$.	**25.** $9\frac{3}{4} \times 21$.
6. $6 \times 2\frac{2}{3}$.	**16.** $6\frac{3}{4} \times 9$.	**26.** $10\frac{1}{2} \times 41$.
7. $7 \times 2\frac{3}{4}$.	**17.** $9 \times 3\frac{3}{5}$.	**27.** $11\frac{2}{3} \times 32$.
8. $8 \times 2\frac{3}{5}$.	**18.** $12 \times 2\frac{7}{8}$.	**28.** $15\frac{4}{5} \times 36$.
9. $6\frac{2}{5} \times 9$.	**19.** $13 \times 3\frac{2}{3}$.	**29.** $16\frac{7}{8} \times 40$.
10. $3\frac{3}{10} \times 10$.	**20.** $16 \times 9\frac{1}{3}$.	**30.** $13\frac{5}{6} \times 27$.

Division of Fractions.

120. When a product of two numbers is equal to 1, each of these two numbers is called the **reciprocal** of the other.

Thus, $5 \times \frac{1}{5} = 1$. Hence, the reciprocal of $\frac{1}{5}$ is 5, and the reciprocal of 5 is $\frac{1}{5}$. Again, $\frac{7}{5} \times \frac{5}{7} = 1$. Therefore, the reciprocal of $\frac{7}{5}$ is $\frac{5}{7}$, and the reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.

121. To multiply by the reciprocal of a number is the same as to divide by the number.

Thus, to multiply by $\frac{1}{3}$, means to separate the multiplicand into three equal parts, and to take one of the parts for the required product; and, to divide by 3 means to separate the dividend into three equal parts, and to take one of the parts for the required quotient.

To multiply by $\frac{2}{3}$ means to separate the multiplicand into three equal parts, and to take two of these parts; and to divide by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$, means to divide the dividend into three equal parts, and to take two of these parts.

Hence, to divide by a whole number or a fraction,

Multiply by its reciprocal.

Thus,

$$\frac{3}{5} \div 2 = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}.$$

$$\frac{3}{5} \div \frac{5}{6} = \frac{6}{5} \times \frac{3}{5} = \frac{18}{25}.$$

Mixed numbers must first be reduced to improper fractions.

Ex. 80. (*Oral.*)

Find the quotients of:

1. $\frac{5}{9} \div 5$.	7. $\frac{18}{27} \div 9$.	13. $\frac{3}{11} \div 6$.	19. $\frac{7}{8} \div 3$.
2. $\frac{8}{11} \div 4$.	8. $\frac{25}{30} \div 5$.	14. $\frac{7}{12} \div 3$.	20. $\frac{7}{8} \div 4$.
3. $\frac{3}{7} \div 2$.	9. $\frac{16}{17} \div 8$.	15. $\frac{12}{17} \div 6$.	21. $\frac{7}{8} \div 5$.
4. $\frac{75}{19} \div 3$.	10. $\frac{8}{10} \div 7$.	16. $\frac{12}{17} \div 3$.	22. $\frac{7}{8} \div 8$.
5. $\frac{14}{20} \div 2$.	11. $\frac{35}{90} \div 5$.	17. $\frac{12}{17} \div 4$.	23. $\frac{7}{8} \div 7$.
6. $\frac{4}{9} \div 5$.	12. $\frac{42}{50} \div 14$.	18. $\frac{12}{17} \div 12$.	24. $\frac{8}{13} \div 4$.

Ex. 81.

Find the quotients of:

1. $\frac{42}{45} \div 12$.
2. $\frac{350}{451} \div 25$.
3. $\frac{88}{95} \div 12$.
4. $\frac{78}{95} \div 13$.
5. $\frac{152}{209} \div 19$.
6. $\frac{7}{24} \div \frac{5}{11}$.
7. $\frac{3}{2} \div \frac{3}{8}$.
8. $2\frac{1}{2} \div \frac{3}{4}$.
9. $\frac{2}{3} \div \frac{4}{5}$.
10. $\frac{7}{8} \div 2\frac{1}{4}$.
11. $6\frac{1}{2} \div 4\frac{1}{6}$.
12. $\frac{4}{5} \div 7\frac{3}{10}$.
13. $3\frac{9}{10} \div 2\frac{3}{5}$.
14. $\frac{3}{4} \div \frac{2}{9}$.
15. $\frac{2}{7} \div \frac{5}{8}$.
16. $\frac{1}{2}$ of $\frac{3}{4} \div \frac{7}{8}$.
17. $\frac{3}{4}$ of $\frac{2}{9} \div \frac{5}{28}$.
18. $\frac{2}{5}$ of $3\frac{1}{2} \div \frac{27}{28}$.
19. $\frac{25}{39} \div \frac{10}{13}$.
20. $\frac{12}{49} \div \frac{2}{7}$.
21. $8\frac{3}{5} \div 6\frac{1}{7}$.
22. $7\frac{1}{9} \div 8\frac{2}{3}$.
23. $\frac{3}{7} \div \frac{15}{19}$.
24. $\frac{17}{68} \div \frac{85}{108}$.
25. $3\frac{31}{48} \div \frac{5}{12}$.
26. $3\frac{1}{6} \div 9\frac{1}{2}$.
27. $9\frac{1}{6} \div 3\frac{1}{2}$.
28. $8 \div \frac{14}{15}$.
29. $19 \div \frac{19}{20}$.
30. $3\frac{3}{7} \div 3\frac{1}{7}$.
31. $\frac{1}{2}$ of $\frac{3}{5} \div \frac{9}{10}$.
32. $1\frac{7}{8}$ of $7\frac{1}{5} \div 2\frac{4}{9}$.
33. $2\frac{1}{5}$ of $1\frac{3}{4} \div 2\frac{1}{3}$.
34. $1\frac{4}{5} \div \frac{3}{25}$ of $\frac{7}{8}$.
35. $\frac{3}{5}$ of $2\frac{1}{2} \div \frac{12}{19}$ of $6\frac{1}{3}$.
36. $\frac{2}{3}$ of $4\frac{1}{2} \div \frac{7}{9}$ of $3\frac{3}{5}$.
37. $2\frac{2}{3}$ of $1\frac{1}{4} \div 5\frac{1}{3}$ of $3\frac{3}{4}$.
38. $2\frac{1}{5}$ of $2\frac{1}{2} \div \frac{3}{14}$ of $3\frac{2}{3}$.
39. $1\frac{1}{2} \div 1\frac{1}{8}$ of $\frac{5}{21}$ of $\frac{8}{23}$.
40. $3\frac{2}{3}$ of $5\frac{1}{4}$ of $7\frac{1}{2} \div 63$.
41. $3\frac{2}{5}$ of $7\frac{1}{7}$ of $1\frac{2}{5} \div 3\frac{1}{5}$.
42. $7\frac{1}{5}$ of $3\frac{1}{3} \div 1\frac{1}{10}$ of $1\frac{5}{7}$.
43. $9 \div \frac{2}{11}$ of $1\frac{5}{17}$ of $4\frac{4}{5}$.
44. $16 \div \frac{7}{15}$ of $2\frac{8}{11}$ of $\frac{11}{35}$.
45. $3\frac{3}{8}$ of $4\frac{4}{5} \div \frac{2}{5}$ of $6\frac{1}{2}$ of $\frac{3}{8}$.

122. When a mixed number is to be divided by a whole number, it is best to divide the integral part of the dividend first, and then the fractional part. If there is a remainder from dividing the integral part, this remainder may be put with the fraction, and the result reduced to an improper fraction, and then divided by the divisor.

Divide $16\frac{4}{5}$ by 4; $16\frac{2}{3}$ by 7.

OPERATION.	OPERATION.
$4\overline{)16\frac{4}{5}}$	$7\overline{)16\frac{2}{3}}$
$4\frac{1}{5}$	$2\frac{8}{21}$

In the first problem we simply divide the whole number 16 by 4, and then the fraction $\frac{4}{5}$ by 4, and obtain the result at once, $4\frac{1}{5}$.

In the second problem we divide the 16 by 7, and obtain the quotient 2 and a remainder 2. The remainder is joined with the $\frac{2}{3}$, making $2\frac{2}{3} = \frac{8}{3}$, and $\frac{8}{3} \div 7 = \frac{8}{21}$.

Ex. 82.

Find the quotients of:

1. $19\frac{1}{2} \div 3$.	6. $34\frac{3}{5} \div 17$.	11. $65\frac{4}{13} \div 9$.
2. $12\frac{4}{5} \div 5$.	7. $31\frac{2}{9} \div 11$.	12. $147\frac{5}{9} \div 13$.
3. $24\frac{2}{3} \div 8$.	8. $37\frac{5}{8} \div 18$.	13. $76\frac{5}{7} \div 19$.
4. $19\frac{3}{7} \div 6$.	9. $45\frac{8}{17} \div 7$.	14. $124\frac{7}{29} \div 6$.
5. $17\frac{5}{9} \div 9$.	10. $57\frac{5}{9} \div 16$.	15. $326\frac{5}{17} \div 15$.

Ex. 83.

1. What must be paid for 24 yards of cloth, at $\$\frac{5}{7}$ per yard?
2. A farmer bought 327 sheep, at $\$4\frac{2}{3}$ a head; required the cost of the flock.
3. At 25 cents a pound, what must be paid for $82\frac{1}{2}$ pounds of butter?
4. A merchant sold $15\frac{2}{7}$ yards of silk, at $\$4\frac{2}{3}$ per yard; what change should he give back from 8 ten-dollar bills?
5. If beefsteak cost 22 cents per pound, and mutton chops 21 cents, how much will a man pay for meat, who eats $\frac{1}{2}$ pound of beefsteak for breakfast, and $1\frac{2}{3}$ pounds of mutton chops for dinner?

6. At $\$\frac{4}{5}$ per yard, how much cloth can be bought for \$25?

7. If $\$19\frac{1}{2}$ be paid for 9 yards of silk, what is the cost per yard?

8. A man walks $37\frac{4}{5}$ miles in 6 hours; how many miles does he walk an hour?

9. A farmer sells $19\frac{1}{2}$ acres of land for \$375; what is the price per acre?

10. A lady pays \$3 for $\frac{8}{9}$ of a yard of silk; what is the price per yard?

11. A man in one year pays \$45.26 for cigars, the average price of which is $6\frac{1}{5}$ cents apiece. How many does he smoke in a year?

12. If $\frac{7}{8}$ of an acre of tillage land cost \$125, what is the price per acre? How many acres can be bought for \$1297?

13. Gideon Lyford earns \$30 per week; what will remain at the close of the week when he has paid for $6\frac{3}{8}$ pounds of butter, at 33 cents a pound, $10\frac{1}{2}$ pounds of mutton, at 20 cents a pound, $8\frac{2}{5}$ pounds of beef, at 25 cents, 3 boxes of strawberries, at 16 cents, 150 pounds of ice, at $\frac{1}{2}$ cent, 20 loaves of bread, at 10 cents, fuel \$2, vegetables \$3?

14. Find the product of $17\frac{3}{5} \times 8\frac{1}{3}$ of $6\frac{3}{10}$.

15. If a man build $\frac{9}{10}$ of a rod of wall in one hour, how much will he build in $\frac{4}{5}$ of an hour?

16. If a ship costs \$16,785, what will $\frac{4}{5}$ of it be worth?

17. If a water-pipe discharges $16\frac{1}{3}$ barrels of water in an hour, how many barrels will it discharge in $9\frac{2}{7}$ hours?

18. For 4 sheep $\$25\frac{3}{8}$ are paid; what is the price per head?

19. A coal dealer paid \$375 freight for transporting coal from Scranton to Hudson. If the price was \$$\frac{3}{4}$ per ton, how many tons were transported?

20. How many pounds of beef, at $18\frac{3}{4}$ cents per pound, can be bought for \$17.48?

21. A farmer hires an equal number of men and boys, and pays for a man and boy \$$2\frac{5}{8}$ a day. If the pay roll is \$84 a day, how many men and boys does he hire?

22. When $8\frac{79}{1125}$ acres of land cost \$1297, what will $\frac{7}{9}$ of an acre cost?

23. A city speculator in land divided $\frac{3}{4}$ of an acre into lots of $\frac{1}{16}$ of an acre each, and sold them all for \$$13,426\frac{2}{3}$. What was the average price per lot?

24. For $\frac{2}{3}$ of $\frac{3}{4}$ of a ship the sum of \$6394 was received; what is the value of the ship?

25. A vessel sails $17\frac{2}{5}$ miles per hour; how many miles will she sail in $26\frac{2}{3}$ hours?

26. George is $13\frac{1}{2}$ years old, Henry is $\frac{3}{4}$ as old as George, and John's age is $1\frac{4}{9}$ that of Henry; what is the age of John?

27. There are $16\frac{1}{2}$ feet in one rod; how many feet are there in $84\frac{5}{9}$ rods?

28. How many feet around a field, each one of whose four sides measures $7\frac{5}{9}$ rods?

29. A schooner sails on the average $175\frac{5}{8}$ miles a day; how far will she sail in a week?

30. At the rate of $8\frac{2}{3}$ miles per hour, how many miles will a ship sail from a quarter past three A.M. to a quarter before six P.M.?

31. Reduce $\frac{3}{7}$ of $\frac{5}{11}$ of $\frac{7}{25}$ of $\frac{35}{49}$ to a simple fraction in its lowest terms.

32. George Ward inherited from his father $\frac{11}{13}$ of a farm containing 377 acres. He divided his share equally among his four sons; how many acres would each one of the sons receive?

33. How many pounds of sugar, at $9\frac{1}{2}$ cents a pound, can be bought for \$1.52?

34. For $23\frac{2}{3}$ baskets of peaches, a grocer gave \$20.59; what was the price per basket?

35. A farmer sold 42 bushels of potatoes for \$26.58; what was the average price per bushel?

36. At $37\frac{1}{2}$ cents per yard, how many yards of lace can be bought for $\$5\frac{1}{4}$?

37. A farmer sold $6\frac{1}{2}$ bushels of apples for $\$4.87\frac{1}{2}$; what was the price per bushel?

38. When $6\frac{1}{2}$ bushels of apples bring \$3.90, what are they worth a peck? (Four pecks make a bushel.)

39. At 60 cents a pound, how many pounds of tea can be bought for \$4.65?

40. If $\frac{3}{10}$ of a yard of cloth cost 80 cents, what should be paid for $\frac{5}{12}$ of a yard?

41. The cost of fencing a lot $8\frac{1}{2}$ rods in circuit is $\$6\frac{4}{5}$, what is the rate per rod?

42. A roll of carpeting containing 202 yards is cut into pieces of $25\frac{1}{4}$ yards each, and each piece is sold for $\$32\frac{1}{2}$. Required the number of pieces, and the price per yard.

43. When $35\frac{3}{4}$ bushels of turnips cost \$28.60, what should be paid for $\frac{1}{2}$ of a bushel?

44. How many yards of cloth can be bought for \$10.80, if $\frac{7}{10}$ of a yard cost 63 cents?

LEAST COMMON DENOMINATOR.

123. A fraction is changed to *higher terms* by multiplication.

Reduce $\frac{8}{4}$ to twelfths.

Multiply both terms by 3; thus,

$$\frac{3\times 8}{3\times 4}=\frac{24}{12}.$$

In either of the two forms $\frac{8}{4}$ or $\frac{24}{12}$ the value of the fraction is 2.

124. Hence, to reduce a fraction to higher terms,

Multiply both terms of the fraction by that number which will change the given denominator to the required denominator.

The required multiplier is found by dividing the required denominator by the denominator of the given fraction.

Ex. 84. (*Oral.*)

Reduce:

1. $\frac{4}{5}$ to 20ths.	8. $\frac{5}{9}$ to 27ths.	15. $\frac{3}{13}$ to 26ths.
2. $\frac{1}{2}$ to 10ths.	9. $\frac{5}{11}$ to 33ds.	16. $\frac{5}{18}$ to 36ths.
3. $\frac{2}{3}$ to 9ths.	10. $\frac{4}{7}$ to 28ths.	17. $\frac{5}{9}$ to 81sts.
4. $\frac{3}{7}$ to 14ths.	11. $\frac{7}{12}$ to 36ths.	18. $\frac{7}{12}$ to 96ths.
5. $\frac{2}{9}$ to 18ths.	12. $\frac{7}{10}$ to 20ths.	19. $\frac{8}{11}$ to 44ths.
6. $\frac{3}{4}$ to 12ths.	13. $\frac{4}{15}$ to 45ths.	20. $\frac{3}{4}$ to 16ths.
7. $\frac{5}{6}$ to 24ths.	14. $\frac{1}{20}$ to 100ths.	21. $\frac{5}{24}$ to 72ds.

125. Similar fractions are fractions that have a common denominator.

All fractions must be expressed as similar fractions before they can be added or subtracted, and in all cases it is best to express them with the least common denominator. (L.C.D.)

126. The least common denominator of two or more fractions is the least common multiple of their denominators.

Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to similar fractions.

The least common multiple of the denominators 2, 3, 4 is 12. It is therefore necessary to reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ to 12ths, by the method explained in § 124, and we have $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$.

127. Hence, to reduce fractions to similar fractions,

Find the least common multiple of the denominators; this will be the required denominator. Divide this denominator by the denominator of each fraction.

Multiply the first numerator by the first quotient, the second numerator by the second quotient, and so on.

The products will be the numerators of the equivalent fractions.

Ex. 85. (*Oral.*)

Reduce to similar fractions:

1. $\frac{1}{2}$, $\frac{1}{3}$.	5. $\frac{1}{4}$, $\frac{1}{5}$.	9. $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{21}$.	13. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$.
2. $\frac{1}{7}$, $\frac{1}{3}$.	6. $\frac{3}{4}$, $\frac{5}{7}$.	10. $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{14}$.	14. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{12}$.
3. $\frac{1}{2}$, $\frac{1}{4}$.	7. $\frac{5}{8}$, $\frac{5}{6}$.	11. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$.	15. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{5}{18}$.
4. $\frac{1}{5}$, $\frac{1}{3}$.	8. $\frac{2}{3}$, $\frac{3}{8}$.	12. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$.	16. $\frac{1}{2}$, $\frac{2}{9}$, $\frac{5}{6}$.

Ex. 86.

Reduce to similar fractions:

1. $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$.	6. $\frac{2}{3}$, $\frac{5}{13}$, $\frac{12}{39}$, $\frac{25}{78}$.
2. $\frac{7}{12}$, $\frac{5}{36}$, $\frac{11}{48}$.	7. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{8}{15}$, $\frac{7}{25}$.
3. $\frac{8}{21}$, $\frac{2}{28}$, $\frac{5}{7}$.	8. $\frac{1}{5}$, $\frac{7}{8}$, $\frac{13}{16}$, $\frac{19}{20}$, $\frac{3}{40}$.
4. $\frac{4}{19}$, $\frac{3}{38}$, $\frac{2}{3}$.	9. $\frac{7}{9}$, $\frac{5}{12}$, $\frac{17}{18}$, $\frac{23}{24}$, $\frac{7}{36}$.
5. $\frac{4}{9}$, $\frac{5}{14}$, $\frac{5}{42}$.	10. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{8}{21}$, $\frac{15}{42}$.

ADDITION OF FRACTIONS.

128. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

These fractions reduced to similar fractions become $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$, and $\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}=2\frac{3}{12}=2\frac{1}{4}$. $2\frac{1}{4}$. *Ans.*

129. Hence, to add fractions,

Reduce the fractions to similar fractions, and write the sum of the numerators over the common denominator.

Add $\frac{7}{8}$, $\frac{5}{12}$, $\frac{8}{15}$.

$$\begin{array}{r|rrr} 2 & 8 & 12 & 15 \\ \hline 2 & 4 & 6 & 15 \\ \hline 3 & 2 & 3 & 15 \\ \hline & 2 & 1 & 5 \end{array}$$

Hence, L. C. D. $= 2^3 \times 3 \times 5 = 120$.

$$\text{Numerators} \ldots . \left\{\begin{array}{r} 105 \\ 50 \\ \underline{64} \end{array}\right.$$

Sum of numerators $= 219$

Therefore, sum of fractions $= \frac{219}{120} = \frac{73}{40} = 1\frac{33}{40}$. $1\frac{33}{40}$. *Ans.*

130. If any of the expressions are integers or mixed numbers, add together separately the integers and the fractions, and find the sum of the results.

Find the sum of $2\frac{3}{20}$, $1\frac{7}{15}$, $5\frac{11}{12}$.

L. C. D. of the fractions $= 2^2 \times 3 \times 5 = 60$.

$$\text{Numerators} \ldots . \left\{\begin{array}{r} 9 \\ 28 \\ \underline{55} \end{array}\right.$$

Sum of numerators $= 92$

Sum of fractions $= \frac{92}{60} = \frac{23}{15} = 1\frac{8}{15}$

Sum of integers $= 2 + 1 + 5 = \underline{8}$

$9\frac{8}{15}$. *Ans.*

Ex. 87. (*Oral.*)

Find the sum of:

1. $\frac{5}{12}$, $\frac{3}{12}$.	3. $\frac{8}{9}$, $\frac{4}{9}$.	5. $\frac{11}{20}$, $\frac{9}{20}$.	7. $7\frac{1}{4}$, $3\frac{1}{4}$.
2. $\frac{9}{16}$, $\frac{7}{16}$.	4. $\frac{8}{15}$, $\frac{2}{15}$.	6. $\frac{21}{30}$, $\frac{19}{30}$.	8. $8\frac{3}{5}$, $4\frac{2}{5}$.

9. $5\frac{3}{8}$, $4\frac{7}{8}$.
10. $9\frac{3}{10}$, $5\frac{3}{10}$.
11. $8\frac{3}{4}$, $5\frac{3}{4}$.
12. $7\frac{7}{12}$, $3\frac{7}{12}$.
13. $\frac{5}{6}$, $\frac{2}{3}$.
14. $\frac{3}{4}$, $\frac{5}{8}$.
15. $\frac{9}{10}$, $\frac{3}{5}$.
16. $\frac{3}{8}$, $\frac{7}{24}$.
17. $\frac{2}{3}$, $\frac{7}{12}$.
18. $\frac{5}{6}$, $\frac{11}{18}$.
19. $3\frac{7}{10}$, $4\frac{3}{20}$.
20. $2\frac{5}{6}$, $3\frac{2}{3}$.
21. $8\frac{5}{12}$, $7\frac{3}{4}$.
22. $6\frac{3}{8}$, $5\frac{1}{2}$.
23. $7\frac{2}{3}$, $4\frac{5}{9}$.
24. $9\frac{3}{10}$, $8\frac{2}{5}$.

Ex. 88.

Find the sum of:

1. $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$.
2. $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$.
3. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$.
4. $\frac{7}{8}$, $\frac{3}{5}$, $\frac{2}{3}$.
5. $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{9}$.
6. $\frac{5}{6}$, $\frac{4}{9}$, $\frac{11}{15}$.
7. $4\frac{5}{6}$, $3\frac{2}{3}$, $6\frac{5}{12}$.
8. $7\frac{3}{4}$, $8\frac{11}{16}$, $9\frac{5}{8}$.
9. $8\frac{7}{15}$, $4\frac{4}{5}$, $3\frac{2}{3}$.
10. $4\frac{5}{9}$, $5\frac{3}{4}$, $6\frac{4}{5}$.
11. $9\frac{5}{6}$, $4\frac{3}{8}$, $8\frac{2}{9}$.
12. $4\frac{14}{15}$, $5\frac{13}{18}$, $6\frac{9}{10}$.
13. $\frac{7}{10}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{11}{20}$.
14. $\frac{3}{8}$, $\frac{5}{6}$, $\frac{11}{24}$, $\frac{7}{12}$.
15. $\frac{7}{10}$, $\frac{8}{15}$, $\frac{11}{18}$, $\frac{3}{5}$.
16. $\frac{11}{16}$, $\frac{13}{20}$, $\frac{3}{4}$, $\frac{7}{15}$.
17. $\frac{5}{9}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{9}{20}$.
18. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{13}{20}$, $\frac{11}{18}$.
19. $\frac{7}{8}$, $\frac{4}{9}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, $\frac{11}{15}$.
20. $\frac{8}{9}$, $\frac{7}{10}$, $\frac{3}{8}$, $\frac{13}{15}$, $\frac{5}{12}$, $\frac{17}{20}$.
21. $\frac{2}{3}$, $\frac{5}{12}$, $\frac{17}{24}$, $\frac{13}{21}$, $\frac{11}{14}$, $\frac{15}{28}$.
22. $\frac{11}{15}$, $\frac{17}{20}$, $\frac{7}{12}$, $\frac{19}{24}$, $\frac{9}{10}$, $\frac{31}{36}$.
23. $\frac{14}{15}$, $\frac{17}{35}$, $\frac{23}{28}$, $\frac{19}{20}$, $\frac{13}{21}$, $\frac{19}{24}$.
24. $\frac{3}{4}$, $\frac{3}{8}$, $\frac{11}{20}$, $\frac{23}{28}$, $\frac{19}{21}$, $\frac{11}{14}$.
25. $24\frac{8}{9}$, $13\frac{4}{5}$, $36\frac{3}{8}$, $60\frac{5}{9}$, $47\frac{3}{5}$.
26. $35\frac{2}{3}$, $17\frac{1}{2}$, $25\frac{3}{4}$, $48\frac{3}{5}$, $18\frac{1}{2}$.
27. $54\frac{3}{5}$, $28\frac{2}{3}$, $16\frac{1}{2}$, $36\frac{3}{4}$, $64\frac{2}{5}$.
28. $36\frac{4}{7}$, $37\frac{2}{5}$, $59\frac{3}{4}$, $54\frac{2}{7}$, $16\frac{4}{5}$.
29. $23\frac{5}{8}$, $32\frac{8}{9}$, $18\frac{3}{4}$, $27\frac{5}{6}$, $28\frac{2}{3}$.
30. $74\frac{8}{15}$, $64\frac{13}{20}$, $48\frac{7}{12}$, $23\frac{11}{24}$, $27\frac{3}{10}$.

SUBTRACTION OF FRACTIONS.

131. From $\frac{19}{24}$ take $\frac{7}{18}$.

$$24 = 2^3 \times 3,$$
$$18 = 2 \times 3^2.$$

Hence, the L.C.D. $= 2^3 \times 3^2 = 72$.

$$\tfrac{19}{24} - \tfrac{7}{18} = \tfrac{57-28}{72} = \tfrac{29}{72}. \qquad \tfrac{29}{72}. \textit{ Ans.}$$

132. Hence, to subtract one fraction from another,

Reduce the fractions to similar fractions.

Subtract the numerator of the subtrahend from the numerator of the minuend.

Write the result over the common denominator.

133. If the terms are mixed numbers, subtract separately the integers and fractions, and unite the results.

Subtract $5\frac{3}{8}$ from $15\frac{3}{4}$.

Here the L.C.D. $= 8$.

$$15\tfrac{3}{4} - 5\tfrac{3}{8} = 10\tfrac{6-3}{8} = 10\tfrac{3}{8}. \qquad 10\tfrac{3}{8}. \textit{ Ans.}$$

Subtract $3\frac{5}{8}$ from $5\frac{5}{12}$.

$$5\tfrac{5}{12} - 3\tfrac{5}{8} = 2\tfrac{10-15}{24} = 1\tfrac{34-15}{24} = 1\tfrac{19}{24}.$$

$1\frac{19}{24}$. *Ans.*

The difference between $5\frac{5}{12}$ and $3\frac{5}{8}$ is $2\frac{10-15}{24}$. Since $\frac{15}{24}$ cannot be subtracted from $\frac{10}{24}$, 1 is taken from 2, and added to $\frac{10}{24}$, making $\frac{34}{24}$.

From 9 take $\frac{19}{24}$.

$$9 = 8\tfrac{24}{24}.$$

$$8\tfrac{24}{24} - \tfrac{19}{24} = 8\tfrac{5}{24}. \qquad 8\tfrac{5}{24}. \textit{ Ans.}$$

Ex. 89.

Find the value of :

1. $\frac{11}{12} - \frac{5}{12}$.
2. $\frac{58}{68} - \frac{44}{68}$.
3. $\frac{83}{84} - \frac{59}{84}$.
4. $\frac{67}{72} - \frac{49}{72}$.
5. $\frac{91}{96} - \frac{73}{96}$.
6. $\frac{5}{6} - \frac{3}{4}$.
7. $\frac{7}{8} - \frac{3}{5}$.
8. $\frac{4}{5} - \frac{1}{6}$.
9. $\frac{7}{10} - \frac{5}{12}$.
10. $\frac{2}{3} - \frac{3}{8}$.
11. $\frac{11}{12} - \frac{3}{4}$.
12. $\frac{5}{7} - \frac{5}{8}$.
13. $\frac{3}{8} - \frac{5}{24}$.
14. $\frac{1}{11} - \frac{1}{13}$.
15. $\frac{11}{12} - \frac{5}{16}$.
16. $\frac{25}{36} - \frac{13}{64}$.
17. $\frac{2}{5} - \frac{1}{8}$.
18. $\frac{14}{15} - \frac{6}{7}$.
19. $\frac{13}{14} - \frac{19}{21}$.
20. $\frac{14}{15} - \frac{41}{50}$.
21. $\frac{19}{30} - \frac{28}{90}$.
22. $\frac{11}{112} - \frac{13}{168}$.
23. $\frac{29}{56} - \frac{3}{14}$.
24. $\frac{16}{19} - \frac{13}{16}$.
25. $14 - \frac{7}{16}$.
26. $21 - \frac{19}{24}$.
27. $20 - \frac{13}{20}$.
28. $42 - \frac{13}{55}$.
29. $25 - \frac{13}{18}$.
30. $21 - \frac{13}{15}$.
31. $14 - \frac{15}{32}$.
32. $13 - \frac{5}{12}$.
33. $24 - 13\frac{5}{8}$.
34. $42 - 15\frac{7}{12}$.
35. $20 - 12\frac{7}{32}$.
36. $84 - 37\frac{17}{36}$.
37. $21\frac{5}{18} - \frac{13}{18}$.
38. $27\frac{5}{8} - \frac{7}{8}$.
39. $42\frac{5}{12} - \frac{7}{12}$.
40. $26\frac{7}{32} - \frac{19}{32}$.
41. $43\frac{17}{24} - \frac{11}{18}$.
42. $27\frac{5}{12} - \frac{13}{18}$.
43. $91\frac{4}{15} - \frac{7}{8}$.
44. $32\frac{11}{20} - \frac{13}{15}$.
45. $83\frac{7}{48} - \frac{13}{16}$.
46. $26\frac{3}{10} - \frac{7}{15}$.
47. $74\frac{5}{12} - \frac{11}{18}$.
48. $68\frac{4}{9} - \frac{5}{7}$.
49. $24\frac{5}{72} - 16\frac{1}{12}$.
50. $92\frac{1}{5} - 73\frac{4}{5}$.
51. $19\frac{5}{72} - 14\frac{13}{72}$.
52. $23\frac{1}{4} - 16\frac{3}{4}$.
53. $15\frac{2}{3} - 12\frac{1}{4}$.
54. $42\frac{1}{2} - 14\frac{1}{6}$.
55. $24\frac{9}{11} - 15\frac{3}{7}$.
56. $72\frac{8}{9} - 28\frac{5}{6}$.
57. $19\frac{1}{3} - 13\frac{1}{2}$.
58. $26\frac{3}{8} - 19\frac{2}{3}$.
59. $45\frac{5}{12} - 26\frac{7}{10}$.
60. $34\frac{3}{8} - 16\frac{8}{9}$.
61. $34\frac{3}{4} - 18\frac{5}{6}$.
62. $64\frac{3}{4} - 28\frac{13}{15}$.
63. $48\frac{2}{7} - 19\frac{5}{6}$.
64. $76\frac{7}{8} - 72\frac{9}{10}$.
65. $97\frac{1}{2} - 32\frac{15}{16}$.
66. $90\frac{1}{2} - 9\frac{5}{8}$.
67. $78\frac{3}{10} - 56\frac{8}{9}$.
68. $96\frac{1}{2} - 49\frac{3}{4}$.
69. $47\frac{3}{4} - 43\frac{11}{12}$.
70. $55\frac{3}{8} - 54\frac{2}{7}$.
71. $69\frac{1}{16} - 67\frac{23}{24}$.
72. $69\frac{2}{7} - 23\frac{17}{18}$.

Ex. 90.

1. A country merchant received on Monday \$$25\frac{3}{4}$, on Tuesday \$$19\frac{7}{8}$, on Wednesday \$$23\frac{2}{3}$, on Thursday \$$32\frac{3}{8}$, on Friday \$$29\frac{1}{2}$, on Saturday \$$37\frac{5}{8}$. What had he left after paying a freight bill of \$$19\frac{3}{8}$, and to his clerk \$$12\frac{1}{2}$?

2. A farmer sold two loads of hay, one for \$$13\frac{1}{2}$ and the other for \$$16\frac{2}{3}$, and received \$25 down. How much is still due?

3. A miner digs $17\frac{2}{3}$, $19\frac{1}{7}$, $18\frac{3}{5}$ ounces of gold. In washing there is a loss of $3\frac{2}{5}$ ounces. How much gold has he left?

4. Henry Cameron had three wheat-fields; the first produced $217\frac{3}{7}$ bushels, the second $309\frac{5}{9}$, the third $419\frac{1}{3}$. He sent $516\frac{2}{3}$ bushels to a flour mill, and sold 193 bushels. How many bushels had he left?

5. From a piece of cloth containing $47\frac{1}{2}$ yards, $22\frac{2}{3}$ yards were sold, and then $5\frac{3}{4}$ yards were sold. How many yards remained?

6. A grocer sold $2\frac{3}{8}$ pounds of tea to one man, $1\frac{1}{2}$ pounds more to a second man than to the first, and to a third man $1\frac{1}{4}$ pounds less than the amount he sold the first and second together. How many pounds did he sell to the second man, and to the third man?

7. Of the prismatic spectrum red occupies $\frac{1}{8}$, orange $\frac{3}{40}$, and yellow $\frac{2}{15}$. What part of the whole do these three colors together occupy?

8. What part of a piece of cloth has a merchant sold, who has cut off and sold $\frac{3}{16}$, $\frac{5}{32}$, $\frac{9}{64}$, and $\frac{7}{40}$ of it?

9. A treasurer has expended $\frac{11}{60}$, $\frac{7}{48}$, $\frac{13}{40}$, $\frac{7}{80}$, and $\frac{5}{24}$ of a given sum. What part of the whole has he left?

10. Of a pole $\frac{1}{5}$ is blue, $\frac{3}{7}$ red, and the rest white. What part of it is white?

11. A jeweller has used $\frac{3}{10}$, $\frac{7}{20}$, and $\frac{11}{40}$ of an ingot of gold. What part of it still remains?

12. A student has read $\frac{5}{21}$, $\frac{2}{7}$, and $\frac{1}{3}$ of a certain book. What part of it has he yet to read?

13. A traveller has gone $\frac{1}{9}$ of a journey on foot, $\frac{2}{15}$ on horseback, $\frac{1}{4}$ by rail, and the rest by coach. What part of the journey has he gone by coach?

14. Of the component elements of albumen $\frac{11}{20}$ is carbon, $\frac{7}{100}$ hydrogen, and $\frac{4}{25}$ nitrogen. What part of the whole do these elements constitute?

15. Add together the greatest and least of the fractions, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{19}{20}$, and subtract this sum from the sum of the other two fractions.

16. How many tons of ore must be raised from a mine so that, on losing $\frac{17}{40}$ in roasting, and $\frac{8}{19}$ of the remainder in smelting, there may be obtained 506 tons of pure metal?

17. A man invested $\frac{2}{5}$ of his capital in bank stock, $\frac{3}{4}$ of the remainder in real estate, and had left \$6000. Find his capital.

18. A man invests $\frac{1}{2}$ of his money in land, $\frac{1}{5}$ in bank stock, $\frac{1}{6}$ in railroad stock, and has \$8000 left. What is his fortune?

19. A owns $\frac{2}{5}$ of a ship, and B the remainder; and $\frac{3}{4}$ of the difference between their shares is \$1500. What is the value of the ship?

Complex Fractions.

134. The quotient $\frac{2}{3} \div \frac{3}{4}$ may be written in the form $\frac{\frac{2}{3}}{\frac{3}{4}}$, in which the dividend is the numerator, and the divisor the denominator of a **complex fraction.**

135. A complex fraction has a fraction in either its numerator or denominator, or in both of them.

The reduction of complex to simple fractions is similar to division of fractions.

Reduce $\frac{\frac{3}{4}}{7}$ to a simple fraction.

Multiply both terms by 4 and we have $\frac{3}{28}$.

Reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction.

Multiply both terms by 12, which is the L.C.M. of 4 and 6, and we have at once $\frac{9}{10}$.

Reduce $\frac{8\frac{3}{4}}{12\frac{5}{6}}$ to a simple fraction.

Multiply both terms by 12 and we have $\frac{105}{154} = \frac{15}{22}$.

Ex. 91.

Reduce to simple fractions:

1. $\frac{6}{8\frac{1}{2}}$.	3. $\frac{9}{12\frac{3}{5}}$.	5. $\frac{4\frac{7}{9}}{7}$.	7. $\frac{\frac{4}{9}}{\frac{7}{10}}$.
2. $\frac{7}{11\frac{3}{4}}$.	4. $\frac{11}{13\frac{4}{7}}$.	6. $\frac{\frac{2}{3}}{\frac{4}{5}}$.	8. $\frac{\frac{10}{11}}{\frac{12}{13}}$.

9. $\dfrac{\frac{6}{7}}{\frac{14}{23}}$.

10. $\dfrac{\frac{15}{19}}{\frac{45}{76}}$.

11. $\dfrac{\frac{13}{14}}{7\frac{4}{5}}$.

12. $\dfrac{5\frac{3}{4}}{6\frac{2}{9}}$.

13. $\dfrac{19\frac{3}{4}}{28\frac{7}{18}}$.

14. $\dfrac{2\frac{8}{11}}{7\frac{3}{4}}$.

15. $\dfrac{23\frac{7}{9}}{24\frac{5}{6}}$.

16. $\dfrac{\frac{3}{4} \text{ of } 3\frac{1}{2}}{\frac{4}{5} \text{ of } 9\frac{2}{3}}$.

17. $\dfrac{\frac{2}{5} \text{ of } 13\frac{1}{4}}{\frac{4}{7} \text{ of } 7\frac{7}{8}}$.

18. $\dfrac{\frac{2}{11} \text{ of } 12\frac{6}{7}}{\frac{3}{8} \text{ of } \dfrac{5}{6\frac{2}{3}}}$.

19. $\dfrac{\frac{3}{4} \text{ of } \frac{2}{7}}{\dfrac{4}{7\frac{1}{2}} \text{ of } \frac{8}{11}}$.

20. $\dfrac{\frac{\frac{2}{3}}{\frac{3}{4}}}{5}$.

21. $\dfrac{2}{\frac{\frac{3}{4}}{\frac{5}{9}}}$.

136. To express one number as a fraction of another.

What fraction of 8 is 5?

Since $1 = \frac{1}{8}$ of 8,

$5 = 5 \times \frac{1}{8}$ of 8.

That is, $5 = \frac{5}{8}$ of 8.

The number which follows "of" is the denominator, and the other number the numerator of the required fraction.

Ex. 92.

What fraction of:

1. 8 is 7?
2. 7 is 8?
3. 6 is 2?
4. 5 is 3?
5. 7 is 15?
6. 15 is 7?
7. $2\frac{1}{2}$ is $\frac{1}{2}$?
8. $\frac{3}{4}$ is $4\frac{1}{4}$?
9. $2\frac{3}{4}$ is $1\frac{1}{4}$?
10. $2\frac{1}{3}$ is $5\frac{2}{3}$?
11. $2\frac{1}{3}$ is $5\frac{1}{4}$?
12. $5\frac{1}{4}$ is $2\frac{1}{3}$?
13. $3\frac{3}{4}$ is $\frac{7}{8}$?
14. $5\frac{1}{4}$ is $4\frac{2}{3}$?
15. $11\frac{3}{4}$ is $5\frac{2}{9}$?
16. $2\frac{17}{19}$ is $7\frac{6}{7}$?
17. $3\frac{11}{12}$ is $9\frac{2}{5}$?
18 $14\frac{2}{3}$ is $4\frac{8}{9}$?

19. $7\frac{3}{4}$ is $2\frac{1}{8}$?

20. $7\frac{1}{2}$ is $\frac{15}{19}$?

21. $\frac{3}{4}$ of $10\frac{1}{3}$ is $\frac{31}{32}$?

22. $\frac{7}{9}$ of $25\frac{7}{8}$ is $\frac{161}{162}$?

23. $\frac{2}{3}$ of $12\frac{3}{4}$ is $\frac{51}{55}$?

24. $\frac{6}{7}$ of $3\frac{1}{6}$ is $\frac{19}{44}$?

25. $\frac{22}{35}$ of $7\frac{1}{2}$ is $3\frac{2}{3}$?

26. $\frac{7}{26}$ is $\frac{1}{4}$ of $\frac{14}{15}$?

27. $\frac{7}{9}$ is $\frac{1}{6}$ of $2\frac{3}{4}$?

28. 33 is $2\frac{1}{3}$ of $2\frac{1}{7}$?

29. $27\frac{17}{24}$ is $2\frac{43}{45}$ of $1\frac{1}{14}$?

30. 36 is $3\frac{3}{8}$ of $6\frac{2}{5}$?

137. To reduce a decimal to a common fraction.

Reduce 0.25 to a common fraction.

$$0.25 = \frac{25}{100} = \frac{1}{4}.$$

138. Hence, to reduce a decimal to a common fraction,

Write the figures of the decimal for the numerator; and 1, *with as many zeros as there are figures in the decimal, for the denominator.*

Ex. 93.

Reduce to common fractions:

1. 0.5.	9. 0.015.	17. 0.7168.	25. 1.6125.
2. 0.06.	10. 0.18.	18. 3.02.	26. 8.0396.
3. 0.15.	11. 0.125.	19. 5.85.	27. 2.18375.
4. 0.025.	12. 0.004.	20. 7.075.	28. 1.0725.
5. 0.7.	13. 0.032.	21. 0.15625.	29. 22.848.
6. 0.19.	14. 0.3125.	22. 0.46875.	30. 1.30125.
7. 0.135.	15. 0.0625.	23. 0.00256.	31. 17.875.
8. 0.005.	16. 0.0425.	24. 0.00375.	32. 2.9375.

139. To reduce a common fraction to a decimal.

Change $\frac{3}{8}$ to a decimal.

$$\begin{array}{r} 8\underline{)3.000} \\ 0.375 \end{array}$$

140. Hence, to reduce a common fraction to a decimal,

Divide the numerator by the denominator.

141. If a fraction, when reduced to its lowest terms, contains in the denominator any other factor than 2 or 5 (the prime factors of 10), the division of the numerator by the denominator will not terminate. In general, it will be sufficient to obtain five decimal places in the quotient. But the number in the fifth place of the quotient must be increased by 1 if the number in the next place of the quotient is five, or greater than five.

Ex. 94.

Change the following fractions to decimals:

1. $\frac{13}{32}$.	5. $11\frac{67}{80}$.	9. $\frac{17}{16}$.	13. $\frac{9}{16}$.
2. $\frac{23}{200}$.	6. $\frac{11}{8}$.	10. $\frac{9}{5}$.	14. $11\frac{18}{625}$.
3. $\frac{39}{4000}$.	7. $\frac{3}{500}$.	11. $\frac{19}{20}$.	15. $14\frac{79}{4000}$.
4. $16\frac{9}{40}$.	8. $5\frac{3}{64}$.	12. $\frac{377}{4}$.	

Express the following as decimals to five places:

16. $\frac{1}{6}$.	19. $\frac{2}{11}$.	22. $\frac{12}{13}$.	25. $\frac{71}{82}$.
17. $\frac{1}{7}$.	20. $\frac{18}{28}$.	23. $\frac{167}{240}$.	26. $\frac{311}{328}$.
18. $\frac{1}{9}$.	21. $\frac{4}{9}$.	24. $\frac{7}{111}$.	27. $4\frac{29}{440}$.

Ex. 95.

Solve the following problems, first changing the common fractions to decimals:

1. A person owed \$24,560. When he has paid \$$8345\frac{3}{10}$, \$$7234\frac{3}{10}$, \$$6472\frac{9}{20}$, how much does he still owe?

2. A man sold $46\frac{17}{20}$ acres of land, at the rate of \$$9\frac{1}{2}$ an acre, and $54\frac{1}{2}$ acres at the rate of \$$2\frac{19}{20}$. How much did he receive for the whole?

3. A merchant purchases 346 pieces of cloth, each containing $32\frac{3}{4}$ yards, at \$$1\frac{3}{4}$ a yard, and sells the whole for \$$2\frac{9}{20}$ a yard. What does he gain?

4. A merchant purchased 8 yards of cloth at \$$6\frac{1}{2}$ a yard. What sum will he gain per yard if he sells the whole piece for \$$56\frac{24}{25}$?

5. A man bought a piece of land for \$$1046\frac{1}{4}$ at the rate of \$$15\frac{1}{2}$ an acre. He sells it for \$$17\frac{1}{5}$ an acre. How much does he gain on the whole?

6. A merchant purchased 15 casks of wine of 25 gallons each. He paid \$980 for the wine, \$$78\frac{3}{4}$ tax, \$$33\frac{3}{5}$ for transportation. He sold it for \$$3\frac{4}{5}$ a gallon. How much did he gain?

7. A speculator purchased 738 acres of land for \$21,294. He sells $\frac{2}{5}$ of his land at the rate of \$$34\frac{3}{4}$ an acre, and the rest at the rate of \$35 per acre. What does he gain?

8. A piece of cloth is $29\frac{3}{4}$ yards in length. How many pieces, each containing $1\frac{55}{64}$ yards, can be cut from it?

9. How many postage-stamps, each containing $\frac{18}{25}$ of a square inch, are in a sheet of $172\frac{4}{5}$ square inches?

10. Of a boat worth \$5600, A, who has $\frac{17}{64}$, sells $\frac{3}{4}$ of his share to B, and B sells $\frac{4}{5}$ of his share to C. Find the value of C's share.

11. From Montreal to Toronto, by the Grand Trunk Railway, the distance is 332 miles. One-half a mile more than $\frac{3}{8}$ of this distance was opened in November, 1855, and the remainder in November, 1856. Find the number of miles opened in 1856.

12. The 36 Israelites who fell in the first assault on Ai, were $\frac{3}{250}$ of the force sent by Joshua. How many were sent by Joshua?

13. What number multiplied by $8\frac{2}{5}$ equals $3\frac{1}{2}+\frac{1}{4}+\frac{16}{25}+\frac{43}{50}$?

14. Multiply the sum of $\frac{9}{10}$ and $\frac{3}{4}$ by their difference.

15. Of the distance from Edinburgh to London by rail, that from Edinburgh to Carlisle is $\frac{1}{4}$, from Carlisle to Preston $\frac{9}{40}$, while that from Preston to London is 210 miles. Find the distance from Edinburgh to London.

16. How many times can a measure holding $\frac{7}{8}$ of a pint be filled from a vessel containing $63\frac{7}{8}$ pints?

17. Of a consignment of guano $\frac{207}{400}$ consisted of carbonate of lime and phosphates of lime and magnesia, and the phosphates made up $\frac{19}{40}$ of the guano. How many parts in a hundred of the guano was carbonate of lime?

18. Of the water of the Dead Sea $\frac{49}{1250}$ is muriate of lime, $\frac{41}{400}$ muriate of magnesia, $\frac{259}{2500}$ muriate of soda, $\frac{1}{2000}$ sulphate of lime. What part of the whole do these ingredients constitute?

Ex. 96.

Oral Exercise in Fractions.

1. From a piece of cloth $\frac{1}{3}$ of it and $\frac{1}{2}$ of it have been cut. What fraction of the cloth is left?

2. To make a yard of cloth, what fraction of a yard must be added to the sum of $\frac{1}{4}$ and $\frac{1}{3}$ of a yard?

3. A boy gave to his sister $\frac{1}{2}$ of an apple, to his brother $\frac{1}{2}$ as much as to his sister, and kept the rest himself. What part of the apple did he keep?

4. A grocer sold $\frac{3}{4}$ of a dozen eggs, and carried home the rest of the dozen. How many did he carry home?

5. What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of a unit?

6. How is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of a unit found?

7. How is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of a number found?

8. At $\frac{2}{3}$ of a dollar per yard, what is the cost of 6 yards of cloth?

9. At $7 per ton, what is the cost of $\frac{1}{3}$ of a ton of coal?

10. Three packages of sugar weigh respectively $2\frac{1}{4}$, $3\frac{1}{2}$, $4\frac{1}{4}$ pounds. What is the weight of the whole?

11. When poultry is worth 20 cents per pound, what must be paid for a turkey weighing $8\frac{1}{2}$ pounds, and a chicken weighing $3\frac{1}{4}$ pounds?

12. From a jar of butter containing $15\frac{1}{2}$ pounds there have been sold $7\frac{3}{4}$ pounds. How many pounds remain in the jar?

13. Change to mixed numbers $\frac{17}{4}$, $\frac{25}{2}$, $\frac{19}{3}$, $\frac{13}{7}$, $\frac{21}{5}$, $\frac{24}{11}$.

14. Express in lowest terms $\frac{12}{16}$, $\frac{8}{12}$, $\frac{16}{20}$, $\frac{56}{63}$, $\frac{36}{42}$, $\frac{84}{108}$.

15. Change to improper fractions $5\frac{1}{2}$, $6\frac{3}{5}$, $8\frac{5}{9}$, $9\frac{4}{7}$, $13\frac{1}{4}$.

16. Reduce $\frac{3}{5}$ to 10ths; to 15ths; to 20ths; to 25ths.

17. A lady gave $\frac{1}{2}$ a dollar to her daughter, and $\frac{1}{5}$ of a dollar to her son. What fraction of a dollar did the daughter receive more than the son?

18. At $\frac{4}{5}$ of a dollar per bushel, how many bushels of apples can be bought for \$3?

19. Four pecks make a bushel. If $2\frac{3}{8}$ pecks be sold from a bushel of cranberries, how many pecks remain?

20. A gentleman bought 2 pairs of gloves at \$$1\frac{1}{2}$ a pair, and 3 pairs of slippers at \$$1\frac{3}{4}$ per pair. He gave a ten-dollar bill in payment. What change should he receive?

21. What part of 2 is 1? of 7 is 3? of 9 is 2? of 12 is 4?

22. A farmer planted 3 bushels of potatoes, and harvested 50 bushels. What fraction of the crop was the seed?

23. From a piece of cloth containing 81 yards there were sold 45 yards. What part of the piece was sold?

24. What part of $\frac{4}{5}$ is $\frac{2}{3}$? of $\frac{1}{2}$ is $\frac{1}{3}$? of $\frac{1}{2}$ is $\frac{1}{5}$?

HINT. Reduce the fractions to similar fractions.

25. What part of 8 is $\frac{1}{2}$? of 7 is $\frac{2}{3}$?

26. In a year there are 365 days. What part of a year are 30 days? 50 days? 75 days? 105 days?

27. Three-fourths of a cord of oak wood costs \$6. What is the cost of $\frac{1}{4}$ of a cord? of a cord?

28. Seven-eighths of a yard of cloth cost 42 cents. Find the cost of $\frac{1}{8}$ of a yard; of 1 yard; of $2\frac{1}{6}$ yards.

29. Four-fifths of a load of wood is sold for \$8. Required the cost of $\frac{1}{5}$ of the load; of the whole load; of $4\frac{5}{8}$ such loads.

30. What is the price of a bushel of turnips, when $\frac{3}{4}$ of a bushel are sold for 30 cents?

31. A farmer divided among his 4 sons $\frac{2}{3}$ of his farm. What part of the farm did each son receive?

32. At the rate of \$10 per week, what is the cost of board per day?

33. How many bushels of carrots, at \$$\frac{2}{3}$ per bushel, can be bought for \$$3\frac{1}{3}$?

34. How many cows are $\frac{4}{5}$ of 20 cows? 16 sheep are $\frac{2}{3}$ of how many sheep?

35. Five-sixths of 12 hens are $\frac{5}{9}$ of how many hens?

36. Three-fourths of a cord of wood, at \$7 per cord, will pay for what part of a ton of coal, at \$9 per ton?

37. The captain of a vessel owns $\frac{1}{2}$ of it, the first mate $\frac{3}{8}$, and the captain's wife $\frac{1}{2}$ of the remainder. What part of the vessel does she own?

38. John Rogers sold to Henry Cook $\frac{1}{2}$ of his woodland, and then bought back $\frac{1}{3}$ of what he had sold. What part of the land did each have then?

39. From a bin of potatoes containing 30 bushels, $5\frac{1}{2}$ bushels, $2\frac{1}{2}$ bushels, $4\frac{3}{4}$ bushels were sold. How many bushels were left in the bin?

40. A bushel of wheat weighs 60 pounds. If a miller takes 3 pounds from each bushel for toll, what part of a bushel does he take?

41. At 20 cents per yard, how many yards of ribbon can be bought for \$2.20?

42. Four-fifths of \$20 is $\frac{2}{3}$ of how much money?

43. Two-thirds of a yard of silk can be bought for \$$\frac{3}{4}$. What is the price per yard? How many yards can be bought for \$$3\frac{3}{8}$?

44. If 5 bushels of oats cost \$2, what will be the cost of 9 bushels at the same rate?

45. If \$$1\frac{1}{2}$ are paid for $\frac{3}{4}$ of a yard of velvet, what will be the cost of $\frac{5}{8}$ of a yard?

46. If $\frac{2}{3}$ of the distance between two towns is $6\frac{4}{5}$ miles, what is the whole distance?

47. If $2\frac{3}{4}$ bushels of apples make a barrel, how many barrels will 11 bushels make?

48. A carpet dealer sold $\frac{2}{3}$ of $\frac{1}{2}$ of a roll of carpet. What part of the roll was left?

49. How many pigs can be bought for \$20, at \$$2\frac{1}{2}$ each?

50. Four quarts make a gallon. When $2\frac{1}{2}$ quarts have been taken from a gallon of vinegar, what part of the gallon has been taken?

51. A drover puts $\frac{1}{2}$ of his cattle in a field, $\frac{2}{5}$ of them in another field, and 10 in a barn. How many cattle has he?

52. A merchant sold $\frac{1}{3}$ of a chest of tea, then $\frac{1}{2}$ of it, and took the rest home. If he took home 12 pounds, how many pounds were there in the chest at first?

53. A cistern has two pipes. By one pipe, 3 gallons of water run into the cistern in a minute, and by the other, 5 gallons run out in a minute. If the cistern contains 42 gallons, and both pipes are open, in how many minutes will it be emptied?

54. A man can perform a certain piece of work in 4 hours, and a boy can do the same work in 6 hours. What part of the work can the man do in 1 hour? What part can the boy do? What part can both together do? How many hours will it take both together to do the work?

55. C can plant an acre of corn in 6 hours, C and D together in 4 hours. What part of an acre can C and D together plant in 1 hour? What part can C plant in 1 hour? What part then can D plant in 1 hour? How many hours will it take D to plant the acre of corn?

56. A fox is 90 rods in advance of a greyhound. The fox runs 60 rods a minute, the greyhound 65. In how many minutes will the fox be overtaken?

57. By selling cigars at $7 a hundred $\frac{3}{11}$ of their cost is gained. Find the price per hundred at which they must be sold, in order to gain $\frac{3}{5}$ of their cost.

58. By selling a farm for $2400, the owner lost $\frac{1}{5}$ of what it cost him. How much did he pay for the farm?

59. How many flowers can be planted along the borders of a flower-bed 12 feet long and 10 feet wide, if the flowers are $\frac{1}{2}$ of a foot apart?

Ex. 97.

1. Reduce to simple fractions, $8\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{7}{8}$, $\dfrac{9\frac{1}{6}}{\frac{1}{2} \text{ of } 7}$, $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2} \text{ of } \frac{2}{3}}$.

2. Find the values of

   $169 - 14\frac{3}{7}$; $1\frac{9}{11} - \frac{1}{3}$ of 4; $76\frac{1}{4} - \frac{2}{3}$ of 19.

3. Six pieces of cloth measure respectively $23\frac{1}{2}$ yards, $19\frac{2}{3}$ yards, $21\frac{5}{6}$ yards, $24\frac{7}{8}$ yards, $35\frac{11}{12}$ yards, $18\frac{3}{4}$ yards. After $39\frac{5}{24}$ yards have been sold from their sum, how much remains?

4. The remainder being 4, the quotient 51, and the divisor 25, it is required to find the dividend.

5. Find the value of $\frac{3}{8}$ of a chest of tea weighing $57\frac{1}{2}$ pounds, at $\$1\frac{1}{2}$ per pound.

6. If a man work $8\frac{1}{2}$ hours in a day he can finish a piece of work in $12\frac{1}{2}$ days. How many hours per day must he work to complete it in $10\frac{5}{8}$ days?

7. A confectioner sells $\frac{2}{3}$ of $\frac{1}{2}$ of a bushel of walnuts. What part of the bushel remains, and what will it bring at 15 cents per quart?

8. What is the value of a basket of 588 eggs, worth 25 cents per dozen?

9. A man starts on a journey 5 hours before the mail coach. How many miles will the coach be ahead of the man after it has run for 12 hours, supposing that he travels at the rate of 3 miles an hour, and the coach 10 miles an hour?

10. **If $\frac{5}{7}$ of $\frac{3}{8}$ of a piece of land cost \$420, what is the value of the whole?**

11. A farmer sold at market 15 sheep at \$$2\frac{5}{8}$ each, and bought 7 yards of cloth at \$$1\frac{4}{5}$ per yard. How much money did he take home?

12. A man walked a distance of 60 miles; for the first 5 hours, at the rate of 3 miles an hour, and during the remainder of the journey he walked at the rate of 4 miles an hour. In how many hours did he complete the journey?

13. The circumference of a fore wheel of a wagon is $6\frac{2}{3}$ feet; that of the hind wheel $8\frac{4}{5}$ feet. In a distance of 20 miles, of 5280 feet each, how many more turns will be made by the former than by the latter?

14. A young man received \$1200 from his father. He spent $\frac{1}{5}$ of the money for clothes, $\frac{1}{2}$ of it in travelling, and invested the remainder in a mortgage. What fraction of the whole was the sum invested?

15. A baker paid \$32 for $\frac{4}{7}$ of a hogshead of molasses. What was the value of $\frac{1}{3}$ of the remainder?

16. A gentleman paid \$125 for keeping 2 horses 12 weeks. What would it cost, at the same rate, to keep one horse $\frac{2}{3}$ of a week?

17. If $\frac{5}{9}$ of a yard of ribbon cost \$$\frac{7}{8}$, what will be the value of $5\frac{2}{3}$ yards?

18. Reduce $\frac{4}{5}$, $\frac{7}{8}$, $\frac{9}{18}$ to decimal fractions, and add the results.

19. The contents of a chest of tea weighing 87.5 pounds are made up into 1 pound, $\frac{1}{2}$ pound, $\frac{1}{4}$ pound packages, an equal number of each. How many packages of each kind?

20. In five successive days a farmer puts into his bin $37\frac{7}{8}$ bushels of potatoes, and on each of these days he sells $19\frac{2}{3}$ bushels. How many bushels have been put into the bin? How many more are in the bin at the end than at the beginning of the five days?

21. A man's weekly income is $\$18\frac{1}{2}$, and his weekly expenses are $\$23\frac{2}{3}$. If he have $\$75\frac{1}{4}$ in reserve, how many weeks can he live without incurring debt?

22. By a leak $87\frac{5}{9}$ barrels of water enter the hold of a boat in 1 hour; the pumps will discharge $58\frac{2}{3}$ barrels in an hour. If she can carry only 875 barrels, in how many hours will she sink?

23. A can mow a field in 10 days, B in 8 days, and C in 5 days. When working together, how many days will they need?

24. A carpenter alone can build a shop in 15 days, and with the help of his son he can build it in 10 days. In how many days will the son alone build the shop?

25. Wales Edwards and George Peters hire a pasture for $14. Edwards puts in 8 horses; Peters puts in 50 sheep. If 21 sheep will eat as much as 2 horses, what must each pay?

26. A flour dealer bought 125 barrels of flour at $\$6\frac{2}{3}$. He sold 97 barrels at $\$7\frac{3}{4}$, and the remainder, being injured, brought only $\$5\frac{4}{5}$. What did he gain?

27. A lady bought $\frac{2}{3}$ of $\frac{4}{5}$ of a yard of ribbon for $\$1\frac{3}{7}$. What was the cost per yard?

28. From two fields 482 bushels of corn are gathered. The first field yields $\frac{1}{2}$ as much as the second. How many bushels does each field yield?

29. A farmer brought to market 3 jars of butter, weighing 26 pounds, 37 pounds, 19 pounds. The empty jars weighed $3\frac{1}{3}$ pounds, $4\frac{1}{2}$ pounds, $5\frac{5}{8}$ pounds. The butter brought \$30. What was the price per pound?

30. From 120 acres of land $32\frac{1}{5}$ acres are sold to one man, and $\frac{1}{3}$ of the remainder to another. How many acres are unsold?

31. If the rent of 3 acres of land for $\frac{2}{3}$ of a year be \$9, what will be the rent of 45 acres for 1 year?

32. If $\frac{5}{9}$ of a ton of coal cost \$4, how many tons can be bought for \$$145\frac{3}{4}$?

33. If 12 horses eat $65\frac{2}{3}$ bushels of oats in 3 months, how many bushels will 7 horses eat in 2 years?

34. The agent for a line of steamers sells $\frac{1}{3}$ of a steamship to one company, $\frac{1}{4}$ of the remainder to a second, and $\frac{1}{2}$ of what is left to a third. What part of the whole ship has the third company?

35. A farmer exchanged 13 loads of oats, of 18 bags each, every bag containing $2\frac{1}{2}$ bushels, for 150 sheep, at \$2.925. What was the price of the oats per bushel?

36. Two men 95.784 miles apart approached each other until they met. One travelled 7.476 miles more than the other. How many miles did each travel?

37. A teacher spent $\frac{3}{5}$ of his salary in board for himself and family, and $\frac{1}{10}$ of it in clothing for himself. The clothing of his wife and child cost $\frac{7}{8}$ as much as his own. At the end of the year \$187 remained. What was the salary?

38. A road to the top of a hill has a rise of $\frac{1}{6}$ of a foot in 100 feet. How many feet is the total elevation of the hill, if the length of the road is 2 miles?

39. A man bequeathes to his wife $\frac{1}{3}$ of his estate; to his daughter, $\frac{1}{5}$ of it; to his son, $\frac{1}{2}$ of the daughter's share; he divides the remainder equally between a hospital and a public library. What part is received by the hospital?

40. If the above estate is worth $150,784, what is the amount received by the hospital?

41. A can build a wall in 7 days, B in 6 days, and C in 5 days. A and B worked together for 2 days, when they were joined by C. How many days will they need to complete the remainder of the work?

42. Find the cost of 75,849 bricks, at $9.75 per M.

43. A lumberman exchanged 50,495 feet of round timber, at $$4\frac{1}{2}$ per M, for pork, at $$20\frac{3}{4}$ per barrel. How many barrels of pork did he receive?

44. For $\frac{7}{8}$ of a bushel of apples $$\frac{3}{4}$ are paid. What will $4\frac{5}{9}$ bushels be worth?

45. Henry Jones bought at a saw-mill 3485 ft. boards, at $7.50 per M; 9872 feet laths, at $0.25 per C; 6492 feet flooring, at $$8\frac{3}{4}$ per M; 8975 feet cherry boards, at $15.05 per M. He paid $152.75 in cash, and the balance in flour, at $9.25 per barrel. Required the number of barrels of flour.

46. A merchant mixed 7 pounds of black tea at 68 cents with 9 pounds of green tea at 75 cents. At what price per pound must he sell the mixture to gain $3.69?

47. Nine men working 10 hours per day will harvest a piece of grain in 8 days. How many days will be needed for the same work by 6 men working 9 hours per day?

48. At \$8.75 per M, how many bricks can be bought for \$393.75?

49. When 1000 bricks cost \$7.20, what is the cost of a single brick?

50. If \$437.645 be paid for 6500 feet of rosewood, what is the cost per M?

51. A sea captain who owned $\frac{3}{5}$ of a ship and cargo, gave to his wife $\frac{1}{3}$ of his share, to his daughter $\frac{1}{2}$ of what his wife received, to his son $\frac{2}{3}$ of the remainder, and equally divided what was still left between two nieces. What part of the whole had each niece?

52. Peter Knowlton sold a farm for \$9786, which was $\frac{5}{8}$ of the sum paid for it. Required the original cost of the farm.

53. A merchant bought a bag of coffee, containing 60 pounds, for \$15. At what advance must he sell it per pound to buy 3 yards of velvet at \$3 per yard with the gain on the coffee?

54. After selling $\frac{2}{3}$ of his sheep to a drover, and $\frac{1}{3}$ of the remainder to his neighbor, a farmer has 150 left. How many were there in the flock at first?

55. A stock broker bought 9 shares in the Northern Pacific Railroad, at \$99$\frac{1}{2}$, and 12 shares in the Illinois Central Railroad, at \$102$\frac{1}{2}$. He sold them all at \$103$\frac{1}{4}$. How much did he gain?

56. A bankrupt's available property can be sold for \$19,780, which will pay $62\frac{1}{2}$ cents on every dollar he owes. How much does he owe?

57. A loaf of bread weighing 2 pounds, when flour is worth \$9.80 per barrel, is sold for 10 cents. What should it bring when flour is worth \$7.84?

58. Divide 0.75 of $17\frac{5}{8}$ by $\frac{4}{5}$ of 0.035.

59. An army of 7844 men has 490,250 pounds of beef. If for every man $1\frac{1}{4}$ pounds daily be allowed, in how many days will the beef be consumed?

60. A seedsman bought $37\frac{2}{3}$ bushels of lawn grass-seed for \$226. He sold 25 bushels at a profit of $\$1\frac{2}{5}$ per bushel. For what price per bushel must he sell the remainder to make his whole gain \$73?

61. The cost of 50 gallons of molasses is \$25. By leakage $\frac{1}{5}$ of it is lost; 20 gallons are sold at $62\frac{1}{2}$ cents. At what rate must the remainder be sold to gain \$5 on the whole?

62. For $\frac{2}{3}$ of a yard of broadcloth at $\$6\frac{1}{2}$ per yard, $1\frac{3}{4}$ yards of cassimere and 50 cents in money were given in exchange. What was the price per yard of the cassimere?

63. A owns $\frac{3}{5}$ of a ship and cargo worth \$25,748, B $\frac{1}{4}$ of the remainder, C $\frac{1}{8}$ of the amount belonging to A and B, and D owns what is still left. Required the amount of D's share?

64. A farmer gives to his eldest son $\frac{12}{25}$ of a farm, and the remainder to his daughter. The difference between their shares is 780 acres. How many acres does the daughter receive?

65. If 1200 pounds can be carried 36 miles for \$14, how many pounds can be carried 24 miles for the same money?

66. When $2\frac{1}{2}$ acres of land cost \$500, what will be paid for 460 acres?

67. Four and four-sevenths tons of cannel coal cost \$64. Required the cost of $13\frac{5}{7}$ tons.

68. Of a certain estate $\frac{1}{5}$ is pasture, $\frac{3}{8}$ land suitable for cultivation, and the remainder, woodland, is 50 acres. How many acres in the estate?

69. If 1.4 bushels of walnuts cost \$1.50, find the value of 7 bushels.

70. A man on the average breathes 17 times in a minute, and takes in at each breath about $\frac{5}{7}$ of a quart of air. How many quarts of air are needed for a man in 1 hour?

71. If the crop of potatoes from an acre is on the average 255 bushels, but the potato beetle destroys $\frac{2}{5}$ of the crop, how many bushels may be expected from $3\frac{2}{7}$ acres?

72. If a miller takes $\frac{1}{16}$ for toll, and a bushel of wheat produces 40 pounds of flour, how many bushels must be carried to the mill to obtain 196 pounds of flour?

73. An expressman carried 100 vases, on the condition that he was to receive $\frac{1}{4}$ of a dollar for every one he carried without breaking, and pay $1\frac{1}{4}$ dollars for every one he broke. He received 16 dollars. How many did he break?

74. A man who rows 4 miles an hour in still water takes $1\frac{1}{5}$ hours to row 4 miles up a river. How many minutes will it take him to row 4 miles down the river?

CHAPTER IX.

COMPOUND QUANTITIES.

142. A quantity expressed with reference to a *single unit* is called a **simple quantity**; but a quantity expressed with reference to *different units* is called a **compound quantity.**

Thus, 20¼ pounds is a simple quantity, but 20 pounds 4 ounces is a compound quantity.

143. The process of changing the *unit* in which a quantity is expressed, without changing the *value* of the quantity, is called **reduction.**

144. If the change be from a higher denomination to a lower, it is called **reduction descending**; if from a lower to a higher, it is called **reduction ascending.**

Thus, 1 yard = 36 inches is an example of reduction descending; and 24 inches = 2 feet is an example of reduction ascending.

UNITS OF LENGTH.

145.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
5½ yards, or 16½ feet,	= 1 rod (rd.).
320 rods, 1760 yards, or 5280 feet,	= 1 mile (mi.).

NOTE. A line = $\frac{1}{12}$ in.; a barleycorn = $\frac{1}{3}$ in.; a hand (used in measuring the height of horses) = 4 in.; a palm = 3 in.; a span = 9 in.; a cubit = 18 in.; a military pace = 2½ ft.; a chain = 4 rds.; a link = $\frac{1}{100}$ chain; a furlong = $\frac{1}{8}$ mi.; a knot (used in navigation) = 6086 ft.; a nautical league = 3 knots; a fathom (used in measuring depths at sea) = 6 ft.; a cable length = 120 fathoms.

Ex. 98. (*Oral.*)

1. How many inches in 1 yd.? in $\frac{1}{2}$ yd.? in $\frac{1}{4}$ yd.?
2. How many yards in 180 in.? in 48 in.? in 45 in.?
3. How many yards in 3 rds.? in 4 rds.? in 5 rds.?
4. How many feet in 2 yds.? in 2 rds.? in 2 rds. 2 yds.?
5. How many rods in 33 ft.? How many yards in 33 ft.?
6. In $\frac{1}{4}$ mi. how many rods? yards? feet?
7. How many rods in 0.4 of a mile? in 0.3? in 0.7?
8. What part of a mile are 160 rds.? 80 rds.? 40 rds.?
9. What part of a foot are 4 in.? 3 in.? 6 in.? 8 in.?
10. What part of a yard are 2 ft.? 1 ft. 6 in.? 2 ft. 6 in.?

Reduction Descending.

146. Change 10 mi. 40 rds. to feet.

```
10 mi. 40 rds.
 320
3200
  40
3240
 16½
 1620
19440
3240
53460
```

10×320 rds. = 3200 rds., to which the 40 rds. are added.

Again, $3240 \times 16\frac{1}{2}$ ft. = 53,460 ft.

The multiplicand and multiplier are interchanged in the operation.

Ex. 99.

Reduce to feet:

1. 3 mi. 5 yds. 2 ft.
2. 40 mi. 5 rds. $2\frac{1}{2}$ yds.
3. 2 mi. 52 rds. 1 ft.

Reduce to inches:

4. 18 mi. 252 rds. 2 yds.
5. 11 mi. 6 rds. 4 yds.
6. 18 mi. 230 rds. 8 ft.

7. 2 yds. 1 ft. 9 in.	**10.** 8 mi. 96 rds. 4 yds.
8. 5 yds. 2 ft. 7 in.	**11.** 2 mi. 80 rds. 2 ft.
9. 170 rds. 3 yds. 9 in.	**12.** 200 rds. 115 yds. 5 in.

Reduction Ascending.

147. Change 53,463 ft. to a compound quantity.

$$\begin{array}{rl}
16\frac{1}{2})\,53463 & \text{ft.} \\
2 & \\
\hline
33)\,106926 & \ldots\ldots \text{ half-feet.} \\
320)\,3240 & \text{rds.}\ \ldots\ 6 \text{ half-feet} = 3 \text{ ft.} \\
\hline
10 & \text{mi.}\ \ldots\ 40 \text{ rds.}
\end{array}$$

10 mi. 40 rds. 3 ft. *Ans.*

There are $16\frac{1}{2}$ ft., or 33 half-feet, in a rod; so the 53,463 ft. are changed to half-feet, and the half-feet to rods, by dividing by 33. The remainder is 6 half-feet = 3 ft.

3240 rds. are changed to miles by dividing by 320, the number of rods in a mile. The remainder is 40 rds.

Reduce 376,985 in. to higher denominations.

$$\begin{array}{r|rl}
12 & 376985 & \text{in.} \\
3 & 31415 & \text{ft.}\ \ldots\ 5 \text{ in.} \\
5\frac{1}{2} & 10471 & \text{yds.}\ \ldots\ 2 \text{ ft.} \\
2 & 2 & \\
\hline
11 & 20942 & \text{half-yards.} \\
320 & 1903 & \text{rds.}\ \ldots\ 9 \text{ half-yards} = 4\frac{1}{2} \text{ yds.} \\
 & 5 & \text{mi.}\ \ldots\ 303 \text{ rds.}
\end{array}$$

The $\frac{1}{2}$ yd. of the $4\frac{1}{2}$ yds. should be reduced to lower denominations, and the result, 1 ft. 6 in., added to the 2 ft. 5 in. Thus,

mi.	rds.	yds.	ft.	in.
5	303	4	2	5
			1	6
5	303	5	0	11

5 mi. 303 rds. 5 yds. 0 ft. 11 in. *Ans.*

Ex. 100.

Reduce to higher denominations:

1. 211 in.	5. 125,899 in.	9. 348,164 in.
2. 33,777 in.	6. 179,875 in.	10. 247,391 in.
3. 142,737 in.	7. 87,476 ft.	11. 99,204 ft.
4. 33,000 ft.	8. 97,378 yds.	12. 11,220 ft.

Compound Addition and Subtraction.

148. Add:

mi.	rds.	yds.	ft.	in.
6	120	3	2	2
18	15	1	1	6
3	215	2	2	8
28	31	$2\frac{1}{2}$	0	4
		$\frac{1}{2}=$	1	6
28	31	2	1	10

28 mi. 31 rds. 2 yds. 1 ft. 10 in. *Ans.*

Write the numbers so that units of the same denomination shall be in the same column. The sum of the inches is 16. Divide the 16 in. by 12 (12 in. = 1 ft.). The result is 1 ft. 4 in. Write 4 under the column of inches, and add 1 to the column of feet.

The sum of the feet, including the 1 ft. from the 16 in., is 6. Divide by 3 (3 ft. = 1 yd.). The result is 2 yds. 0 ft. Write 0 under the column of feet, and add 2 to the yards.

The sum of the yards, including the 2 yds. from the 6 ft., is 8. Divide by $5\frac{1}{2}$ ($5\frac{1}{2}$ yds. = 1 rd.). The result is 1 rd. $2\frac{1}{2}$ yds. Write $2\frac{1}{2}$ under the column of yards, and add 1 to the rods.

The sum of the rods, including the 1 rd. from the 8 yds., is 351. Divide by 320 (320 rds. = 1 mi.). The result is 1 mi. 31 rds. Write 31 under the column of rods, and add 1 to the miles.

The sum of the miles, including the 1 mi. from the 351 rds., is 28. The $\frac{1}{2}$ yd. is changed to 1 ft. 6 in., and added to 0 ft. 4 in.

149. Take 4 mi. 110 rds. 5 yds. 2 ft. from 6 mi. 25 rds. 4 yds. 2 ft.

mi.	rds.	yds.	ft.	
6	25	4	2	
4	110	5	1	
1	234	$4\frac{1}{2}$	1	
		$\frac{1}{2}$	=1	6 in.
1	234	4	2	6

1 mi. 234 rds. 4 yds. 2 ft. 6 in. *Ans.*

Write the numbers so that units of the same denomination shall be in the same column.

Since 5 yds. are more than 4 yds., 1 rd. is reduced to yards, and the result added to 4 yds., making $9\frac{1}{2}$ yds. Then $9\frac{1}{2}$ yds. $-$ 5 yds. $= 4\frac{1}{2}$ yds., which is written under the column of yards.

Since the minuend has been increased by 1 rd., 1 rd. must be added to the 110 rds. of the subtrahend. (See § 48, page 37.)

Since 111 rds. are more than 25 rds., 1 mi. is reduced to rods, and the result added to 25 rds., making 345 rds. Then 345 rds. $-$ 111 rds. $=$ 234 rds.

The 234 is written under the column of rods. The 4 mi. are increased by 1 mi., and the result taken from 6 mi. The $\frac{1}{2}$ yd. is changed to 1 ft. 6 in.

Ex. 101.

Add:

1.

yds.	ft.	in.
15	1	7
23	2	9
35	0	6
7	2	11

2.

rds.	yds.	ft.
23	3	1
18	4	2
27	0	2
6	4	0

3.

mi.	rds.	yds.
17	23	4
9	17	2
23	0	3
11	35	1

4.

mi.	rds.	yds.
37	14	2
28	16	2
19	10	4
10	56	3

5.

mi.	rds.	ft.
23	119	15
19	173	11
8	65	12
32	147	8

6.

mi.	rds.	ft.	in.
7	95	8	9
8	96	7	8
3	98	9	9
6	87	8	7

Find the difference between :

	yds.	ft.	in.		rds.	yds.	ft.		mi.	rds.	ft.
7.	14	1	4	8.	22	2	0	9.	23	76	1
	10	2	11		19	3	2		6	157	2

	mi.	rds.	ft.	in.		mi.	rds.	yds.	ft.		mi.	rds.	yds.
10.	17	125	1	10	11.	7	0	0	0	12.	13	33	2
	8	187	2	11		3	64	3	2		9	32	4

Compound Multiplication and Division.

150. Multiply 37 yds. 2 ft. 11 in. by 4.

yds.	ft.	in.
37	2	11
		4
151	2	8

4×11 in. = 44 in. = 3 ft. 8 in. Write the 8 in. under the column of inches.

4×2 ft. = 8 ft.; 8 ft. with the 3 ft. added are 11 ft. = 3 yds. 2 ft. Write the 2 ft. under the column of feet.

4×37 yds. = 148 yds.; and 148 yds. with the 3 yds. added = 151 yds.

151 yds. 2 ft. 8 in. *Ans.*

NOTE. When the multiplier is the product of two factors, multiply by one of the factors, and the resulting product by the other.

151. Divide 121 yds. 2 ft. by 73.

```
     yds.  ft.
73)  121   2(1 yd. 2 ft.
      73
      48
       3
     144
       2
     146
     146
```

The remainder from dividing 121 yds. by 73 is 48 yds., which are reduced to feet by multiplying by 3 (3 ft. = 1 yd.). The result with the 2 ft. added is 146 feet.

There is no remainder from dividing 146 ft. by 73.

1 yd. 2 ft. *Ans.*

Divide 10 ft. 11 in. by 2 ft. 8 in.

Reduce both quantities to inches.

10 ft. 11 in. = 131 in. $\frac{131}{32} = 4\frac{3}{32}$.
2 ft. 8 in. = 32 in.

$4\frac{3}{32}$. *Ans.*

Ex. 102.

1. Multiply 33 yds. 2 ft. 11 in. by 17.
2. Multiply 23 rds. 3 yds. 2 ft. by 100.
3. Divide 15 yds. 1 ft. 9 in. by 3.
4. Divide 289 yds. 2 ft. 9 in. by 213.
5. Divide 150 mi. 178 rds. 3 yds. by 9.
6. Multiply 3 mi. 72 rds. 9 ft. by 11.
7. Multiply 150 rds. 2 yds. 1 ft. by 235.
8. Divide 33 mi. 40 rds. by 200.
9. Divide 200 mi. 56 rds. 3 yds. 2 ft. by 121.
10. Multiply 11 mi. 200 rds. by 14.
11. Multiply 52 mi. 1021 yds. by 47.
12. Divide 43 mi. 280 rds. by 24.

Fractions of Simple and Compound Quantities.

152. Express $\frac{2}{3}$ of a mile in rods, feet, and inches.

$\frac{2}{3}$ mi. $= \frac{2}{3}$ of 320 rds. $= 213\frac{1}{3}$ rds.

$\frac{1}{3}$ rd. $= \frac{1}{3}$ of $16\frac{1}{2}$ ft. $= 5\frac{1}{2}$ ft.

$\frac{1}{2}$ ft. $= \frac{1}{2}$ of 12 in. $= 6$ in.

213 rds. 5 ft. 6 in. *Ans.*

Express 0.6275 of a mile in rods, feet, and inches.

$$\begin{array}{r l}
0.6275 & \\
\underline{320} & \quad 0.6275 \text{ mi.} = 0.6275 \text{ of } 320 \text{ rds.} = 200.8 \text{ rds.} \\
12\,5500 & \quad 0.8 \text{ rd.} = 0.8 \text{ of } 16\tfrac{1}{2} \text{ ft.} = 13.2 \text{ ft.} \\
\underline{188\,25} & \quad 0.2 \text{ ft.} = 0.2 \text{ of } 12 \text{ in.} = 2.4 \text{ in.} \\
200.8 \text{ rds.} & \\
\underline{16\tfrac{1}{2}} & \\
13.2 \text{ ft.} & \\
\underline{12} & \\
2.4 \text{ in.} &
\end{array}$$

200 rds. 13 ft. 2.4 in. *Ans.*

Find the value of $\frac{5}{9}$ of 3 rds. 14 ft. 7 in.

$$\begin{array}{r r r r}
 & \text{rds.} & \text{ft.} & \text{in.} \\
 & 3 & 14 & 7 \\
 & & & \underline{5} \\
9) & 19 & 6 & 11 \\
 & 2 & 2 & 7\frac{4}{9}
\end{array}$$

Here we multiply by the numerator of the fraction, and divide the product by the denominator.

2 rds. 2 ft. $7\frac{4}{9}$ in. *Ans.*

NOTE. When the multiplier is a mixed number, multiply by the integer and the fraction separately, and add the resulting products.

Ex. 103.

Fine the value of:

1. $\frac{5}{9}$ of a mile.
2. $\frac{2}{7}$ of a mile.
3. $\frac{5}{9}$ mi. − $\frac{5}{7}$ rd.
4. $\frac{3}{16}$ mi. + $\frac{2}{3}$ of 40 rds. + $\frac{3}{8}$ yd.
5. 0.475 of a mile.
6. 0.3975 of a mile.
7. 0.01284 of 14 miles.
8. 3.726 mi. − 33.57 rds.
9. Find $\frac{3}{5}$ of 5 mi. 89 rds. 3 yds. 2 ft.
10. Take $\frac{3}{5}$ of 4 mi. from $\frac{7}{8}$ of 3 mi. 18 rds. 3 yds. 2 ft.
11. Add 0.525 mi., 0.125 rd., 0.5 yd., 0.16 ft.

To express one Quantity as the Fraction of another.

153. Express 145 rds. 2 yds. 1 ft. 6 in. as the fraction of a mile.

$$6 \text{ in.} = \tfrac{6}{12} \text{ ft.} = \tfrac{1}{2} \text{ ft.}$$

$$1\tfrac{1}{2} \text{ ft.} = \frac{1\frac{1}{2}}{3} \text{ yds.} = \tfrac{1}{2} \text{ yd.}$$

$$2\tfrac{1}{2} \text{ yds.} = \frac{2\frac{1}{2}}{5\frac{1}{2}} \text{ rds.} = \tfrac{5}{11} \text{ rd.}$$

$$145\tfrac{5}{11} \text{ rds.} = \frac{145\frac{5}{11}}{320} = \frac{1600}{320 \times 11} \text{ mi.} = \tfrac{5}{11} \text{ mi.}$$

$\tfrac{5}{11}$ of a mile. *Ans.*

Express 120 rds. 3 yds. 1 ft. 6.72 in. as the decimal of a mile.

12	6.72 in.
3	1.56 ft.
5.5	3.52 yds.
320	120.64 rds.
	0.377 mi.

6.72 in. ÷ 12 = 0.56 ft., and this added to the 1 ft. gives 1.56 ft. 1.56 ft. ÷ 3 = 0.52 yds., and this added to 3 yds. gives 3.52 yds. 3.52 yds. ÷ 5.5 gives 0.64 rds., and this added to 120 rds. gives 120.64 rds. 120.64 rds. ÷ 320 gives 0.377 mi.

0.377 of a mile. *Ans.*

Note. The quotient in any case need not be carried beyond the *fifth* decimal place, and the required answer will be sufficiently accurate for all practical purposes.

154. Express 1 yd. 2 ft. 3 in. as the fraction of 5 yds. 1 ft. 3 in.

1 yd. 2 ft. 3 in.:

$$3 \text{ in.} = \tfrac{3}{12} \text{ ft.} = \tfrac{1}{4} \text{ ft.}$$

$$2\tfrac{1}{4} \text{ ft.} = \frac{2\frac{1}{4}}{3} \text{ yd.} = \tfrac{3}{4} \text{ yd.}$$

$1\tfrac{3}{4}$ yds.

5 yds. 1 ft. 3 in.

$$3 \text{ in.} = \tfrac{3}{12} \text{ ft.} = \tfrac{1}{4} \text{ ft.}$$

$$1\tfrac{1}{4} \text{ ft.} = \frac{1\frac{1}{4}}{3} \text{ yd.} = \tfrac{5}{12} \text{ yd.}$$

$5\tfrac{5}{12}$ yds.

$$\frac{1\frac{3}{4}}{5\frac{5}{12}} = \tfrac{21}{65}.$$

$\tfrac{21}{65}$. *Ans.*

NOTE. If the answer to the last problem is to be expressed as a decimal fraction, first find the answer as a common fraction, and reduce this common fraction to a decimal fraction.

Ex. 104.

Express:

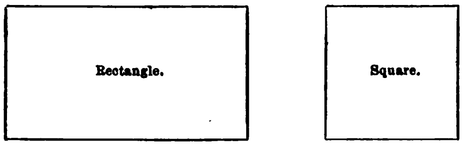
1. 125 rds. 4 yds. 2 ft. 6 in. as the fraction of a mile.
2. 1 yd. 2 ft. 3 in. as the fraction of 5 yds.
3. 51 rds. 1 yd. 3.6 in. as the decimal of a mile.
4. $\frac{1}{2}$ rd. + $\frac{1}{4}$ yd. as the fraction of a mile.
5. 3 mi. 53 rds. 4 yds. 1.2 ft. as the decimal of 5 mi. 89 rds. 3 yds. 2 ft.
6. 2 mi. 138 rds. 1 yd. as the fraction of 3 mi. 265 rds. $3\frac{1}{2}$ yds.
7. 233 rds. 9 ft. 10.8 in. as the decimal of a mile.
8. 3 mi. 242 rds. $2\frac{3}{4}$ yds. as the decimal of 7 mi. 160 rds.
9. 2 ft. $7\frac{1}{2}$ in. as the decimal of 100 yds.
10. 11 rds. 4 yds. $4\frac{1}{2}$ in. as the fraction of a mile.
11. $\frac{7}{8}$ rd. + $\frac{3}{8}$ yd. + $\frac{3}{16}$ ft. as the fraction of a rod.
12. 195 yds. 1 ft. 8 in. as the fraction of $\frac{1}{8}$ of a mile.
13. 1 mi. 232 rds. 4 yds. 1 ft. 6 in. as the fraction of 8 mi. 204 rds. 0 yd. 1 ft. 6 in.
14. 127 rds. 3 ft. 3.6 in. as the decimal of a mile.
15. 261 rds. 4 yds. 1 ft. 6 in. as the fraction of a mile.
16. $\frac{4}{5}$ of the difference between 3 yds. 2 ft. 11 in. and 10 yds. 7 in. as the fraction of 16 yds.
17. 7 rds. 1 ft. 3.17 in. as the decimal of 76 rds. 2 yds. 5 in.
18. 248 rds. 4 yds. 2 ft. 8 in. as the fraction of 2 mi.

Measures of Surface.

155. A surface has two dimensions, *length* and *breadth*.

156. If a surface is flat and has four square corners, it is called a **rectangle**.

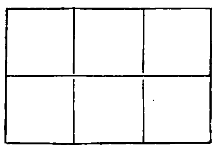
157. If a rectangle has its four sides equal, it is called a **square**.



158. The unit of surface is a square each side of which is a linear unit.

159. The area of a surface is the number of square units it contains.

160. Suppose the rectangle in the margin is 3 in. long and 2 in. wide. If lines are drawn as represented in the figure, the surface will be divided into **square inches**. There will be 2 horizontal rows of 3 square inches each; that is, in all, 2×3 square inches. Hence,



Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express its area in **square units** *of the same name as the linear unit of the sides.*

Conversely, the number of square units in a rectangle divided by the number of linear units in one side will give the number of linear units in its adjacent side.

UNITS OF SURFACE.

161.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards, or $272\frac{1}{4}$ square feet,	= 1 square rod (sq. rd.).
160 square rods, or 10 square chains,	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

A square of flooring or roofing	= 100 sq. ft.
A section of land	= 1 mile square.
A township	= 36 sq. mi.

The units of surface measure are obtained by squaring the units of linear measure. Thus,

$$144 = 12^2;\ 9 = 3^2;\ 30\tfrac{1}{4} = (5\tfrac{1}{2})^2;\ 272\tfrac{1}{4} = (16\tfrac{1}{2})^2$$

Ex. 105. (*Oral.*)

1. How many square feet of surface in a blackboard 4 ft. wide and 9 ft. long?

2. If a slate is 8 in. wide and has a surface of 80 sq. in., what is the length of the slate?

3. How many square inches in $\frac{1}{4}$ of a square foot? in $\frac{3}{4}$? in $\frac{1}{3}$? in $\frac{5}{6}$?

4. How many square feet in 3 sq. yds.? in 5 sq. yds.?

5. How many square inches in a board 4 in. long and 3 in. wide?

6. A square yard of carpet is 3 ft. long and 3 ft. wide; how many feet in it?

7. How many square feet in a yard of carpet 2 ft. wide? $2\frac{1}{2}$ ft. wide?

8. How many square feet in a room 12 ft. by 15 ft.?

9. How many yards of carpet 2 ft. wide will be required to cover the floor of the above room, if the strips run lengthwise of the room?

10. How many square yards in 81 sq. ft.?

11. How many square rods in $\frac{5}{8}$ of an acre?

12. What part of an acre are 40 sq. rds.? 80? 100?

Ex. 106.

1. Reduce 5 A. 147 sq. rds. to square rods.

2. How many square inches in 9 sq. yds. 7 sq. ft.?

3. Reduce 33,796 sq. in. to square yards.

4. Reduce 153 A. 87 sq. rds. to square inches.

5. In 67,413 sq. yds. how many acres?

6. In a rectangular field 49 yds. long and 16 yds. wide, how many square feet?

7. How many tiles 1 ft. square will be needed to pave a hall 20 ft. long and 9 ft. wide?

8. How much greater is the area of a lot 50 rds. square than that of a lot containing 50 sq. rds.?

9. How many square yards in a square lot measuring 142 ft. on a side?

10. Ingrain carpet is 3 ft. wide. How many yards will be required for a room 27 ft. long and 18 ft. wide?

11. From each corner of a square, the side of which is 2 ft. 5 in., a square measuring 5 in. on a side is cut out. Find the area of the remainder of the figure.

12. Find the value of 0.45 of an acre.

13. Reduce $\frac{17}{24}$ of a square mile to lower denominations.

14. Reduce 80 sq. rds. 2.42 sq. yds. to the decimal of an acre.

15. Add $\frac{4}{5}$ of an acre, $\frac{5}{9}$ sq. rd., and $\frac{2}{3}$ sq. yd.

16. Add $\frac{2}{3}$ of an acre and $\frac{4}{9}$ of a square rod.

17. From $\frac{2}{11}$ of a square rod take $\frac{3}{4}$ of a square yard.

18. Find $\frac{5}{7}$ of 9 A. 70 sq. rds. 15 sq. yds. 7 sq. ft. 19 sq. in.

19. A side of Russell Square in London is 660 ft. How many acres does it contain?

20. A garden 76 yds. long and 56 yds. broad, enclosed by a wall, has a border 4 ft. wide within the wall, and within this a path 5 ft. wide, the middle being grass. Find the areas of the border, path, and grass, respectively.

21. Subtract 18 A. 117 sq. rds. $17\frac{1}{2}$ sq. yds. from 367 A. 120 sq. rds.

22. What fraction of 223 sq. yds. 7 sq. ft. 120 sq. in. are 31 sq. yds. 8 sq. ft. 120 sq. in.?

23. How many planks, each 13 ft. long and $10\frac{1}{2}$ in. wide, will be required for the construction of a platform 54 yds. long and 21 yds. broad?

24. The area of a rectangular field is 33 sq. rds. 1 sq. yd. 6 sq. ft. 108 sq. in., and the length is 9 rds. 1 ft. 6 in. What is the width?

Carpeting Rooms.

In determining the number of yards of carpeting required for a room, we first decide whether the strips shall run lengthwise or across the room, and then find the number of strips needed. The number of yards in a strip, including the waste in matching the pattern, multiplied by the number of strips will give the required number of yards.

25. How many yards of carpet $2\frac{1}{4}$ ft. wide will cover a floor 18 ft. long and 15 ft. wide, if the strips run across the room?

26. How many yards of carpeting $\frac{3}{4}$ of a yard wide will be required for a floor 26 ft. long, $15\frac{3}{4}$ ft. wide, if the strips run lengthwise. How many if the strips run across the room? How much will be turned under in each case?

27. How many yards of carpeting $\frac{7}{8}$ of a yard wide will be required for a room $8\frac{1}{2}$ yds. long and 17 ft. wide, if the strips run lengthwise and there is a waste of $\frac{1}{16}$ of a yard in each strip in matching patterns?

28. Find the cost of carpet 30 inches wide, at $1.25 per yard, for a room 18 ft. by 14 ft., if the strips run lengthwise. If the strips run across the room.

29. Find the cost of carpeting $\frac{3}{4}$ of a yard wide, at $2.75 per yard, for a room 34 ft. 8 in. by 13 ft. 3 in., if the strips run lengthwise, and if there be a waste of $\frac{1}{4}$ of a yard on each strip in matching the pattern.

30. Which way must the strips of carpet $\frac{3}{4}$ of a yard wide run in order to carpet most economically a room 20 ft. 6 in. long and 19 ft. 6 in. wide, if there be no waste for matching the pattern?

Papering and Plastering.

The area of the **four walls** of a room is equal to that of a rectangle whose length is equal to the **circuit** of the room, and whose breadth is equal to the **height** of the room.

31. How many yards of plastering in the four walls of a room 14 ft. 3 in. long, 13 ft. 4 in. wide, and 7 ft. high, if no allowance is made for doors and windows?

Hint. 14 ft. 3 in. = $14\frac{1}{4}$ ft., and 13 ft. 4 in. = $13\frac{1}{3}$ ft.

32. Find the yards of plastering in the walls of a room $21\frac{3}{4}$ ft. long, $16\frac{1}{2}$ ft. wide, and 11 ft. high, if 12 sq. yds. be allowed for doors, windows, and base-boards?

33. How many square yards of plastering in the walls and ceiling of a room 30 ft. 8 in. long, 26 ft. 5 in. wide, 10 ft. 6 in. high, if 24 sq. yds. be allowed for doors, windows, and base-boards?

34. What will be the cost of plastering the walls and ceiling of a room 27 ft. 4 in. long, 20 ft. wide, and 12 ft. 6 in. high, at 27 cents per square yard, if 20 sq. yds. be deducted for doors, windows, and base-boards?

35. Find the cost of whitening the ceiling and walls of a room 14 ft. 4 in. wide, 15 ft. 6 in. long, and 10 ft. 6 in. high, at 5 cents per square yard, allowing 9 sq. yds. for doors and windows.

36. Find the cost of papering a room 32 ft. long, 22 ft. wide, 13 ft. high, with paper 18 in. wide, 8 yds. in a roll, at $1.25 a roll, if 50 sq. yds. be allowed for doors, windows, and base-boards.

SURVEYORS' CHAIN.

162. Surveyors use, in measuring distances, a chain 4 rds. long and containing 100 links.

The links can be written as decimals of a chain.

Ex. 107. (*Oral.*)

1. How many chains make a mile?
2. In 10 mi. how many chains?
3. How many rods in 9 chains?
4. How many yards in 1 chain? how many feet?
5. How many chains in 48 rods?
6. In 0.75 of a mile how many chains?
7. How many chains in $\frac{3}{8}$ of a mile?
8. If a square field measures 1 chain on a side, how many square rods does it contain? How many square rods then in a square chain?
9. If a square chain contains 16 sq. rds., how many square chains make an acre?
10. What part of an acre are 4 sq. chains? 5 sq. chains? 8 sq. chains?
11. What part of a chain are 50 links? 25 links? 20 links?
12. How many square rods in a square garden-plot measuring 75 links on a side?
13. How many inches long is a link?
14. The distance between two places is found to be 320 chains. Express the distance in miles.

Ex. 108.

1. Reduce 38 chains 80 links to the decimal of a mile.

2. The four sides of a field are 23 chains 19 links, 17 chains 34 links, 6 chains 85 links, and 24 chains 62 links. How many yards around the field?

3. One field contains 3 sq. chains, and another is 3 chains square. How many acres in both fields together?

4. A field is crossed by a driveway 15 links wide, and 13 chains 43 links long. How many square rods in the driveway?

5. From a field of 4 A. a rectangular piece 3 chains 25 links long and 2 chains 75 links wide is reserved. How much of the field is left?

6. The sides of a triangular field measure $21\frac{1}{2}$ chains, 14 chains 11 links, and 8 chains 10 links respectively. By how many yards is the longest side less than the sum of the other two?

Board Measure.

163. Boards one inch or less in thickness are sold by the square foot.

Boards more than one inch in thickness, and all squared lumber, are sold by the number of square feet of boards one inch in thickness to which they are equivalent.

Thus, a board 16 ft. long, 1 ft. wide, and 1 in. thick, contains 16 ft. *board measure.* If only $\frac{7}{8}$, $\frac{3}{4}$, or $\frac{1}{2}$ of an inch thick, it still contains 16 ft.; but, if $1\frac{1}{4}$ in. thick, it contains 20 ft. board measure.

Ex. 109.

How many feet board measure in :

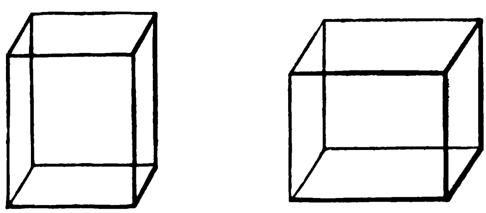
1. A board 18 ft. long, 6 in. wide, $\frac{7}{8}$ in. thick?
2. A board 16 ft. long, 12 in. wide, 1 in. thick?
3. Forty boards 14 ft. long, 10 in. wide, $\frac{1}{2}$ in. thick?
4. A stick of square timber 8 in. by 9 in., and 30 ft. long?
5. Six joists, each 3 in. by 4 in., and 11 ft. long?
6. Ten joists, each 6 in. by 4 in., and 14 ft. long?
7. A stick of square timber 10 in. by 12 in., and 36 ft. long?
8. Ten 2-in. planks, each 13 ft. long, 15 in. wide?
9. Thirty 3-in. planks, each 12 ft. long, 10 in. wide?
10. A board 24 ft. long, 23 in. wide at one end, and 17 in. at the other, and $1\frac{1}{2}$ in. thick?

    HINT. The average width is $\frac{23+17}{2}$ in.

11. In a stick of timber 40 ft. long and 15 in. square?
12. Ten 4-in. planks 16 ft. long and 10 in. wide?

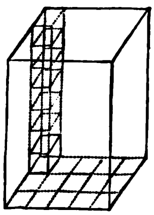
MEASURES OF VOLUME.

164. A rectangular solid is a solid bounded by six rectangles.



165. If the rectangles are all squares, the solid is called a cube.

166. In the figure represented in the margin, let the length contain 5, the breadth 3, and the height 7 in.



The base may be divided into square inches; there will be three rows of 5 sq. in. each; in all 15 sq. in. Upon each square inch may be placed a pile of 7 cu. in., so that the solid will contain 15×7 cu. in.; that is, $3 \times 5 \times 7$ cu. in.

Hence, to find the volume of a rectangular solid,

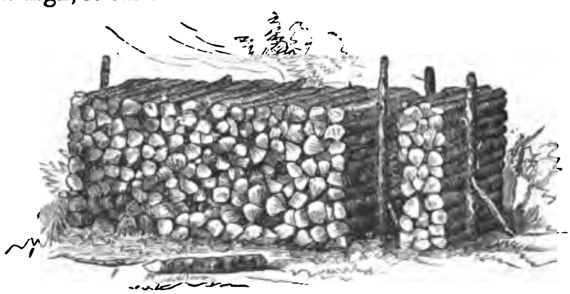
Express its length, breadth, and height in the same linear unit; the product of these numbers will express its volume **in cubic units** *of the same name as the linear unit.*

Units of Volume.

167. 1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

The units of volume are cubes of the linear units. Thus, $1728 = 12^3$, $27 = 3^3$.

168. In measuring wood, a pile 8 ft. long, 4 ft. wide, and 4 ft. high, is called a **cord.**



169. A **cord foot** is one foot in length of such a pile. Hence,

1 cord foot (cd. ft.) = 16 cu. ft.
8 cord feet = 1 cord (cd.).

Ex. 110. (*Oral.*)

1. A brick is 2 by 4 by 8 in. Find its volume.
2. How many cubic feet in 2 cu. yds.?
3. How many cubic yards in 81 cu. ft.?
4. Twelve cubic feet are what part of a cubic yard?
5. How many cords in 40 cd. ft.? in 16 cd. ft.
6. How many cords in a pile of wood 32 ft. long, 4 ft. wide, and 4 ft. high?
7. How many loads of earth, each 1 cu. yd., must be removed in digging a ditch 21 ft. long, 3 ft. wide, and 3 ft. deep?
8. How many cubic feet in a stick of timber 12 in. wide, 9 in. thick, and 24 ft. long?
9. How many feet board measure in a cubic foot?
10. How many cubic feet in a stick of timber 16 in. wide, 9 in. thick, and 21 ft. long?

Ex. 111.

1. Reduce 15 cu. yds. 13 cu. ft. to cubic inches.
2. Reduce 150,000 cu. in. to cubic yards.
3. Subtract 28 cu. yds. 25 cu. ft. 1500 cu. in. from 47 cu. yds. 13 cu. ft. 1236 cu. in.
4. Multiply 17 cu. yds. 17 cu. ft. 187 cu. in. by 11.
5. Divide 22 cu. yds. 10 cu. ft. 933 cu. in. by 7.
6. How many cubic inches can be cut out of a cubic foot?
7. How many cubic feet of water will a cistern hold whose three dimensions are each 4 ft.?

8. How many cubic inches in a rectangular stone post 3 ft. high, 1 ft. wide, and 1 ft. thick?

9. Find the value of 0.975 of a cubic yard.

10. Reduce 13 cu. ft. 864 cu. in. to the decimal of a cubic yard.

11. A man bought 52 cu. yds. 18 cu. ft. 984 cu. in. of stone for the cellar of a house, and $\frac{3}{4}$ as much for the cellar of a second house. How much did he buy for both?

12. How many cords of wood in a pile 50 ft. long, 6 ft. high, and 4 ft. wide?

13. How many cords of wood in a pile 42 ft. long, $6\frac{1}{2}$ ft. high, and 8 ft. wide?

14. What must be the length of a load of wood that is 4 ft. wide, $5\frac{1}{8}$ ft. high, to contain 2 cds.?

15. A cubic foot of wood weighs 20 pounds. Find the weight of 10 boards, each 30 ft. long, 1 ft. wide, and 1 in. thick.

Units of Capacity.

Dry Measure.

170.

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

Liquid Measure.

171.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

$31\frac{1}{2}$ gallons = 1 barrel (bbl.).
2 barrels = 1 hogshead (hhd.).

NOTE. The gallon of liquid measure contains 231 cu. in. The bushel of dry measure contains 2150.42 cu. in. Therefore the quart of liquid measure contains $57\frac{3}{4}$ cu. in., and the quart of dry measure $67\frac{1}{5}$ cu. in.

Ex. 112. (*Oral.*)

1. How many pints in 16 gi.? in 37 gi.?
2. How many pints in 2 qts.? in 7 qts. 1 pt.?
3. How many quarts in 2 pks.? in 3 pks. 3 qts.?
4. How many quarts in 3 bu.?
5. How many baskets holding $2\frac{1}{2}$ pks. each will 10 bu. of apples fill?
6. If a pint of milk cost 4 cts., what will a gallon cost?
7. How many times will a gallon of water fill a half-pint cup?
8. How many pint bottles will be required to hold 5 gals. 2 qts. of cider?
9. If 4 qts. of blueberries cost 32 cts., what will a bushel cost at the same rate?
10. A 2-gal. measure of molasses lacks 3 pts. of being full. What is the molasses worth at 80 cts. a gallon?
11. If a horse eats 4 qts. of corn a day, how many days will a bushel last him?
12. If a quart of berries is worth 10 cts., what is a peck worth?

Ex. 113.

1. Reduce 440 pts. to pecks, and 109 pts. to gallons.
2. Reduce 2024 pts. to bushels.

Add:

	gals.	qts.	pts.		bu.	pks.	qts.		bu.	pks.	qts
3.	13	2	1	4.	5	1	3	5.	17	2	4
	2	3	0		6	1	1		11	3	4
	15	0	0		2	0	0		3	0	0
	7	1	1		3	0	2		18	3	4

Subtract:

	gals.	qts.	pts.		bu.	pks.	qts.		bu.	pks.	qts.
6.	4	1	0	**7.**	56	1	0	**8.**	27	1	1
	3	2	1		27	3	0		18	1	3

9. Multiply 15 gals. 2 qts. 1 pt. by $130\frac{3}{5}$.

10. Divide 34 bu. 3 pks. 4 qts. by 9.

11. How many pint bottles will be required to hold 63 gals. of wine?

12. How many pint packages can a seedsman make from 3 bu. 3 pks. 6 qts. of peas?

13. In one season a market-gardener sold 3758 baskets of strawberries averaging 1 pt. each. How many bushels did he sell?

14. A farmer having a flock of 87 fowls feeds them daily 2 bu. 1 pk. 1 pt. of grain. What is the average amount for each fowl?

15. A lady in one month gave to her 4 canaries $\frac{3}{4}$ of a quart of seed, to her parrot $2\frac{1}{3}$ qts., and to 3 mocking-birds $1\frac{4}{5}$ qts. How much seed did she give to all the birds together?

16. A gardener raised $\frac{4}{5}$ of a bushel of Lima beans, and $\frac{7}{8}$ of a bushel of Caseknife beans. He sold 3 pks. 6 qts. 1 pt.; how many had he left?

17. A merchant receives 37 boxes of oranges, amounting to 25 bu. 3 pks. 7 qts. Only $\frac{5}{7}$ of the fruit was fit to sell; how many bushels had to be thrown away?

18. A tank is 30 ft. 3 in. long, 16 ft. 4 in. wide, and 6 ft. 4 in. deep. Find how many gallons it will hold.

19. Find the number of bushels in a bin that is 6 ft. long, 5 ft. wide, 4 ft. deep.

20. How many gallons will a cistern hold that is 5 ft. square and 6 ft. deep?

Units of Weight.

Avoirdupois Weight.

172.

16 drams (drs.)	= 1 ounce (oz.).
16 ounces	= 1 pound (lb.).
100 pounds	= 1 hundred-weight (cwt.).
20 hundred-weight	= 1 ton (t.).
112 pounds	= 1 long hundred-weight.
2240 pounds	= 1 long ton.

Note. Avoirdupois weight is used for weighing all articles except gold, silver, and jewels.

In the United States custom house and in wholesale transactions in coal and iron the long ton is used.

The pound avoirdupois contains 7000 grains.

Ex. 114. (*Oral.*)

1. How many ounces in 2 lbs.? in 5 lbs.?
2. How many ounces in $\frac{1}{4}$ of a pound? in $\frac{3}{4}$ of a pound? in $\frac{1}{8}$ of a pound? in $\frac{5}{8}$ of a pound? in $\frac{7}{8}$ of a pound?
3. What part of a pound are 4 oz.? 2 oz.? 8 oz.? 6 oz.? 12 oz.?
4. How many pounds in 48 oz.? in 36 oz.? in 24 oz.?
5. How many hundred-weight in 2 t.? in 3 t.?
6. How many 4-oz. packages of nutmegs can be put up from $2\frac{1}{4}$ lbs. of nutmegs?
7. If hay is $20 a ton, how many pounds can be bought for $5? $7? $10?
8. If hay is $16 a ton, what are 750 lbs. worth?
9. What part of a pound is $\frac{4}{5}$ of an ounce?

10. If butter is 25 cts. a pound, and hay is $16 a ton, how many pounds of butter will it take to pay for 1500 lbs. of hay?

Ex. 115.

1. Reduce 12,484 oz. to higher denominations.

2. Reduce 7 cwt. 64 lbs. to ounces.

3. Reduce 95,784 oz. to higher denominations.

4. A bushel of wheat weighs 60 lbs. How many bushels in $1\frac{1}{2}$ t.?

5. A cubic foot of water weighs 1000 oz. In 1800 cu. ft. of water how many tons?

6. What is the difference in pounds between 27 long tons of coal and 27 short tons of coal?

7. Find the value of $\frac{4}{15}$ of a ton.

8. What fraction of a pound is 0.00006 of a ton?

9. Add $\frac{4}{5}$ t., $\frac{1}{4}$ cwt., $\frac{3}{4}$ lb.

10. Reduce 8 cwt. 34 lbs. to the decimal of a ton.

11. Find the value of 0.472875 of a ton.

12. Reduce 12 cwt. 80 lbs. 6 oz. to the decimal of a ton.

13. A farmer sold in one week 5.825 t. of hay. On Monday he sold 1350 lbs.; on Tuesday, $\frac{1}{2}$ t.; on Wednesday, $1\frac{1}{8}$ t.; on Thursday, 1.415 t.; on Friday, $1\frac{3}{4}$ t. What part of a ton did he sell on Saturday?

14. A grocer sold in one day 17 cwt. 83 lbs. 6 oz. of loaf sugar, 13 cwt. 95 lbs. 12 oz. of coffee sugar, 15 cwt. 78 lbs. 15 oz. of brown sugar. Required the whole amount sold.

15. A grocer has 7 cwt. 57 lbs. 12 oz. of Java coffee, 5 cwt. 39 lbs. 10 oz. of Mocha. After mixing the two kinds of coffee, he sells from the mixture 10 cwt. 97 lbs. 9 oz. How much coffee has he left?

16. A butcher receives from the West every day, Sundays excepted, 9 cwt. 81 lbs. 7 oz. of beef. How much does he receive per week?

17. A man puts into his cellar 17 loads of coal, averaging 1 t. 387 lbs. a load. Required the whole amount.

18. Divide 19 t. 17 cwt. 58 lbs. by 9.

19. A farmer sells 4 oxen whose united weight is 2 t. 7 cwt. 29 lbs. 13 oz. What is their average weight?

20. Find $\frac{3}{4}$ of 8 t. 16 cwt. $24\frac{3}{4}$ lbs.

21. Divide 15 t. 17 cwt. 29 lbs. 7 oz. by $\frac{4}{5}$.

Troy Weight.

173.

24 grains (grs.)	=	1 pennyweight (dwt.).
20 pennyweights	=	1 ounce (oz.).
12 ounces	=	1 pound (lb.).

NOTE. Troy weight is used for weighing gold, silver, and jewels. The pound Troy contains 5760 grs.

Ex. 116. (*Oral.*)

1. How many grains in 2 dwt.? in 2 dwt. 9 grs.? in 3 dwt. 7 grs.?

2. How many pennyweights in 24 grs.?

3. How many pennyweights in 1 oz.? in 2 oz.? in 2 oz. 8 dwt.? in 5 oz. 17 dwt.?

4. How many ounces in 40 dwt.? in 100 dwt.? in 60 dwt.

5. How many ounces in 1 lb.? in 5 lbs.? in 10 lbs.? in 3 lbs. 6 oz.? in 4 lbs. 9 oz.?

6. How many pounds in 12 oz.? in 48 oz.? in 72 oz.? in 80 oz.? in 90 oz.?

7. If 1 dwt. of gold is worth $1.50, find the value of 1 oz. of gold; 1 lb. of gold.

8. How many spoons weighing 25 dwt. each can be made from 1 lb. 3 oz. of silver?

9. If 10 dwt. of silver are worth 70 cts., find the value of 1 lb. of silver.

Ex. 117.

1. Reduce 3 lbs. 9 oz. 18 dwt. 17 grs. to grains.

2. Reduce 25 lbs. 9 oz. 5 dwt. to pennyweights.

3. Reduce 3420 dwt. to higher denominations.

4. What is the difference in weight between 3 doz. silver tablespoons weighing 5 lbs. 9 oz. 8 dwt. and 3 doz. silver teaspoons weighing 1 lb. 9 oz. 16 dwt. 18 grs.?

5. Required the weight of 8 silver teapots, each weighing 3 lbs. 9 oz. 18 dwt. 13 grs.

6. When 12 tankards weigh 36 lbs. 8 oz. 14 dwt. 16 grs., what is their average weight?

7. Find the value of $\frac{8}{9}$ of a pound.

8. Reduce $\frac{4}{9}$ of a grain to the fraction of an ounce.

9. Reduce 7 oz. 10 dwt. to the fraction of a pound.

10. Add 0.475 lbs., 0.75 dwt., 0.125 oz., 0.374 lbs.

11. From 0.675 lbs. subtract 5.25 oz.

12. Reduce 1 oz. 7 dwt. 18 grs. to the decimal of a pound.

13. Reduce $\frac{3}{4}$ dwt. to the fraction of a pound.

14. Reduce 4 oz. 4 dwt. to the fraction of a pound.

15. What decimal of a pound is $\frac{7}{25}$ lb. — $\frac{3}{4}$ oz.?

174. In preparing medicines, apothecaries use the following:

Apothecaries' Weight.

20 grains (grs.)	= 1 scruple (℈).
3 scruples	= 1 dram (ʒ).
8 drams	= 1 ounce (℥).
12 ounces	= 1 lb.

Apothecaries' Measure.

60 minims (♏)	= 1 dram (♏ lx.).
8 drams	= 1 ounce (fl. drm. viij.).
16 ounces	= 1 pint (fl. oz. xvj.).

Ex. 118.

1. In 4 lbs. 8 ℥ 4 ʒ 2 ℈ how many grains?

2. In 7864 grs. how many pounds?

3. A patient is required to take daily 2 ʒ 2 ℈ of bark. How many weeks will 7 lbs. of bark last him?

4. Find the amount of 0.4 lb. 0.25 ℥ 0.375 ʒ 0.648 ℈ 2.147 grs.

5. Subtract 3 ℥ 7 ʒ 12 grs. from 9 ℥ 6 ʒ 1 ℈ 16 grs., and reduce the result to the decimal of a pound.

6. How many grains in 1 lb. of apothecaries' weight?

7. What part of a pound avoirdupois is a pound troy or a pound apothecaries' weight?

8. What part of an ounce avoirdupois is an ounce troy or an ounce apothecaries' weight?

UNITS OF TIME.

175.	60 seconds (sec.)	= 1 minute (min.).
	60 minutes	= 1 hour (hr.).
	24 hours	= 1 day (dy.).
	7 days	= 1 week (wk.).
	365 days (or 52 wks. 1 dy.)	= 1 common year (yr.).
	366 days	= 1 leap-year.
	100 years	= 1 century.

The names of the months called calendar months, and the number of days in each are:

	dys.		dys.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March	31	9. September (Sept.)	30
4. April	30	10. October (Oct.)	31
5. May	31	11. November (Nov.)	30
6. June	30	12. December (Dec.)	31

NOTE. The number of days in each month may be easily remembered by committing the following lines:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
Which has but twenty-eight, in fine,
Till leap-year gives it twenty-nine."

A solar year is 365 dys. 5 hrs. 48 min. 50 sec.; that is, nearly $365\frac{1}{4}$ days. As there are 365 days in a common year, a common year lacks nearly $\frac{1}{4}$ of a day of being a solar year, and this defect is made up by reckoning for some years (leap-years) 366 days.

Whenever the number representing the year is divisible by 4 and not by 100, or is divisible by 400, that year is a leap-year. Thus, 1884, a leap-year; 1885, not a leap-year; the year 1800, not a leap-year; the year 2000, a leap-year.

Ex. 119. (*Oral.*)

1. How many seconds in 2 min.? in 3 min.?
2. How many minutes in 2 hrs.? in 3 hrs.? in 60 sec.? in 120 sec.?
3. How many hours in 2 dys.? in 3 dys.? in 120 min.? in 360 min.?
4. How many days from Jan. 1 to Feb. 17, both days inclusive?
5. How many months from Aug. 9 to Nov. 9? from March 5 to Sept. 5? from April 4 to Oct. 4?
6. If a man can do a piece of work in 30 min., how many hours will it take him to do four times as much?
7. If a man can walk a mile in 15 min., how many hours will it take him to walk 24 mi.?
8. At the rate of 3 mi. an hour, how far will a man walk in 45 min.?
9. If a man earns $12 a week, and pays for expenses $12 per month of 4 wks., how much will he save in 20 wks.?
10. If a man walks $\frac{1}{4}$ of a mile in 5 min., how many hours, at that rate, will it take him to walk 4 mi.?

Ex. 120.

1. Reduce 4 yrs. 39 dys. 17 hrs. 22 min. 18 sec. to seconds.
2. In 48,967,349 sec. how many years?
3. Find the exact length of the lunar month which contains 2,551,443 sec.

4. How many seconds more are there in the 3 spring months than in the 3 autumn months?

5. Reduce $\frac{4}{5}$ of a year to days.

6. Find the value of 0.375 yr. 0.142 dy. 0.27 min.

7. What part of a day are 12 hrs. 15 min. 25 sec.?

8. What part of 2 dys. 7 hrs. 18 min. are 1 dy. 3 hrs. 15 min.?

9. How much greater is the quotient of 100 yrs. 25 dys. 12 hrs. 27 min. 28 sec. divided by 4 than the product of 4 yrs. 17 dys. 9 hrs. 12 min. 18 sec. multiplied by 5?

10. Find the number of days, reckoning from noon of the one to noon of the other, between Feb. 24 and June 23, 1884; also between Dec. 25, 1884, and May 25, 1885.

11. How many hours from noon of the 4th to midnight of the 7th of July, 1885?

12. Divide 11 wks. 6 dys. 18 hrs. by 9.

13. Divide 2 yrs. 135 dys. 17 hrs. by 72.

14. From 5 yrs. 17 hrs. take 2 yrs. 138 dys. 22 hrs.

15. Find the value of 3.1725 dys.

16. Find the value of 21.325 of a year.

17. Express 9 dys. 3 hrs. as the decimal of a week.

18. Express 13 hrs. 15 min. 17 sec. as the fraction of 6 dys. 1 hr. 48 min. 7 sec.

19. Express 3 dys. 20 hrs. 35 min. 33 sec. as the decimal of 27 dys. 13 hrs. 22 min. 30 sec.

20. Find the value of 5.58 yrs.

DIFFERENCE BETWEEN TWO DATES.

176. Find the difference between April 3, 1885 and May 7, 1837.

yrs.	mos.	dys.
1885	4	3
1837	5	7
47	10	26

In finding the difference between long dates, 30 days are considered a month. As April is the fourth, and May the fifth, month, we write 4 and 5 instead of the names of these months.

In finding the difference between short dates, the exact number of days is generally counted.

Ex. 121.

1. On the 1st of January, 1885, how much time had passed since the discovery of the Island of San Salvador, Oct. 12, 1492?
2. At the birth of Lafayette, Sept. 6, 1757, what was the age of George Washington, born Feb. 22, 1732?
3. If a note is dated March 5, 1885, and has 3 mos. 3 dys. to run, when is the note due?
4. If a note is discounted Feb. 1, and is due April 22, how many months and days has it to run?
5. Find the exact number of days from Sept. 23 to Jan. 11.

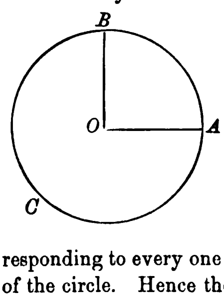
   NOTE. In finding the difference of these dates, the 23d of September is not counted, but the 11th of January is.
6. Find the exact number of days between March 5 and July 4.
7. Find the exact number of days between June 3 and Nov. 1.
8. Find the exact number of days between Feb. 3 and June 3, of a common year.
9. Find the difference between June 7, 1885, and July 4, 1776.

Angular Measure.

177. A circle is a plane figure bounded by a curved line called the **circumference,** all points of which are equally distant from a point within called the **centre.**

178. A line drawn through the centre and terminated by the circumference is called a **diameter;** and half the diameter is called the **radius.** Any part of the circumference is called an **arc.**

If a straight line fixed at one end is revolved, the other end will describe the circumference of a circle; and the difference in direction between the position of the straight line at the start and the straight line at any other position, is the angular magnitude described by the moving straight line. Thus, if OA revolve about O as a fixed point, the extremity A will describe the circumference ABC. When OA has reached the position OB, the part of the circumference between A and B is described by A, and the part of the angular magnitude about the point O, between OA and OB, is described by OA. The angle AOB is such a part of the angular magnitude about O as AB is of the circumference.



The circumference of every circle is divided into 360 equal parts, called *degrees*, and corresponding to every one of these equal parts is an angle at the centre of the circle. Hence the whole angular magnitude about any point in a plane is divided into 360 equal parts called *degrees*, and the number of degrees in the angle formed by two lines drawn from the centre of a circle is the same as the number of degrees in that part of the circumference which is intercepted by these two lines.

179. An angle of 90°, as AOB, is called a **right angle.** An angle greater than a right angle is called an **obtuse angle,** and an angle less than a right angle is called an **acute angle.**

Units of Angular Measure.

180.

60 seconds ($''$)	=	1 minute ($'$).
60 minutes	=	1 degree ($^\circ$).
360 degrees	=	1 revolution.

Note. A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

Ex. 122.

1. Reduce $49^\circ\ 37'\ 29''$ to seconds.
2. In $13{,}978''$ how many degrees?
3. Find the value of $\frac{4}{7}$ of 360°.
4. What part of the whole angular magnitude about a point is $\frac{3}{5}$ of a second?
5. Find the sum of 45.425°, $0.115'$, $0.255''$.
6. Change 0.471 of a minute to the decimal of a degree.
7. What part of $7^\circ\ 35'\ 15''$ are $3^\circ\ 20'\ 45''$?
8. Divide $17^\circ\ 27'\ 13''$ by 5; multiply $8^\circ\ 19'\ 47''$ by 8; and find the difference between the results.
9. From $7^\circ\ 0'\ 18''$ subtract $3^\circ\ 47'\ 36''$.
10. The latitude of New York is $40^\circ\ 42'\ 43''$ North; the latitude of Boston is $42^\circ\ 21'\ 30''$ North. Find their difference in latitude.
11. The latitude of New Orleans is $29^\circ\ 57'\ 46''$ North; the latitude of Rio Janeiro is $22^\circ\ 56'$ South. Find their difference in latitude.

Hint. Their difference in latitude is found by taking the sum of their latitudes.

Currency.

181. The coins of the United States are: 20-dollar, 10-dollar, 5-dollar, 3-dollar, $2\frac{1}{2}$-dollar, and 1-dollar gold coins; 1-dollar, 50-cent, 25-cent, and 10-cent silver coins; 5-cent and 3-cent nickel coins; and 2-cent and 1-cent bronze coins.

182. As any sum of money can be expressed in United States currency as dollars and decimal fractions of a dollar, it is always best to treat United States money as a simple quantity.

183. The same is true of French, Italian, German, Russian, and Austrian currency.

184.

French Currency:	100 centimes	= 1 franc (fr.)	= \$0.193.
Italian Currency:	100 centissimi	= 1 lira	= \$0.193.
German Currency:	100 pfennigs	= 1 mark	= \$0.238.
Russian Currency:	100 kopecks	= 1 rouble	= \$0.734.
Austrian Currency:	100 kreutzers	= 1 florin (fl.)	= \$0.453.

English Currency.

185.

4 farthings = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).
20 shillings = 1 pound (£).

A guinea = 21 *s.*	A crown = 5 *s.*
A sovereign = 20 *s.*	A florin = 2 *s.*

A sovereign = \$4.866½.

Ex. 123. (*Oral.*)

1. How many shillings in 48*d.*? in 60*d.*?
2. How many pence in 5*s.*? in 10*s.*? in a sovereign?
3. How many shillings in £$3\frac{1}{2}$? in £$2\frac{3}{4}$?

4. How many pounds in 95 *s.*? in 100 *s.*?

5. How many pence in a crown? in a florin?

6. How many shillings in a guinea? in a half-sovereign?

7. What part of a pound are 4 *s.*? 5 *s.*? 8 *s.*?

8. What part of a shilling are 6 *d.*? 4 *d.*? 3 *d.*?

9. How many pence in 1 *s.* 3 *d.*? in 2 *s.* 6 *d.*? in 3 *s.* 2 *d.*?

Ex. 124.

1. Reduce £432 15 *s.* 10 *d.* to pence.

2. Change 4238 farthings to higher denominations.

3. Express in dollars the value of £18.

4. Express in English money $60.83.

5. Express in United States money £3 16 *s.*

6. How many sovereigns are equal in value to $389.32?

7. Reduce $\frac{4}{5}$ *s.* to the fraction of a guinea.

8. What part of 13 *s.* 2 *d.* 1 farthing are 9 *s.* 10 *d.* 2 farthings?

9. Find the value of £5.375.

10. Reduce 6 *s.* 5 *d.* 3.04 farthings to the decimal of a pound.

11. Express in pounds £5 9 *s.* 3 *d.*

12. Add £0.75, 0.125 guineas, 0.54 *s.*, 0.55 *d.*

Miscellaneous.

186.

Numbers.	*Paper.*
12 units = 1 dozen.	24 sheets = 1 quire.
12 dozen = 1 gross.	20 quires = 1 ream.
12 gross = 1 great gross.	2 reams = 1 bundle.
20 units = 1 score.	5 bundles = 1 bale.

187. *Weights.*

A bushel of corn or rye	= 56 lbs.	A bushel of barley	= 48 lbs.
A bushel of corn meal, rye meal, or cracked corn,	= 50 lbs.	A bushel of timothy-seed	= 45 lbs.
A bushel of wheat	= 60 lbs.	A stone of iron or lead	= 14 lbs.
A bushel of potatoes	= 60 lbs.	A pig of iron or lead	= $21\frac{1}{2}$ stone.
A bushel of beans	= 60 lbs.	A fother of iron or lead	= 8 pigs.
A bushel of oats	= 32 lbs.		

The weight of a bushel of potatoes, corn, etc., varies slightly in different States, but the weights here given are those generally adopted in business transactions.

A barrel of flour	= 196 lbs.
A barrel of pork or beef	= 200 lbs.
A cask of lime	= 240 lbs.
A cental of grain	= 100 lbs.
A quintal of fish	= 100 lbs.

Books.

188. A book formed of sheets folded in

2 leaves is a folio;
4 leaves is a quarto;
8 leaves is an octavo;
12 leaves is a duodecimo;
16 leaves is a 16mo.

Ex. 125.

1. How many barrels in 75 t. of beef?
2. In a car-load of 36,000 lbs. of wheat, how many bushels?
3. Find the weight of 27 bu. of potatoes.
4. How much paper will be used by an author who sends to a semi-weekly paper 6 sheets of manuscript twice a week for a year?

5. Reduce $\frac{7}{8}$ of a quire to the fraction of a bundle.

6. Reduce 2 bundles 6 quires 6 sheets to the fraction of 2 bales 1 bundle.

7. Reduce 3 bundles 7 quires 18 sheets to the decimal fraction of a bale.

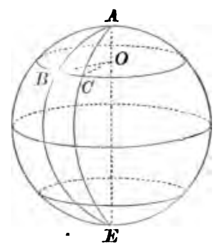
8. A button manufactory makes 96 dozen buttons a day. How many great gross will it make in 24 wks.?

9. Find the weight of 103 bu. 3 pks. 4 qts. of barley.

10. In 5 t. 624 lbs. of oats, how many bushels?

Longitude and Time.

189. A **meridian** is any line drawn straight around the earth, and passing through both poles.



190. The **longitude** of a place is the angle of inclination of the two planes which are supposed to pass through the centre of the earth, and contain, the one the meridian of that place, and the other the standard meridian. Thus, the longitude of *C*, reckoned from meridian *ABE*, is the angle *BOC*. A plane passing through *ABE* divides the earth into Eastern and Western Hemispheres. Places on the Eastern Hemisphere are in East Longitude; on the Western Hemisphere, in West Longitude.

191. As the earth turns upon its axis once in twenty-four hours, a point on the earth's surface will describe a circumference (360°) in twenty-four hours. Therefore longitude may be reckoned in *time* as well as in degrees.

In one hour a point on the earth's surface describes $\frac{1}{24}$ of

$360° = 15°$; in one minute $\frac{1}{60}$ of $15° = 15'$; and in one second $\frac{1}{60}$ of $15' = 15''$.

Again, since it requires one hour (60 min.) for a point to pass over 15°, to pass over 1° it requires $\frac{1}{15}$ of 60 min. = 4 min.; and to pass over 1′ it requires $\frac{1}{60}$ of 4 min. = 4 sec.

192. Express 20° 36′ 15″ of longitude in time.

$$15\underline{)20°\ 36'\ 15''}$$
$$1\text{ hr. } 22\text{ min. } 25\text{ sec.}$$

Since 15° longitude give 1 hr. in time, 15′ longitude 1 min., and 15″ longitude 1 sec., divide 20° 36′ 15″ by 15, as in compound division, and the quotient will be the time required.

193. Express 1 hr. 4 min, 4 sec. in degrees.

$$\begin{array}{r} 1\text{ hr. } 4\text{ min. } 4\text{ sec.} \\ 15 \\ \hline 16°\ 1'\ 0'' \end{array}$$

Since 1 hr. of time equals 15° of longitude, 1 min. of time 15′, and 1 sec. of time 15″, multiply 1 hr. 4 min. 4 sec. by 15, as in compound multiplication, and the product will be the longitude required.

194. Hence, if longitude is expressed in **degrees, divide** by 15; the quotient is the equivalent *number of hours, minutes, and seconds.*

195. If longitude is expressed in **hours, minutes, and seconds, multiply** by 15; the product is the equivalent *number of degrees, minutes, and seconds.*

Ex. 126.

1. The difference in time between New York and Paris is 5 hrs. 5 min. 20 sec. What is the difference in longitude?
2. Boston is 71° 3′ and San Francisco 122° 26′ west of Greenwich. What is the difference in clock-time between the two cities?

3. The difference in clock-time between New York and Canton is 12 hrs. 28 min. 12 sec. Find the difference in longitude.

4. The difference in longitude between Cincinnati and Boston is 13° 26′. Find the difference in time.

5. New York is 74° and Cincinnati is 84° 30′ west longitude. Find the difference in time.

6. The difference in time between Canton and Cincinnati is 13 hrs. 10 min. 8 sec. Find the difference in longitude.

7. The difference in longitude between New York and Canton is 187° 3′. What is the difference in time?

8. Find the difference in time between Philadelphia, longitude 75° 10′ West, and Buenos Ayres, longitude 58° 22′ West.

9. The difference of time between St. Petersburgh and New Orleans is 8 hrs. 1 min. 16 sec. What is the difference in longitude?

10. Find the difference in time between the Cape of Good Hope, longitude 18° 28′ East, and Halifax, longitude 63° 36′ West?

196. Since the sun *appears* to move from east to west, sunrise will occur earlier at all points east, and later at all points west, of a given place. Hence, clock-time will be later in all places east, and earlier in all places west, of a given meridian.

Therefore, if the time of a place be given,

To find the time of a place **east, add** to the given time the difference of time between the two places.

To find the time of a place **west, subtract** from the given time the difference of time between the two places.

To Find the Difference in Clock-Time when the Difference in Longitude is known.

When it is noon at Boston (long. 71° 3′ 30″ West), what is the time at Paris (long. 2° 20′ 22″ East)?

$$\begin{array}{l} 71^\circ\ 3'\ 30''\ \text{W.} \\ \underline{\ \ 2^\circ\ 20'\ 22''\ \text{E.}} \\ 73^\circ\ 23'\ 52'' \ldots \text{difference in longitude.} \end{array}$$

$$15\,\underline{)\,73^\circ\ 23'\ 52''\qquad\qquad}$$
$$4\text{ hrs. } 53\text{ min. } 35.47\text{ sec.}$$

6 min. 24½ sec. before 5 P.M. *Ans.*

Since Boston is west and Paris is east of the meridian of Greenwich, the difference between their longitudes is found by taking the sum of their longitudes.

Their difference in longitude, 73° 23′ 52″, is equivalent to 4 hrs. 53 min. 35.47 sec., and as Paris is *east* of Boston, the time at Paris is found by *adding* the 4 hrs. 53 min. 35.47 sec. to the time at Boston.

Ex. 127.

1. When it is noon at Chicago, what is the hour at New York, the difference in longitude being 13° 37′?
2. What is the time in London when it is half-past 3 in the afternoon at Constantinople, Constantinople being 29° east of London?
3. The longitude of New York is 74° West, that of Paris is 2° 20′ East. When it is 15 minutes past 10 A.M. in New York, what is the time in Paris?
4. The longitude of Boston is 71° 3′ and that of New York 74° West. What is the time in Boston when it is midnight in New York?
5. The difference in longitude between San Francisco and Chicago is 34° 49′. What time is it at San Francisco when it is 9 o'clock P.M. at Chicago?

6. Paris is 45° 10′ east of Rio Janeiro. What time is it at Rio Janeiro when it is 7 o'clock P.M. at Paris?

7. If the sun rises at half-past 4, when it is sunrise at Richmond, Va., what is the time at Rouen, France, the difference of longitude being 78° 46′?

8. The French residents in Calcutta wish to unite with the people of Paris in a celebration to occur at 3 o'clock P.M. Paris is 2° 20′ East, Calcutta, 88° 27′ East. At what hour must the festivities begin in Calcutta?

NOTE. **Standard Time** is the clock-time of some selected meridian. Eastern standard time is the clock-time of the meridian 75° west of Greenwich, and is five hours slow of Greenwich time. Central standard time is the clock-time of 90° west of Greenwich, and is just one hour slow of Eastern standard time. Mountain standard time is the clock-time of the meridian of 105°, and is one hour slower than that of 90°. Western standard time is the clock-time of the meridian of 120°, and is one hour slower than that of 105°. The railroads and many cities and towns of the United States have adopted standard time.

MISCELLANEOUS EXAMPLES.

Ex. 128.

1. Find the amount of the following bill:

To 5 yds. ribbon @ $62\frac{1}{2}$ cts. . .	$.	
" 3 yds. satin @ $4.375 . . .		
" 4 papers pins @ $12\frac{1}{2}$ cts. . .		
" 1 sun-hat @ 75 cts. . . .	.	

How much change out of a 20-dollar bill should the purchaser receive?

Make out the bills for:

2.
27	yds. of flannel	at	80 cts. a yard;
32	yds. of calico	at	11 cts. a yard;
$3\frac{1}{3}$	doz. of stockings	at	$2 per dozen;
6	pairs of gloves	at	84 cts. a pair;
4	collars	at	35 cts. each.

3.
10	lbs. of sugar	@	10 cts. a pound;
6	lbs. of tea	@	88 cts. a pound;
8	lbs. of coffee	@	32 cts. a pound;
12	lbs. of currants	@	$11\frac{1}{2}$ cts. a pound;
10	lbs. of rice	@	9 cts. a pound.

4.
$18\frac{1}{2}$	lbs. of beef	@	22 cts. a pound;
$10\frac{1}{4}$	lbs. of mutton	@	21 cts. a pound;
$7\frac{1}{2}$	lbs. of pork	@	17 cts. a pound;
16	lbs. of veal	@	16 cts. a pound;
$14\frac{3}{4}$	lbs. of ham	@	20 cts. a pound.

5.
$5\frac{1}{2}$	lbs. of soap	@	$9\frac{1}{2}$ cts. a pound;
$3\frac{1}{2}$	lbs. of candles	@	13 cts. a pound;
2	lbs. of butter	@	35 cts. a pound;
56	lbs. of rice	@	$4\frac{1}{2}$ cts. a pound.

6.
7	doz. and 4 eggs	@	18 cts. per dozen;
19	lbs. of soap	@	11 cts. per pound;
18	lbs. of butter	@	28 cts. per pound;
$13\frac{1}{2}$	lbs. of cheese	@	15 cts. per pound;
$\frac{1}{2}$	lb. pepper	@	$2\frac{1}{2}$ cts. per ounce.

7.
$12\frac{1}{2}$	t. of hay	@	$18 a ton;
66	bu. of rye	@	$1.26 a bushel;
102	bu. of barley	@	78 cts. a bushel;
5	bbls. of flour	@	$6.60 a barrel.

8. A man walks 1 mi. 47 rds. in 20 min. How many hours will it take him to walk 41 mi. 92 rds.?

9. Required the cubic feet of a box 6 ft. 6 in. long, 4 ft. 9 in. wide, and 3 ft. 3 in. deep.

10. What will be the weight of a wall of brick-work 10 ft. long, $1\frac{1}{2}$ ft. thick, $4\frac{1}{6}$ ft. high, if each cubic foot weighs 120 lbs.?

11. How many cubic yards of earth will be cut out of a drain 420 ft. long, 2 ft. wide, and 4 ft. deep?

12. What will be the expense of glazing a window of 16 squares, each $1\frac{1}{6}$ ft. long and $\frac{5}{6}$ ft. wide, at $1.08 per square foot?

13. What length must be cut off an inch-board 9 in. wide to obtain 4 ft. board measure?

14. How many boards each $11\frac{1}{2}$ ft. long, and 10 in. wide will be required for the flooring of a room 23 ft. long and 17 ft. 6 in. wide?

15. A farm of $22\frac{1}{2}$ acres is divided into house-lots measuring 75 yds. in length by 33 yds. in breadth. How many lots are there?

16. At 9 cts. per cubic foot, what will be the cost of a block of stone 9 ft. long, $5\frac{1}{2}$ ft. wide, and 4 ft. thick?

17. At 50 cts. an ounce, what is the value of a silver cup weighing 15 oz. 12 dwt. 12 grs.?

18. If the cost of making a barrel of flour into bread is $2.20, and flour is worth $9 a barrel, what should a baker receive for a loaf containing $1\frac{3}{4}$ lbs. of flour?

19. At $30 per M., what is the value of a stick of timber 24 ft. long, and 2 ft. square at the end?

20. A schoolroom is 44 ft. long, $28\frac{1}{2}$ ft. wide, and 13 ft. high. What will be the cost of painting the four walls and the ceiling, at the rate of 18 cents a square yard, making no allowance for doors and windows?

21. A druggist pays 50 cts. a pound avoirdupois for chloride of potash, and retails it in powders containing 1 ℈ 5 grs., at 5 cts. each. How much will he gain on $5\frac{1}{2}$ lbs.?

22. Find the entire surface of a block of marble 8 ft. long, 2 ft. wide, $1\frac{1}{2}$ ft. thick.

23. How many revolutions will be made by a wheel $3\frac{3}{4}$ yds. in circumference in passing over 198 mi.?

24. When an ounce of gold is worth $16.25, what must be paid for $\frac{1}{25}$ of a pound?

25. If candles $8\frac{1}{2}$ in. long are worth 9 cts. a half-dozen, and candles $10\frac{1}{4}$ in. long are worth 11 cts. a half-dozen, which is the better kind to buy?

26. How many silver spoons weighing 1 oz. 18 dwt. 12 grs. each can be made from 23 oz. 2 dwt. of silver?

27. An apprentice 14 yrs. 11 mos. 14 dys. old is to serve his employer until he is 21 yrs. of age. How long is he to stay with his employer?

28. What is the rate per hour of a horse that travels 18 mi. 1620 yds. in 3 hrs. 45 min.?

29. At 15 cts. a yard, what will be the cost of fencing a rectangular field 325 yds. long and 215 yds. wide?

30. What will be the width of carpeting, if 120 yds. are necessary to cover a floor 30 ft. long and $22\frac{1}{2}$ ft. wide?

31. When the mercury in the tube of a barometer is 30 in. high, the pressure of the atmosphere is about 15 lbs. for every square inch. What will be its pressure when the mercury stands 25 in. high?

32. A cistern containing 60 hhds. of water has two pipes open, by one of which 3 gals. of water per minute run in, and by the other 9 gals. run out. In how many hours will the cistern be emptied?

33. How many pounds of cement will be required to plaster an open cistern whose dimensions are $4\frac{1}{2}$ ft. long, $3\frac{1}{3}$ ft. wide, and $2\frac{2}{3}$ ft. deep, if the cement on a square foot weighs $6\frac{3}{4}$ lbs.?

34. How many tons of water will the cistern in example 33 hold, if a cubic foot of water weighs 1000 oz.?

35. What will be the cost of covering with paper $\frac{1}{2}$ yd. wide the four walls of a room 21 ft. long, 16 ft. wide, and 10 ft. high, if the cost of the paper is $12\frac{1}{2}$ cts. per yard, and no allowance is made for doors and windows?

36. What is the breadth of a rectangular field containing $7\frac{1}{2}$ A., if the length is 242 yds.?

37. A certain watch gains $3\frac{1}{2}$ sec. in 24 hrs., and another loses $2\frac{1}{2}$ sec. in the same time. If both be set right on Monday at noon, what will be the difference between them at 6 o'clock (true time) the next Saturday evening?

38. A milkman paid a farmer $3.20 for ten 2-gal. cans of milk. He lost 20 qts. At what price per quart must he retail the remainder to gain 8 cts. a gallon?

39. A man who had $\frac{3}{8}$ of a square mile of woodland sold $12\frac{1}{2}$ sq. rds. How much had he left?

40. How many yards of Florence silk $\frac{5}{8}$ yd. wide will be required to line 19 yds. of camel's hair cloth $1\frac{1}{2}$ yds. wide?

41. How many days from Sept. 16, 1882, to Feb. 12, 1884?

42. A miller makes 154 bbls. of flour from $885\frac{1}{2}$ bu. of wheat. How many bushels on the average are required for a barrel of flour?

43. What must be the length of a walk $2\frac{1}{4}$ ft. wide to contain 38 sq. ft.?

44. A cistern 7 ft. long and 5 ft. wide contains 105 cu. ft. What is its depth? how many gallons of water will it hold?

45. What must be paid for a pile of wood 25 ft. long, 3 ft. high, and 4 ft. wide at $5.50 per cord?

46. Each person on the average breathes 28 cu. ft. of air in an hour. How many hours will the air in a room 14 ft. long, 12 ft. wide, and 6 ft. high last 12 men?

47. Sound travels at the rate of 1130 feet a second. How long after the flash will the clap of thunder come when the cloud is 2 mi. 1000 yds. distant?

48. There are 9 oz. of iron in the blood of 1 man. How many men would furnish iron enough in their veins to make a ploughshare weighing $22\frac{1}{2}$ lbs.?

49. The fore wheel of a carriage is 4 ft. 7 in. in circumference, and the hind wheel 5 ft. 6 in. How many more times will the fore wheel turn than the hind wheel in going a distance of 1 mi.?

50. A boarding-house uses 3 pks. of potatoes daily. At $87\frac{1}{2}$ cts. per bushel, what will be the expense for potatoes during October, November, and December?

CHAPTER X.

PERCENTAGE.

197. 1. A boy gave away 6 marbles out of every hundred he had. How many did he give away out of 400? of 600? of 1000?

2. A man had 300 sheep, and sold 10 out of every hundred. How many did he sell?

3. A man sold 7 tons of coal out of every hundred he had. How many did he sell out of 900 tons?

4. A man sells 11 lbs. of sugar out of every hundred he has. How many pounds will he sell out of 500? How many pounds will he have left?

198. In considering the increase or decrease of quantities, we usually employ the **number 100 as the representative of the quantity considered.**

199. Instead of using the phrases 6 *in every hundred*, 10 *in every hundred*, 7 *in every hundred*, 11 *in every hundred*, we say 6 *per cent*, 10 *per cent*, 7 *per cent*, 11 *per cent*. The words *per cent* therefore mean *hundredths*.

200. The symbol % is used for the words *per cent*.

How many hundredths of a number are:

2%?	$6\frac{1}{2}$%?	20%?	$12\frac{1}{2}$%?
5%?	7%?	25%?	$33\frac{1}{3}$%?

How many per cent of a number is:

0.20?	0.75?	$0.12\frac{1}{2}$?	1.40?
0.15?	$0.06\frac{1}{4}$?	0.50?	2.25?

What common fraction of a number (in its lowest terms) is:

10%?	25%?	$8\frac{1}{3}$%?	$12\frac{1}{2}$%?	125%?
20%?	50%?	$6\frac{1}{4}$%?	$66\frac{2}{3}$%?	160%?
$16\frac{2}{3}$%?	75%?	$33\frac{1}{3}$%?	100%?	175%?

Express as hundredths and as per cent:

$\frac{1}{2}$;	$\frac{2}{3}$;	$\frac{4}{5}$;	$\frac{3}{8}$;	$\frac{7}{12}$;	$\frac{5}{18}$;	$\frac{7}{40}$;
$\frac{1}{3}$;	$\frac{3}{4}$;	$\frac{5}{6}$;	$\frac{5}{9}$;	$\frac{1}{15}$;	$\frac{7}{20}$;	$\frac{7}{50}$;
$\frac{1}{4}$;	$\frac{3}{5}$;	$\frac{3}{7}$;	$\frac{9}{10}$;	$\frac{3}{16}$;	$\frac{7}{25}$;	$\frac{7}{100}$.

201. Express $\frac{1}{2}$% as hundredths and as a common fraction.

$$\frac{1}{2}\% = 0.00\frac{1}{2} = \frac{1}{200}.$$

In like manner express as hundredths and also as common fractions:

$\frac{1}{4}$%;	$\frac{3}{5}$%;	$\frac{1}{7}$%;	$\frac{3}{8}$%;	$\frac{1}{10}$%;
$\frac{1}{5}$%;	$\frac{4}{5}$%;	$\frac{3}{7}$%;	$\frac{5}{8}$%;	$\frac{3}{10}$%;
$\frac{3}{4}$%;	$\frac{1}{6}$%;	$\frac{5}{7}$%;	$\frac{1}{9}$%;	$\frac{7}{10}$%;
$\frac{2}{5}$%;	$\frac{5}{6}$%;	$\frac{1}{8}$%;	$\frac{4}{9}$%;	$\frac{9}{10}$%.

202. Find 8% of 250 bu. of corn.

8% of a number is $\frac{8}{100}$ of the number; and $\frac{8}{100}$ of 250 bu. = 20 bu.

20 bu. *Ans.*

Find 20% of 80 yds. of cloth.

20% = $\frac{20}{100}$ = $\frac{1}{5}$; and $\frac{1}{5}$ of 80 yds. is 16 yds.

16 yds. *Ans.*

Ex. 129. (*Oral.*)

Find:

1. 4% of 400 sheep.
2. 5% of 1000 bricks.
3. 8% of 200 ft. of board.
4. 6% of 90 dys.
5. 10% of 150 cds. of wood.
6. 20% of 250 prs. of gloves.
7. 25% of 120 horses.
8. 80% of 400 A.
9. $6\frac{1}{4}\%$ of 320 rds.
10. $12\frac{1}{2}\%$ of 400 melons.
11. $66\frac{2}{3}\%$ of 300 oranges.
12. $8\frac{1}{3}\%$ of 1 doz. eggs.
13. 75% of 40 hens.
14. 60% of 20 girls.
15. $16\frac{2}{3}\%$ of 60 lbs. of butter.
16. $37\frac{1}{2}\%$ of 120 gals. of syrup.
17. $62\frac{1}{2}\%$ of 800 soldiers.
18. $\frac{4}{5}\%$ of 500 bu. of wheat.
19. $\frac{1}{2}\%$ of 4000 yds. of cloth.
20. $\frac{1}{5}\%$ of 100 dollars.

Ex. 130.

Find:

1. 9% of 1297.
2. $2\frac{1}{2}\%$ of 4300.
3. $\frac{1}{2}$ of 1% of 1346.
4. 12% of 6072.
5. $\frac{1}{2}\%$ of 150,975.
6. $1\frac{1}{10}\%$ of 1984.
7. 150% of 1050.
8. 100% of 7968.
9. A farmer having a flock of 1200 sheep lost 37% of them. What per cent of them, and how many sheep, had he left?
10. If copper ore yields 6% of pure metal, how many pounds of copper will be obtained from 1 t. of ore?

11. If a man buys 24 A. of land at \$84 an acre, what must be the annual income that the investment may yield 10% ?

12. A grocer bought 40 cwt. of sugar for \$240. 4% of it is wasted, and the remainder is retailed so that there is neither loss or gain. What is the retail price per pound?

13. A stone-mason contracted to dig a cellar 45 ft. long, 36 ft. wide, and 6 ft. deep at 25 cts. a cubic yard. He lost 5% of his contract price. What was his loss?

14. A coal-dealer bought 25,784 t. of coal at \$5 a ton. He sold 40% of it at \$7, 20% at \$8.50, and the remainder at \$4.50. How much did he gain?

15. A gentleman owns 2 farms. The first contains 360 A., and the number of acres in the second is 150% of the number of acres in the first. Find the number in the second farm.

203. What per cent of 20 is 5?

1% of 20 $= \frac{1}{100}$ of 20 $= 0.2$; therefore 5 is as many times 1% of 20 as 0.2 is contained in 5. $\frac{5}{0.2} = \frac{50}{2} = 25$.

25%. *Ans.*

Ex. 131. (*Oral.*)

What per cent of:

1. 16 is 8?
2. 20 is 5?
3. 25 is 15?
4. 48 is 8?
5. 100 is $12\frac{1}{2}$?
6. 2 is $\frac{1}{2}$?
7. 3 is $\frac{2}{3}$?
8. $\frac{4}{9}$ is $\frac{2}{9}$?
9. \$72 are \$18?
10. \$52 are \$39?
11. 50 qts. are 5 qts.?
12. 66 gals. are 6 gals.?
13. 480 dys. are 24 dys.?
14. 90 cds. are 9 cds.?
15. 80 men are 50 men?

Ex. 132.

1. From a school of 150 scholars, 50 are absent. What per cent of the whole is the number present?
2. In a school numbering 200 the daily average attendance is 160. What is the per cent of attendance? The number absent on the average is what per cent of the number present?
3. A person bought a house and lot for \$6000, paying \$5000 for the house. The value of the lot is what per cent of the value of the house?
4. From a peck of corn a crop of $48\frac{1}{4}$ bu. was raised. What per cent was the increase?
5. From $67\frac{1}{2}$ bu. of corn, 6 bu. 3 pks. are sold. What per cent of the whole is sold?
6. A house worth \$8000 rents for \$720 a year. What per cent of its value does it rent for?
7. From Delhi to Bombay the distance is 720 miles, and from Delhi to Madras 1080 miles. What per cent of the distance to Madras is the distance to Bombay?
8. Westminster Hall is 270 ft. long and 75 ft. broad. What per cent of the length is the breadth?
9. The Peak of Teneriffe is 12,232 ft. high. What per cent of a mile is its height?
10. The Danube is 1630 miles long, and the Missouri from its source to the Gulf of Mexico is 4000 miles long. What per cent of the length of the Missouri is the length of the Danube?
11. What per cent of 7 hrs. and 30 min. are $6\frac{3}{4}$ min.?
12. What per cent of 3 wks. and 4 dys. are 3 dys. and $10\frac{1}{2}$ hrs.?

204. What is the number of which 15 is 5% ?

If 15 is 5%, then 15 is $\frac{5}{100}$ or $\frac{1}{20}$ of the number, and, if 15 is $\frac{1}{20}$ of the number, the number itself will be $20 \times 15 = 300$.

300. *Ans.*

Ex. 133. (*Oral.*)

What is the number of which :

1. 10 is 20% ?	**6.** $\frac{1}{2}$ is $16\frac{2}{3}$% ?	**11.** $\frac{7}{8}$ is 175% ?
2. 3 is 10% ?	**7.** $\frac{3}{4}$ is 50% ?	**12.** 17 is 34% ?
3. 8 is 25% ?	**8.** $\frac{3}{7}$ is 75% ?	**13.** 50 is $62\frac{1}{2}$% ?
4. 4 is $6\frac{1}{4}$% ?	**9.** 60 is 60% ?	**14.** 300 is 0.3% ?
5. 5 is $8\frac{1}{3}$% ?	**10.** 50 is 40% ?	**15.** 20 is $\frac{1}{5}$% ?

Ex. 134.

1. 10.08 is 16% of what number?
2. 24 is $7\frac{1}{2}$% of what number?
3. 10.94 is $\frac{4}{5}$% of what number?
4. 2500 is $12\frac{1}{2}$% of what number?
5. 960 is $33\frac{1}{3}$% of what number?
6. 6000 is 20% of what number?
7. 990 is 110% of what number?
8. 810 is 90% of what number?
9. 980 is 175% of what number?
10. A city in 5 yrs. increased 12,000 in population, a gain of 25%. What was the population at the beginning and end of the 5 yrs.?
11. A schoolboy in one week read 450 lines of Latin, which was 75% of the number in the book. How many lines had he still to read?

12. A boy sold chestnuts at $12\frac{1}{2}$ cts. a quart, which was 200% of their cost. What did they cost a bushel?

13. A clerk spent 60% of his salary for board, 20% of it for clothes, 11% for books, and saved $117. What was his salary?

14. At Christmas a lady gave her daughter an atlas worth $27, and $\frac{2}{3}$ of the cost of the atlas was 90% of the sum paid for an engraving. What was the sum paid for the engraving?

15. A sea-captain owning 60% of a vessel gave to his son 50% of his share, which was worth $6000. What was the value of the vessel?

16. A gentleman worth $50,000 gave 30% of his property to his son, and this gift was 80% of the property which the son already owned. Find the amount the son was worth after receiving his father's gift.

205. By selling a horse for $90, a man gains 20% of its cost. Find the cost.

He gets the cost and 20% of the cost, or 120% of the cost. The question therefore is, $90 is 120% of how many dollars?

A man sold a horse for $90, and lost 25% of the cost. What did the horse cost?

He got the cost minus 25% of the cost, or 75% of the cost. The question therefore is, $90 is 75% of how many dollars?

Ex. 135. (*Oral.*)

1. 36 is $12\frac{1}{2}$% more than what number?
2. 65 is $6\frac{1}{4}$% less than what number?
3. 68 is $6\frac{1}{4}$% more than what number?
4. 75 is $12\frac{1}{2}$% less than what number?
5. By selling a hat for $5.40, I sell it for 20% more than the cost. What was the cost?

6. A manufacturer sells mowing-machines at $125 apiece, and gains 40%. What do they cost?

7. Sold a carriage for $240, which was 20% more than the cost. What was the cost?

8. 64 is $33\frac{1}{3}$% more than what number?

9. What number diminished by 5% of itself equals 190?

10. What number diminished by 10% of itself equals 180?

Ex. 136.

1. 874 is $33\frac{1}{3}$% less than what number?

2. 1740 is 20% more than what number?

3. 40% of 4000 is 20% less than what number?

4. What number diminished by 15% of the number equals 5100?

5. What fraction increased by 25% of itself equals $\frac{15}{16}$?

6. 7500 is $33\frac{1}{3}$% less than what number?

7. A drover sold 250 sheep for $1150, which was 15% more than they cost. Find the cost of the sheep per head.

8. At a forced sale, a bankrupt sold his house for $8000, which was 20% less than its real value. If the house had been sold for $12,000, what per cent above its real value would it have brought?

9. A flock of sheep has been increased by 250% of its number, and now numbers 1050. What is the original number?

10. If 20% be lost on a ton of rye-straw sold for $19.20, what is the cost of the straw?

PROFIT AND LOSS.

206. The difference between the buying and selling prices of goods is called **profit** or **loss**, according as the selling-price is more or less than the buying-price.

Ex. 137.

1. A horse which cost $80 was sold for $60. Find the actual loss and the loss per cent.

   NOTE. Gain or loss is so much per cent on the **cost** of the goods.

2. Flour that cost $10 per barrel was sold for $12 per barrel. Find the gain per cent.

3. If milk is bought at 4 cts. a quart, and sold at 6 cts., what is the gain per cent?

4. Goods that cost $40 were sold at 20% below cost. What was the actual loss?

5. Velvet is sold for $3.75 per yard, at a gain of 25%. Find the cost of the velvet.

6. By selling cloth at $1.60 a yard, a merchant loses 20%. What is the cost?

7. Five cords of wood costing $20 were sold at $7 per cord. What was the gain per cent?

8. A carpenter paid $5000 for a house; spent in repairs a sum equal to 80% of the purchase-price; and then sold the house for $12,000. How much did he gain, and what per cent of the whole cost?

9. In selling 32 yds. of cloth, a merchant made $6.40, which was 16% of the cost. What did the cloth cost a yard?

10. Goods were sold for \$1615.12½, at a gain of 9½%. What did they cost?

11. If tea sold at 84 cts. a pound gives a profit of 20%, what would be the profit if it were sold at 75 cts. a pound?

12. A trader's profits were \$1980 in the year 1880. This sum was 20% more than his profits in 1881. Find his profits in 1881.

13. A cord of wood costing \$4.50 sold for \$9. What was the gain per cent?

14. A house-lot was sold for \$1850, at an advance of 15% on its cost. What would have been the gain per cent if it had been sold for \$2210?

15. A manufacturer owning ⅗ of a factory sold 12½% of his share, at 10% above cost, for \$1100. What is the cost of the factory?

16. What per cent is made in buying coal by the long ton, at \$5 a ton, and selling it by the short ton, at the same price?

17. Corn cultivated at an expense of 28 cts. a bushel is sold at 1 ct. a pound. What is the gain per cent?

18. What per cent advantage is there in buying opium by the pound avoirdupois, and selling it by the pound apothecaries' weight?

19. A grocer lost 5% in selling a 50-lb. tub of butter for \$15.20. What did the butter cost per pound?

20. Ten cows were sold for \$690, at a gain of 15%. For how much per head, on the average, should they have been sold to gain 20%?

21. For what price per dozen must gloves be bought in order that, by selling them at $1.75 per pair, there may be a gain of 25%?

22. A merchant lost 25% by selling flour at $6 per barrel. If he had sold it at $9 per barrel, what would have been the gain per cent?

23. A fruit-grower sent to New York 300 peck baskets of peaches, valued at 75 cts. each. Sixty baskets were spoiled on the journey. At what rate per basket must he sell the remainder to make 20% proft on the entire value of his fruit?

24. Sold goods at a loss of 20%, and actual loss of $57.50. What was the prime cost?

25. Find the selling-price of goods by which there is a loss of 2% and an actual loss of $54.50.

26. How many pounds of cheese bought at 9 cts. a pound must be sold at 12 cts. a pound to gain $30?

27. Sold steel at $25.44 a ton with a profit of 6% and a total profit of $103.32. What quantity was sold?

Commission and Brokerage.

207. The **commission** paid to an agent for his services is generally reckoned at a rate per cent.

208. The sum left after the payment of the commission and other expenses is called **net proceeds.**

209. Commission paid to a broker is called **brokerage.**

210. In **selling,** the commission is reckoned on **the money received;** in **buying,** the commission is reckoned on **the money paid.**

Ex. 138.

1. A commission-merchant sold 90 bbls. of flour at $6 a barrel, and received 5% commission. What was his commission?

2. A commission of $121.29 was charged for selling $1866 worth of goods. What was the rate of commission?

3. A grain-dealer charged 7% commission for selling a quantity of wheat, and received for his commission $109.20. What was the total amount received for the wheat?

4. A real-estate broker sold a house on $6\frac{1}{4}$% commission, and sent to the owner as net proceeds $3060. What was the broker's commission, and what sum was received for the house?

   HINT. The broker received $6\frac{1}{4}$%, and the owner $93\frac{3}{4}$%, of the sum the house sold for. Hence the question is, $3060 is $93\frac{3}{4}$% of what sum?

5. A New York merchant sent $1295.32 to New Orleans to be expended in cotton. The broker in New Orleans charged 6% commission. What sum was paid for cotton?

   HINT. The broker received 6% commission on the money invested in cotton. Therefore, the question is: $1295.32 is 106% of what sum?

6. If $5125 include the amount expended for wool and $2\frac{1}{2}$% commission to the purchasing agent, how much money does the agent lay out in wool?

7. A lawyer collected 75% of a debt of $1260, and charged 5% commission on the sum collected. What did the creditor receive?

8. An agent sold 420 bu. of corn at 60 cts. a bushel, and the commission was $7.56. What rate of commission was charged for selling?

9. A land agent charged 4% for selling 750 A. of land at $20 an acre. What was his commission?

10. How many yards of cloth, at 45 cts. a yard, can an agent buy with the commission received from the sale of 180 bu. of potates at 50 cts. a bushel, his rate of commission for selling the potatoes being $1\frac{1}{2}$%?

11. A man bought a horse for $225, which sum was half of his commission, at $2\frac{1}{2}$%, on the sale of a farm. What did the farm bring?

12. A young man selling tea on $2\frac{3}{4}$% commission sent to his employer $875.25 as the net proceeds of one week's sales. What did the average daily sales amount to?

13. A St. Louis merchant received $150 as his commission, at $2\frac{1}{2}$% for purchasing 1200 bbls. of flour. What was the price paid per barrel?

14. A broker sold for a farmer 12,000 lbs. of pork, at $8\frac{1}{8}$ cts. per pound. He charged 3% commission for selling, and paid $37.60 for freight. How many feet of pine boards, at $25 per M., can the broker buy with the net proceeds, if he charges 1% commission for buying?

15. A broker is offered a commission of $5\frac{1}{2}$% for selling wool and guaranteeing payment, or a commission of $3\frac{3}{4}$% without guaranteeing payment. He accepts the $5\frac{1}{2}$% commission, and guarantees the payment. The sales amount to $8500, and the bad debts to $147.75. How much did he gain by his choice?

INSURANCE.

211. In insurance a payment called a **premium of insurance** is made for a guaranty of a specified sum of money in the event of loss from fire or accident, and is reckoned at a rate per cent on the amount insured.

212. In life-insurance an annual payment is made in order to secure a specified sum of money in the event of death, or at the end of a fixed period of time.

213. The deed of contract is called the **policy of insurance.**

Ex. 139.

1. Find the cost of insuring property worth $15,000, if $\frac{2}{3}$ of the value is insured at $\frac{4}{5}\%$.
2. Find the cost of insuring $\frac{5}{6}$ of the value of 6000 bbls. of flour worth $9.60 a barrel, the insurance being reckoned at $\frac{1}{2}\%$.
3. A stock of goods worth $12,000 was insured for $\frac{7}{8}$ of its value at $\frac{3}{4}\%$. If the whole stock were burned, what would be the loss to the owner, including the premium paid for insurance?
4. After three annual payments of $337.50, premium at $1\frac{1}{2}\%$ on $\frac{3}{4}$ of the value of a mill, it was burned. Find the loss to the insurance company.
5. At $\frac{4}{5}\%$, how much insurance can be effected upon a store for $108?
6. What annual premium at $1\frac{1}{3}\%$ must be paid on a life-insurance of $6000?
7. At the rate of $17 upon $1000, what annual premium will be paid on a life-insurance of $6700?
8. The annual premium paid for life-insurance at $1\frac{3}{4}\%$ is $70. What is the sum insured?

TAXES AND DUTIES.

214. *Taxes* on property are reckoned at a rate per cent on the assessed value of the property; and *duties* on imported goods are sometimes reckoned at a rate per cent on the cost in the country from which they are imported.

Ex. A tax of $18,000 is levied upon a town which contains 800 polls, assessed at $1.50 each, and which has taxable property valued at $1,100,000. It is estimated that the town will receive from the state $3600 as its share of the railroad tax. Find the rate of taxation and the tax paid by Brown, whose property is assessed at $5960, and who pays for 1 poll.

The amount of poll-taxes = 800 × $1.50 = $1200
The amount from the state = $3600
The sum from state and polls = $4800
Sum levied on property = $18,000 − $4800 = $13,200.
The rate = $13,200 ÷ $1,100,000 = $0.012.
That is, the tax is 12 mills on a dollar, or $12 on $1000.
Therefore Brown's property-tax is 0.012 of $5960 = $71.52.
Total tax = $71.52 + $1.50 = $73.02.

Ex. 140.

1. If the assessed valuation of a town is $784,750, and the town has 260 polls, paying $1.25 each, what is the rate when the tax levy is $16,020 besides the estimated amount to be received from the state?

2. A district schoolhouse is to cost $3500, and the property of the district is assessed at $210,000. What is the rate, and what tax must be paid on property assessed at $3798.60?

3. In a city of 2000 polls, each paying $1.50, the sum of $111,000 is to be raised by taxation on property assessed at $9,000,000. What is the tax of a man who pays for 4 polls, and tax on property assessed at $25,670?

4. What is the rate of taxation when $710.92 is the tax upon $50,780?

5. If a tax of $12,350 is to be raised, and the collector receives 5% for collecting the taxes, what sum must be levied?

6. A town-hall is to be built at a cost of $11,400. What sum must be assessed if the collector receives 5% for collecting the taxes, and what will be the rate if the assessed valuation of the town is $800,000?

7. Find the duty, at 15%, on 95 cases of indigo, each weighing 190 lbs., and invoiced at 75 cts. per pound.

8. After deducting 20% for leakage, what will be the duty on 40 hhds. of molasses, of 84 gals. each, if the molasses is invoiced at 90 cts. a gallon, and the duty is 30%?

9. On 15 doz. bottles of sherry wine there is paid $1.25 per dozen for transportation, and $1.50 per dozen for duty. What is the whole cost of importation?

10. A Boston merchant received from Paris:

325 yds. of silk	@	$2.25 a yard;
296 yds. of lace	@	1.50 a yard;
480 yds. of ribbon	@	0.50 a yard;
45 doz. gloves	@	15.00 a doz.

If the duty on silk, ribbon, and lace is 35%, and on gloves 25%, what is the whole amount of the duties?

11. If the duty is \$2.50 a gallon on cologne-water, what must be paid on 75 doz. pint bottles, if there is an allowance of 5% for breakage?

12. What is the invoice cost of goods upon which \$625 duty are paid, if the duty is reckoned at 25%?

13. What will be paid by a grocer importing 120 chests of tea, containing 79 lbs. each, invoiced at 75 cts. per pound, if the duty is 15%?

Ex. 141.

Miscellaneous Examples.

1. Of what number is 450 nine per cent?

2. What is the excess of 5% of 1500 over $\frac{1}{2}$% of 7000?

3. What per cent of 9000 is 45?

4. Five hundred and sixty is 12% more than what number?

5. Seven hundred and fifty-two is 6% less than what number?

6. There is a difference of 893 between a certain number and 6% of the number. Find the number.

7. What per cent of 25 lbs. are 3 lbs. 4 oz.?

8. The difference between 50$\frac{1}{2}$% and 75$\frac{1}{4}$% of a number is 99. Find the number. Let the example be proved.

9. A merchant sold cloth at \$4.20 per yard, and gained 20%. If it had been sold at \$3.60, what actual gain, and what gain per cent, would have been made?

10. By how much does $\frac{6}{7}$% exceed $\frac{4}{5}$%?

11. At an average price of 55 cts. per bushel, and a charge of $2\frac{1}{2}\%$ commission, how many bushels of grain can be bought for \$4510?

HINT. First find the cost of 1 bu., including commission.

12. A landau was sold for \$488, at a gain of 22%. Required the cost.

13. A milkman's gallon measure was too small by $\frac{1}{2}$ gi. What was the rate per cent of fraudulent gain?

14. A merchant paid \$112.50 for 75 yds. of silk, of which 15 yds. were worthless. At what price per yard must the remainder be sold to gain 20% on the purchase-price of the whole?

15. For selling goods, an agent received \$106.83 commission, $2\frac{1}{4}\%$ for selling, $2\frac{3}{4}\%$ for guaranteeing payment. What sum was received for the goods?

16. A dealer bought 70 bags of wool at \$32 a bag; 10% of it proved unsalable. For what price per bag must he sell the rest to realize 15% on his purchase?

17. A lady paid for investing money \$9.37$\frac{1}{2}$ brokerage, rate $\frac{1}{8}\%$. Required the amount invested.

18. From a stack of hay, 7 t. 11 cwt. were sold, which was $75\frac{1}{2}\%$ of the whole. What did the stack contain before the sale?

19. A carriage worth \$250 was bought for \$50 less, and sold for \$25 more, than its value. What was the rate of gain on the price paid?

20. A man left 30% of his estate to his wife, 50% of the remainder to his son, 75% of the residue to his daughter, and the balance, \$546, to a family servant. Required the value of the estate.

21. What per cent of $\frac{9}{11}$ is $\frac{3}{22}$? of $\frac{7}{18}$ is $\frac{14}{39}$?

22. A man sold 36 horses for $200 each: on half of them he gained 20%, and on half he lost 10%. What was his gain per cent on the whole sale?

23. A gentleman sent to a broker $1281.25 to be invested in land at $62.50 an acre. A commission of $2\frac{1}{2}$% being charged for buying, how many acres were bought?

24. The dimensions 10, 8, and 6, of a rectangular bin being increased 10%, what will be the rate per cent of increase in capacity?

25. One-half of a stock of goods valued at $612.60 was sold for $\frac{2}{3}$ of the value of the whole stock. What was the gain per cent?

26. A roll of 140 yds. of carpet was sold for $72, at a loss of 10%. What should it have brought per yard to insure a gain of 15%?

27. A railroad company with $9,000,000 capital declares a dividend of $360,000. What sum will be received on 120 shares of $100 each?

28. Ten per cent of a roll of carpet having been sold to one man, 10% of the remainder to another, 30.375 yds. are left. How many yards were there at first?

29. At an annual premium of $405, rate $1\frac{1}{2}$%, $\frac{3}{4}$ of the value of a mill is insured. What is the entire value of the mill?

30. A broker buying cotton at $\frac{3}{4}$% commission retained $75 for his commission, and paid $25 for storage. What sum was sent by his employers to cover the whole expense of investment?

31. What sum must be insured upon a library to cover its entire value, $18,000, and the premium at $1\frac{4}{5}\%$?

HINT. If 100 be taken to represent the sum to be insured, then $1\frac{4}{5}$ will represent the premium; and $100 - 1\frac{4}{5}$, that is, $98\frac{1}{5}$, will represent the value of the library. Hence the sum to be insured will be $\$18,000 \div 0.98\frac{1}{5} = \$18,329.94$.

32. A merchant placed 80% of his year's profits in a bank; having drawn out 20% of this deposit, $2880 remained. What were his profits for the year?

33. Required the tax-rate, in a city appropriating for public expenses $147,000, to be assessed on property worth $35,000,000.

34. A lady bought a house for $7965, which rented for $841.85. The taxes were $50; repairs, $75. What rate per cent did the investment yield?

35. A premium of $960 was paid for full insurance on a ship and cargo, at $1\frac{1}{2}\%$. The cost of the cargo was 60% of the cost of the ship. What was the value of each?

36. Find the entire cost of 4000 bbls. of flour purchased by an agent, at $7 a barrel, who charged 3% commission, and paid $315 for freight.

37. How many barrels of flour can be bought for $5924.38 by an agent who pays $7 a barrel for the flour, charges 3% commission, and pays $315 for the freight?

38. The insurance on $\frac{2}{3}$ the value of a hotel and furniture cost $300. The rate being 75 cts. on $100, what was the value of the property?

39. What is the duty, at $25\frac{3}{4}$ cts. per gallon, on 48 bbls. of turpentine, 31 gals. making a barrel, and 5% being allowed for leakage?

CHAPTER XI.

INTEREST AND DISCOUNT.

215. Interest is the payment made for the use of money.

The interest to be paid for the use of a given sum of money differs from the payments considered in the last chapter, in that it depends upon the *time* for which the sum is loaned as well as on the *rate per cent* charged.

216. The sum loaned is called the **principal.** The principal and interest together is called the **amount.**

SIMPLE INTEREST.

217. If 100 be taken as the representative of the principal, the rate will represent *the interest for one year;* the product of *the rate by the number of years* will represent *the whole interest.*

Thus, if the time be 4 yrs., and the rate per cent 5, the interest will be represented by 20, and the amount by 120.

Find the interest on \$512 for 2 yrs. 4 mos., at 6%.

$$\begin{array}{rl}
\$512 & \\
0.06 & \\
\hline
\$30.72 & = \text{interest for 1 yr.} \\
2\frac{1}{3} & = \text{2 yrs. 4 mos.} \\
\hline
1024 & \\
6144 & \\
\hline
\$71.68 &
\end{array}$$

\$71.68. *Ans.*

218. In most business transactions the time for which interest is required is 1, 2, 3, or 4 months (30 dys. being

reckoned 1 mo.), and the rate of interest is 6%, that is, $\frac{1}{2}$% a month.

Hence the interest at 6% on a given sum for 2 mos. (or 60 dys.) is found by moving the decimal-point two places to the left; for 1 mo., 3 mos., 4 mos., by moving the decimal-point two places to the left, and multiplying by $\frac{1}{2}$, $1\frac{1}{2}$, and 2 respectively.

Thus, the interest on \$2500 for 2 mos. is \$25.00; for 1 mo., \$12.50; for 3 mos., \$37.50; for 4 mos., \$50.

Find the interest on \$1120 for 3 yrs. 2 mos. 18 dys., at 6%.

The interest at 6% for 1 yr. = 0.06 of the principal.
The interest for 1 mo. is $\frac{1}{12}$ of 0.06 = 0.005 of the principal.
The interest for 1 dy. is $\frac{1}{30}$ of 0.005 = $\frac{1}{6}$ of 0.001 of the principal.
Hence the interest for

$$\begin{array}{rll} 3 \text{ yrs.} & = 3 \times 0.06 & = 0.18 \\ 2 \text{ mos.} & = 2 \times 0.005 & = 0.01 \\ 18 \text{ dys.} & = 18 \times \frac{1}{6} \text{ of } 0.001 & = \underline{0.003} \\ 3 \text{ yrs. } 2 \text{ mos. } 18 \text{ dys.} & & = 0.193 \text{ of the principal.} \end{array}$$

And 0.193 of \$1120 = \$216.16.

\$216.16. *Ans.*

219. The **six per cent method** may be employed for any rate per cent by first finding the interest at 6%, and then taking such a part of the interest as the given rate is of six per cent.

Thus, the interest at $4\frac{1}{2}$% = $\frac{4\frac{1}{2}}{6}$ = $\frac{3}{4}$ of the interest at 6%. In this case, we should diminish the interest at 6% by $\frac{1}{4}$ of itself. The interest at 8% is $\frac{8}{6}$ = $\frac{4}{3}$ of the interest at 6%. In this case, we increase the interest at 6% by $\frac{1}{3}$ of itself.

220. To compute interest for days at 6%, we move the decimal-point in the principal *three places to the left*, and multiply by one-sixth of the number of days.

Find the interest for \$8080 for 93 dys., at 6%.

$$\begin{array}{r} \$8.08 \\ 15\frac{1}{2} \\ \hline 404 \\ 4040 \\ 808 \\ \hline \$125.24 \end{array}$$

By moving the decimal-point three places to the left, we have \$8.08; and $\frac{1}{6}$ of 93 dys. = $15\frac{1}{2}$. Therefore, multiplying \$8.08 by $15\frac{1}{2}$, we obtain the required interest.

\$125.24. *Ans.*

221. For any other rate, find the interest at 6%, and then increase or diminish this interest by such a fraction of itself as the given rate is greater or less than 6%.

Ex. 142.

Find the interest of:

1. \$51.25 for 30 dys., at 6%
2. \$2581 for 60 dys., at 6%.
3. \$1261 for 90 dys., at 6%.
4. \$1250.60 for 4 mos., at 6%.
5. \$3020 for 3 mos., at 6%.
6. \$2300 for 3 mos., at 6%.
7. \$275 for 2 mos., at 6%.
8. \$5000 for 1 mo., at 6%.
9. \$1361 for 2 yrs., at 5%.
10. \$675.90 for 5 yrs., at $3\frac{1}{2}$%.
11. \$775.83 for 3 yrs. 9 mos., at $2\frac{1}{2}$%.
12. \$533.33$\frac{1}{3}$ for 10 mos., at $4\frac{1}{2}$%.
13. \$250.60 for 3 yrs. 6 mos., at $4\frac{1}{4}$%.
14. \$575.87$\frac{1}{2}$ for 1 yr. 10 mos. 15 dys., at 5%.
15. \$760 for 2 yrs. 11 mos. 27 dys., at $4\frac{1}{2}$%.

16. \$725.40 for 5 mos. 27 dys., at $5\frac{1}{4}\%$.

17. \$547.60 from Feb. 20 to Dec. 5, at $3\frac{1}{4}\%$.

18. \$1750 from May 5, 1884, to June 21, 1885, at $5\frac{1}{2}\%$.

19. \$1517 from Jan. 5 to July 1, at $4\frac{1}{2}\%$.

20. \$476.50 from July 5, 1884, to Feb. 9, 1885, at 4%.

21. \$319.20 from April 7 to Aug. 31, at $3\frac{1}{4}\%$.

22. \$6460 from June 15, 1883, to May 7, 1885, at $4\frac{1}{4}\%$.

23. \$150 from Aug. 5, 1883, to March 17, 1885, at 7%.

24. \$527.20 from Jan. 1 to Nov. 20, at $4\frac{1}{2}\%$.

25. \$1250 from Nov. 15, 1884, to March 1, 1885, at 5%.

26. \$624.36 from March 5 to Dec. 20, at $7\frac{3}{10}\%$.

Find the amount of:

27. \$1100 for 3 yrs. 4 mos., at 5%.

28. \$1290.50 for 60 dys., at 6%.

29. \$1275 for 3 yrs. 2 mos. 15 dys., at 8%.

30. \$250.80 for 10 mos. 10 dys., at $3\frac{1}{2}\%$.

31. \$377.65 for 1 yr. 3 mos., at 5%.

32. \$7234.25 for 22 yrs. 2 mos. 20 dys., at $4\frac{1}{2}\%$.

33. \$6130 from May 6 to Oct. 24, at $3\frac{3}{4}\%$.

34. \$258.85 from March 6 to June 24, at 5%.

35. \$25.62 for 33 dys., at 6%.

36. \$85.85 for 1 yr. 7 mos. 21 dys., at 6%.

37. \$600 for 93 dys., at 4%.

38. \$350 from Sept. 21, 1884, to March 5, 1885, at 4%.

39. \$1226 from Oct. 4, 1884, to May 6, 1885, at 5%.

40. \$342.42 from Feb. 5, 1884, to March 15, 1885, at 7%.

41. \$360.50 from Aug. 1, 1884, to March 3, 1885, at $6\frac{1}{2}$%.

42. \$504.25 from Jan. 8 to March 10, at $6\frac{1}{4}$%.

43. \$1240 from Mar. 3 to Aug. 28, at 7%.

NOTE. In business, a year is reckoned at 360 days in computing interest *for a time less than a year expressed in months and days;* hence the interest is $\frac{5}{365}$ or $\frac{1}{73}$ too great. But national governments take the number of days between the two given dates, and reckon for the interest such a part of a year's interest as this number of days is of 365 days.

222. It is often required to find the rate, time, or principal, when two of these and the interest (or amount) are given.

223. When the principal, interest (or amount), and time are given, to find the rate per cent.

At what rate per cent will \$320 produce \$48 in 3 yrs.?

Interest on \$320 for 3 yrs. is \$48.
Interest on \$320 for 1 yr. is $\frac{1}{3}$ of \$48.
Interest on \$1 for 1 yr. is $\frac{1}{320}$ of $\frac{1}{3}$ of \$48 = \$0.05.
But \$0.05 = 5% of \$1. 5%. *Ans.*

At what rate per cent will \$8000 amount to \$9277.78 in 2 yrs. 6 mos. 20 dys.?

Interest is \$9277.78 − \$8000 = \$1277.78.
Time is 2 yrs. 6 mos. 20 dys. = $2\frac{5}{9}$ yrs.
Interest on \$8000 for $2\frac{5}{9}$ yrs. = \$1277.78.

$$\text{Interest on \$8000 for 1 yr.} = \frac{\$1277.78}{2\frac{5}{9}}.$$

$$\text{Interest on \$1 for 1 yr.} = \frac{\$1277.78}{2\frac{5}{9}\times 8000} = \$0.06\tfrac{1}{4}.$$

But \$0.06¼ = 6¼% of \$1. $6\frac{1}{4}$%. *Ans.*

Ex. 143.

Find the rate per cent:

1. When the interest on \$500 for 1 yr. 6 mos. is \$67.50.
2. When the interest on \$250 for 2 yrs. is \$52.50.
3. When \$500 amount to \$754 in 9 yrs.
4. When the interest on \$725 for 12 yrs. is \$141.37½.
5. When \$880 amount to \$899.25 for 7 mos.
6. When the interest on \$424 for 2 yrs. 6 mos. is \$26.50.
7. When the interest on \$255.50 from April 1 to June 20 is \$2.80.
8. When \$175 amount to \$203.35 for 3 yrs. 7 mos. 6 dys.
9. When a sum of money is doubled in 16 yrs.
10. When an investment for 6 yrs. produces a sum equal to $\frac{3}{5}$ of the capital.

224. When the principal, interest (or amount), and rate per cent are given, to find the time.

In what time will the interest on \$793.87½ be \$11.96½, at 5½%?

Interest on \$793.875 at 5½% for 1 yr. = \$43.663.

Therefore the number of years will be $\frac{11.965}{43.663} = 0.274$.

And 0.274 yr. = 3 mos. 9 dys.

3 mos. 9 dys. *Ans.*

Ex. 144.

Find the time in which:

1. The interest on \$225 will be \$36, at 4%
2. \$440 will amount to \$505.45, at 4¼%.

3. \$250 will double itself, at $2\frac{1}{2}\%$.
4. \$225 will amount to \$256.50, at $3\frac{1}{2}\%$.
5. \$50 will amount to \$85, at 6%.
6. The interest on \$4260 will be \$873.30, at 6%.
7. \$1005.34 will amount to \$1156.14, at $4\frac{1}{2}\%$.
8. \$1587.75 will amount to \$1611.68, at $5\frac{1}{2}\%$.
9. A sum of money will double itself, at 6%.
10. \$1000 will amount to \$1125, at 4%.

225. When the interest, time, and rate are given, to find the principal.

What principal will in 8 yrs. 6 mos. produce \$100 interest, at 5%?

8 yrs. 6 mos. = 8.5 yrs.

Interest for 1 yr. $= \frac{\$100}{8.5} = \$11.765.$

Interest on \$1 for 1 yr. at 5% = 0.05 of \$1.

Hence principal required $= \frac{\$11.765}{0.05} = \$235.30.$

\$235.30. *Ans.*

Ex. 145.

Find the principal that will:

1. Produce \$180 interest in 3 yrs., at 4%.
2. Produce \$189 interest in 3 yrs., at $6\frac{1}{4}\%$.
3. Produce \$3493.20 interest in 3 yrs. 5 mos., at 6%.
4. Produce \$10.70 interest in 5 mos., at 4%.
5. Produce \$75.40 interest in 3 yrs. 4 mos., at $4\frac{1}{2}\%$.
6. Produce \$75.05 interest in 3 mos. 2 dys., at $4\frac{1}{4}\%$.

7. Produce $\$1746.60$ interest in 3 yrs. 5 mos., at 6%.

8. Produce $\$64.46$ interest in 6 yrs., at $4\frac{1}{2}$%.

9. Produce $\$80.62\frac{1}{2}$ interest in 3 yrs. 9 mos., at 4%.

10. Produce $\$669.64$ interest in 2 yrs. 7 mos. 24 dys., at 6%.

226. When the amount, time, and rate are given, to find the principal.

Find the principal that will amount to $\$748.12\frac{1}{2}$ in 3 yrs. 6 mos., at 4%.

3 yrs. 6 mos. = $3\frac{1}{2}$ yrs.
Let the principal be represented by 100.
The interest will be represented by $3\frac{1}{2} \times 4 = 14$.
The amount will be represented by $100 + 14 = 114$.
Hence the principal = $\frac{100}{114}$ of $\$748.125 = \$656.25$.

$656.25. *Ans.*

Ex. 146.

Find the principal that will amount:

1. To $1680 in 3 yrs., at 4%.
2. To $962 in $4\frac{1}{2}$ yrs., at $4\frac{1}{2}$%.
3. To $725.47 in 2 yrs. 3 mos., at $3\frac{1}{2}$%.
4. To $3215.83 in 4 yrs. 6 mos., at 3%.
5. To $595.20 in 8 mos., at 6%.
6. To $1275.75 in 1 yr. 1 mo., at 5%.
7. To $2053.32 in 3 yrs. 5 mos., at 6%.
8. To $131.88 in 2 yrs. 11 mos. 15 dys., at 6%.
9. To $37.02 in 2 yrs. 3 mos. 18 dys., at 5%.
10. To $2359.38 in 2 yrs. 7 mos. 24 dys., at $4\frac{1}{2}$%.

Bank Discount.

227. When the holder of a promissory note sells the note to a bank, or other purchaser, the sum paid by the bank is called the **proceeds** or **avails** of the note, and the difference between the sum named in the note and the proceeds is called the **discount**.

228. Discount is reckoned at so much per cent, and the per cent is called the **rate of discount**.

229. Questions in bank discount are calculated like questions in simple interest, the terms used being *discount* instead of *interest*, and *rate of discount* instead of *rate of interest*.

NOTE. The sum named in the note should be written in words, and is called the *face* of the note. The person signing the note is called the *maker*; a person who writes his name on the back of the note is called an *indorser*, and is responsible for the payment of the note.

A note, to be legal, must contain the words "*value received*"; to be negotiable, must be made payable to the *bearer*, or *to the order* of some person who must indorse the note.

When a note bears interest, the discount is computed on the *amount* of the note.

A note is *nominally* due at the expiration of the time named in the note, but it does not *mature*, that is, become *legally* due, until three days after this time. These three days are called **days of grace**. And the discount is computed on the time between the day the note is discounted and the day of its maturity.

When the time is expressed in *days*, the day of maturity is found by counting forward from the date of the note the *number of days* named in the note, and the three days of grace. When the time is in *months*, the day of maturity is found by counting the *number of calendar months*, and the three days of grace. When a note falls due on Sunday, or a legal holiday, it is payable on the day previous.

A **protest** is a notice in writing by a notary public to the indorsers that a note has not been paid. If a note be not protested on the last day of grace the indorsers are released from their obligation.

230. Find the *day of maturity*, the *time to run* (from the day the note is discounted), the *discount*, and the *proceeds* of the following notes:

$610.25. BOSTON, June 12, 1885.

Sixty days after date I promise to pay to the order of Edwin Ginn six hundred ten and $\frac{25}{100}$ dollars, for value received.

Discounted at 6%, July 1. SAMUEL HALE.

Counting 60 dys. from June 12, we have 18 in June, 31 in July, and 11 in August.

Therefore the note becomes due Aug. $^{11}/_{14}$ (11 denotes the day it is nominally due, and 14 the day it is legally due).

The time to run is 30 dys. in July and 14 in August, that is, 44 dys.

The discount is the interest on \$610.25 for 44 dys., at 6%. Therefore the discount is $7\frac{1}{3} \times \$0.61025 = \$4.48$.

The proceeds $= \$610.25 - \$4.48 = \$605.77$.

Due Aug. 14; discount, $4.48; proceeds, $605.77. *Ans.*

$1050. CHICAGO, Feb. 13, 1885.

Six months from date we jointly and severally promise to pay to the order of George Hall ten hundred and fifty dollars, for value received, with interest at six per cent.

Discounted at 8%, May 13. JAMES BLAKE.
HENRY SHAW.

Interest on note for 6 mos. $= \$31.50$.

Amount of note when due is $\$1050 + \$31.50 = \$1081.50$.

Day of maturity, Aug. $^{13}/_{16}$.

Time to run, 3 mos. 3 dys.

Discount on \$1081.50, at 8%, for 3 mos. 3 dys. $= \$22.35$.

Proceeds, $\$1081.50 - \$22.35 = \$1059.15$.

Due Aug. 16; discount, $22.35; proceeds, $1059.15. *Ans.*

Ex. 147.

Find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*, on the following notes:

1. $2250. CONCORD, N.H., Jan. 1, 1885.

Four months from date I promise to pay to the order of George Marston twenty-two hundred and fifty dollars, for value received.

Discounted at 7%, Jan. 12. SIMON STEVENS.

2. $432.55. NEW YORK, Jan. 3, 1885.

Sixty days from date I promise to pay James Wilson, or order, four hundred thirty-two and $\frac{55}{100}$ dollars, value received.

Discounted at $6\frac{1}{2}$%, Jan. 6. JOHN ALLEN.

3. $670.35. ST. LOUIS, Jan. 6, 1885.

Ninety days from date I promise to pay to the order of Peter Holmes six hundred seventy and $\frac{35}{100}$ dollars, value received.

Discounted at 7%, Jan. 26. ROBERT DAY.

4. $1304.90. CINCINNATI, Jan. 25, 1885.

Five months after date I promise to pay to the order of John Shannon thirteen hundred four and $\frac{90}{100}$ dollars, for value received, with interest at six per cent.

Discounted at $4\frac{1}{2}$%, March 15. CHARLES HILLMAN.

5. $2260. BALTIMORE, MD., June 19, 1885.

Sixty days from date I promise to pay to the order of John Morrison twenty-two hundred and sixty dollars, value received.

Discounted at $5\frac{1}{2}$%, July 16. FRANK HOWE.

6. \$645. AUSTIN, TEX., July 28, 1885.

Thirty days from date I promise to pay to the order of John Moses six hundred and forty-five dollars, value received.

Discounted at 6%, Aug. 3. RICHARD SMITH.

7. \$1000. SAVANNAH, GA., Oct. 4, 1884.

Six months after date I promise to pay to John Proctor, or order, one thousand dollars, value received, with interest at seven per cent.

Discounted at 8%, Dec. 31. JAMES WHITRIDGE.

8. \$2912.60. PHILADELPHIA, Feb. 19, 1885.

Ninety days after date I promise to pay to the order of George Wright twenty-nine hundred twelve and $\frac{60}{100}$ dollars, value received.

Discounted at 6%, March 1. PETER BURKE.

9. \$455.04. CHARLESTON, S.C., Sept. 2, 1885.

Four months from date I promise to pay to the order of Edmund Horne four hundred fifty-five and $\frac{4}{100}$ dollars, value received.

Discounted at $5\frac{1}{2}$%, Sept. 16. PAUL WEST.

10. \$1140. NEW ORLEANS, LA., July 1, 1885.

Ninety days after date I promise to pay to the order of William Whitridge eleven hundred and forty dollars, value received.

Discounted at $7\frac{1}{2}$%, Aug. 15. JOHN CLEMENT.

11. \$10,089.25. DENVER, COL., Oct. 14, 1885.

Ninety days after date I promise to pay to the order of John Higgins ten thousand eighty-nine and $\frac{25}{100}$ dollars, value received.

Discounted at 10%, Dec. 1. JOHN KELLEY.

231. To determine the face of a note that will yield a given sum when discounted.

For how much must a four-months' note without interest be made that it may yield $1000 when discounted at a bank at 6%?

The discount on $1 for 4 mos. 3 dys. is $0.0205.
Proceeds of $1 is $1 − $0.0205 = $0.9795 = 0.9795 of $1.
Therefore the face required is $ 1000 ÷ 0.9795 = $1020.93.

Ex. 148.

1. Find the face of a note for 30 dys. that will realize $600 when discounted at $6\frac{1}{2}$%.
2. Find the face of a note for 60 dys. that will realize $8000 when discounted at 8%.
3. Find the face of a four-months' note that will realize $800 when discounted at $5\frac{1}{2}$%.
4. Find the face of a note for 90 dys. that will realize $1700 when discounted at 7%.
5. Find the face of a two-months' note that will realize $900 when discounted at $7\frac{3}{10}$%.
6. Find the face of a three-months' note that will realize $2200 when discounted at 7%.

Present Worth and Discount.

232. The *present worth* of a sum of money due at the end of a fixed time is the sum that, put at interest for the fixed time, will amount to the given sum.

Thus, $100 will in 2 yrs., at 6%, amount to $112. And $112 to be paid at the end of 2 yrs. is equal in value to $100 paid now. Hence $100 is regarded as the present worth of $112 to be paid in 2 yrs.

233. The difference between a given sum and its present worth is called its *true discount.*

The operation of finding the present worth of a sum of money at a fixed rate of interest is the same as the operation of finding the principal, when the amount, time, and rate per cent are given (§ 226).

Find the present worth and discount of $1000 due at the end of 3 yrs. 6 mos., at 6%.

The interest of $1 for 3 yrs. 6 mos., at 6%, is $3\frac{1}{2} \times 6$ cts. = 21 cts.
Therefore the amount of $1 for the given time = $1.21 = 1.21 of $1.
Hence the present worth is $1000 ÷ 1.21 = $826.45.
The discount is $1000 − $826.45 = $173.55.

Ex. 149.

Find the present worth and discount of:

1. $1667.31 due in 2 yrs., at $4\frac{1}{2}$%.
2. $8522.66 due in 1 yr. 6 mos., at 6%.
3. $500 due in 11 mos., at 5%.
4. $1208.25 due in 7 mos., at 5%.
5. $1430.40 due in 16 mos., at $3\frac{1}{2}$%.
6. $1356.80 due in 1 yr. 4 mos., at $4\frac{1}{2}$%.
7. $1148.34 due in 2 yrs. 3 mos., at $5\frac{1}{3}$%.
8. $1250.26 due in 8 mos., at $7\frac{3}{10}$%.
9. $2120 due in 1 yr., at 6%.
10. $1456 due in 8 mos. 12 dys., at 8%.
11. $715.20 due in 1 yr. 4 mos., at $3\frac{1}{2}$%.
12. $5000 due in 3 yrs. 6 mos., at 6%.

Ex. 150.

1. A note for \$3000, dated April 1, 1884, payable on demand, with interest at 7%, bears the following indorsements: May 6, \$600; July 5, \$676.11; Oct. 18, \$966. What is due Jan. 1, 1885?
2. A note for \$1237.50, dated April 17, 1884, payable on demand, bears the following indorsements: June 5, \$253; Aug. 20, \$274.50; Nov. 17, \$420. What is due Jan. 1, 1885, reckoning interest at 6%?
3. A note for \$775.50, dated May 15, 1884, payable on demand, bears the following indorsements: July 21, \$150; Oct. 10, \$250; Feb. 24, 1885, \$100. What is due May 15, 1885, reckoning interest at 6%?
4. A note for \$1670.50, dated July 1, 1884, payable on demand, with interest at $6\frac{1}{2}$%, bears the following indorsements: Aug. 20, \$315; Sept. 21, \$360.50; Oct. 5, \$400; Dec. 1, \$160. What is due Jan. 1, 1885?

236. When a note that contains the words "**with interest**" runs longer than a year, and partial payments have been made, the interest is computed by a rule adopted by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest

Find the net amount of a bill of:

13. \$136, discounts being 50, 10, and 5.
14. \$164.50, discounts being $\frac{2}{3}$ and 30.
15. \$15, discounts being 40, 5, 25, and $17\frac{1}{2}$.

233. The difference between a given sum and its present worth is called its *true discount.*

The operation of finding the present worth of a sum of money at a fixed rate of interest is the same as the operation of finding the principal, when the amount, time, and rate per cent are given (§ 226).

Find the present worth and discount of $1000 due at the end of 3 yrs. 6 mos., at 6%.

The interest of $1 for 3 yrs. 6 mos., at 6%, is $3\frac{1}{2} \times 6$ cts. = 21 cts.
Therefore the amount of $1 for the given time = $1.21 = 1.21 of $1.
Hence the present worth is $1000 ÷ 1.21 = $826.45.
The discount is $1000 − $826.45 = $173.55.

Ex. 149.

Find the present worth and discount of:

1. $1667.31 due in 2 yrs., at $4\frac{1}{2}$%.
2. $8522.66 due in 1 yr. 6 mos., at 6%.
3. $500 due in 11 mos., at 5%.
4. $1208.25 due in 7 mos., at 5%.
5. $1430.40 due in 16 mos., at $3\frac{1}{2}$%.
6. $1356.80 due in 1 yr. 4 mos., at $4\frac{1}{2}$%.
7. $1148.34 due in 2 yrs. 3 mos., at $5\frac{1}{3}$%.
8. $1250.26 due in 8 mos., at $7\frac{3}{10}$%.
9. $2120 due in 1 yr., at 6%.
10. $1456 due in 8 mos. 12 dys., at 8%.

Balance due, $96.25

This method is in accordance with what is called the **Merchants' Rule.**

Ex. 150.

1. A note for $3000, dated April 1, 1884, payable on demand, with interest at 7%, bears the following indorsements: May 6, $600; July 5, $676.11; Oct. 18, $966. What is due Jan. 1, 1885?
2. A note for $1237.50, dated April 17, 1884, payable on demand, bears the following indorsements: June 5, $253; Aug. 20, $274.50; Nov. 17, $420. What is due Jan. 1, 1885, reckoning interest at 6%?
3. A note for $775.50, dated May 15, 1884, payable on demand, bears the following indorsements: July 21, $150; Oct. 10, $250; Feb. 24, 1885, $100. What is due May 15, 1885, reckoning interest at 6%?
4. A note for $1670.50, dated July 1, 1884, payable on demand, with interest at $6\frac{1}{2}$%, bears the following indorsements: Aug. 20, $315; Sept. 21, $360.50; Oct. 5, $400; Dec. 1, $160. What is due Jan. 1, 1885?

236. When a note that contains the words "**with interest**" runs longer than a year, and partial payments have been made, the interest is computed by a rule adopted by the Supreme Court of the United States, and therefore called

THE UNITED STATES RULE.

Find the amount of the principal to the time when the payment, or sum of the payments, equals or exceeds the interest.

From this amount deduct the payment or sum of the payments.

Consider the remainder as a new principal, and proceed as before.

Ex. A note of $1520, dated May 20, 1884, and drawing interest at 6%, had payments indorsed upon it as follows: Oct. 2, 1884, $300; Feb. 26, 1885, $25; April 2, 1885, $570; Aug. 8, 1885, $600. Find the amount due Dec. 6, 1885.

yr.	mos.	dys.				
1884	10	2			$1520	1st principal.
1884	5	20			.022	
	4	12	.022		$33.44	1st interest.
					1520.00	
$300.					$1553.44	
					300.00	1st payment.
1885	2	26			$1253.44	2d principal.
1884	10	2			.024	
	4	24	.024	$25	$30.08	2d interest.
					$1253.44	2d principal.
$25.					.006	
				$570	$7.52	3d interest.
1885	4	2			30.08	2d interest.
1885	2	26			1253.44	
	1	6	.006		$1291.04	
					595.00	2d & 3d payments.
$570.					$696.04	3d principal.
					.021	
1885	8	8			$14.62	4th interest.
1885	4	2			696.04	
	4	6	.021		$710.66	
					600.00	4th payment.
$600.					$110.66	4th principal.
					.019$\frac{2}{3}$	
1885	12	6			$2.18	5th interest.
1885	8	8			110.66	
	3	28	.019$\frac{2}{3}$		$112.84	$112.84. *Ans.*

In the first place, find the difference in time between each pair of consecutive dates. At the right of the result in each case put the corresponding decimal multiplier for the interest at 6%, and put the corresponding payment below.

Generally, it can be determined *mentally* whether one or more payments must be taken to make a sum equal to or greater than the interest. If two or more payments are required, the corresponding decimal multipliers may be added, and the result taken for the multiplier. Thus, it is evident that .024 of \$1253.44 is more than \$25; therefore $.024 + .006 = .03$ may be taken for the multiplier, which will give for the interest \$37.60. To this the principal is added, and from the amount the sum of the payments is subtracted.

When the rate is greater or less than 6%, the several *interests* must be increased or diminished according to the given rate.

Ex. 151.

1. A note of \$1000, dated Jan. 22, 1884, and drawing interest at 6%, had payments indorsed upon it as follows: May 20, 1884, \$50; July 20, 1884, \$162.50; Dec. 23, 1884, \$72.50. Find the balance due March 1, 1885.

2. A note of \$3325, dated Jan. 15, 1884, and drawing interest at $6\frac{1}{2}$%, had payments indorsed upon it as follows: June 24, 1884, \$100; Sept. 2, 1884, \$1250; Jan. 31, 1885, \$1400. Find the balance due May 12, 1886.

3. A note of \$2280, dated Jan. 22, 1883, and drawing interest at 7%, had payments indorsed upon it as follows: Jan. 10, 1884, \$1000; Aug. 31, 1884, \$250; Jan. 15, 1885, \$600; March 4, 1885, \$430. Find the balance due June 15, 1885.

Compound Interest.

237. When a note contains the words "**with interest annually,**" and the interest is not paid at the time it is due, the interest is usually added to the principal; and new principals are thus formed at regular intervals of time.

238. The interest may be compounded with the principal (that is, made a part of the principal) annually, semi-annually, quarterly, etc., according to agreement.

Ex. Find the compound interest of \$800 for 2 yrs. 3 mos. 15 dys., at 7%.

$$\begin{array}{lrl}
 & \$800 & \\
 & .07 & \\ \cline{2-2}
 & \$56 & \text{1st interest.} \\
 & 800 & \\ \cline{2-2}
 & \$856 & \text{2d principal.} \\
 & .07 & \\ \cline{2-2}
 & \$59.92 & \text{2d interest.} \\
 & 856.00 & \\ \cline{2-2}
 & \$915.92 & \text{3d principal.} \\
\text{3 mos. 15 dys.} & \left\{\begin{array}{r} .0175 \\ 6\overline{)16.03} \\ 2.67 \end{array}\right. & \\ \cline{2-2}
 & \$18.70 & \text{3d interest.} \\
 & 915.92 & \\ \cline{2-2}
 & \$934.62 & \text{amount.} \\
 & 800.00 & \\ \cline{2-2}
 & \$134.62 & \text{interest.}
\end{array}$$

\$134.62. *Ans.*

239. If the given time be not an integral number of years, the amount is found for the number of entire years, and then the amount of this for the fractional part of a year.

Ex. 152.

1. Find the compound interest on \$125 for 3 yrs., at $2\frac{1}{2}$%.
2. Find the amount of \$87.50 for 3 yrs., at 4% per annum, at compound interest.
3. Compare the simple and compound interest on \$21.50, at the end of 4 yrs., at 5%.

4. What will a debt of \$4250 amount to, if left standing for 2 yrs. 6 mos., at 5% per annum, compound interest?

5. Find the compound interest on \$104 for 1 yr. 9 mos., at 5%.

6. Find the compound interest on \$1800 for 2 yrs. 3 mos. 15 dys., at $3\frac{1}{2}$%.

7. Find the compound interest on \$4500 for 3 yrs. 6 mos., at 4%.

If the interest be payable semi-annually, quarterly, etc., the half, quarter, etc., of the rate per cent, must be used, and the amount obtained for each half-year, quarter-year, etc.

8. Find the compound interest on \$4000 for 2 yrs. 6 mos., at 5% per annum, interest payable semi-annually.

9. Find the compound interest on \$1001.50 for 1 yr. 3 mos., at 6%, interest payable semi-annually.

10. Find the compound interest on \$4000 for 1 yr. 3 mos., at 4% per annum, interest payable quarterly.

Ex. What principal will produce in 2 yrs. \$650.14, compound interest at 6%?

Amount of \$1 for 1 yr., at 6%, is $1.06 \times \$1$.
Amount of \$1 for 2 yrs. is $1.06 \times 1.06 \times \$1 = (1.06)^2 \times \$1$.
That is, the amount of \$1 for 2 yrs., at 6%, is \$1.1236.
Interest is $\$1.1236 - \$1 = \$0.1236 = 0.1236$ of \$1.
The principal required is $\$650.14 \div 0.1236 = \$5260$.

\$5260. *Ans.*

11. What principal will amount to \$275.62 in 2 yrs., at 5% compound interest?

12. What principal will amount to \$620.32 in 3 yrs., at 6% compound interest?

ANNUAL INTEREST.

240. Annual Interest is simple interest on the principal **and on each year's interest** from the time each interest is due until settlement.

(1) Find the interest due Aug. 4, 1885, on a note dated June 4, 1881, for $1700, with interest payable annually, at 6%.

yrs.	mos.	dys.		
1885	8	4	$1700.00	
1881	6	4	0.25	
4	2		$425.00	Interest for 4 yrs. 2 mos.
yrs.	mos.			
3	2		$1700.00	
2	2		.06	
1	2		$102.00	Annual interest.
	2		.06	
6	8 = 6⅔ yrs.		$6.12	
			6⅔	
			4.08	
			36.72	
			$40.80	Interest on annual int.
			425.00	
			$465.80	Total interest due.

$465.80. *Ans.*

The first year's interest, $102, remains overdue 3 yrs. 2 mos., the second year's 2 yrs. 2 mos., the third year's 1 yr. 2 mos., and the fourth year's 2 mos. Now the interest on $102 for the sum of these periods, 6⅔ yrs., is $40.80. Hence the total interest is $465.80.

13. Find the amount due May 17, 1885, on a note dated May 17, 1881, for $700, at 6% annual interest.

14. Find the amount due May 27, 1885, on a note dated Jan. 4, 1883, for $431, at 5½% annual interest.

15. Find the amount due May 19, 1885, on a note dated Dec. 26, 1881, for $612.30, at 5% annual interest.

16. Find the amount due Jan. 16, 1885, on a note dated Jan. 8, 1883, for \$623.04, at 5% annual interest.

17. Find the amount due Jan. 18, 1885, on a note dated Jan. 8, 1881, for \$575, at 6% annual interest.

Stocks and Bonds.

241. The name **stock** is applied to the capital of banks, railroads, and other incorporated companies.

The capital of a company is usually divided into **shares,** of which the *original value* is \$100, or some other fixed sum; but the *market-value* at any time is estimated by the current price per share.

When the market-value of stock is equal to its original value, it is said to be **at par.** In quotations of stocks, par is generally represented by 100; and when stock is quoted at above 100, it is said to be at a premium; below 100, at a discount. The premium or discount is the difference between the quotation and 100.

Thus, when the price of a stock on a given day is 91, or, as it is commonly expressed, when the stock is *at* 91, the meaning is, that \$100 stock costs on that day \$91 money; or that, if 100 be the representative of any quantity of stock, 91 will represent the corresponding value in money. In this case the stock is said to be 9% discount.

The buying and selling of stocks is conducted through the agency of stock-brokers, who receive a brokerage on the stock. The brokerage is generally reckoned at $\frac{1}{8}$ of 1% on the *par value* of the stock. Thus, if a broker buy stock for a person at 91, that person pays $91\frac{1}{8}$.

(1) How much would be received for 52 shares of stock, \$100 each, at $89\frac{1}{2}$?

$\frac{1}{8}$ will represent the brokerage.

$89\frac{1}{2} - \frac{1}{8} = 89\frac{3}{8}$, price to the seller.

Hence 1 share will bring \$$89\frac{3}{8}$; and 52 shares, $52 \times$ \$$89\frac{3}{8}$ = \$4647.50.

\$4647.50. *Ans.*

(2) What amount of stock, at $84\frac{5}{8}$, including brokerage, may be bought for \$$9393.37\frac{1}{2}$?

Since \$$0.84\frac{5}{8}$, or $0.84\frac{5}{8}$ of \$1, buys \$1 stock, the amount bought for \$$9393.37\frac{1}{2}$ will be $\frac{\$9393.37\frac{1}{2}}{0.84\frac{5}{8}} = \$11,100.$

\$11,100. *Ans.*

(3) What is the quoted price of stock when \$42,464.25 is paid for \$46,600 stock?

\$46,600 stock costs \$42,464.25.

\$1 stock costs $\frac{1}{46600}$ of \$42,464.25 = \$$0.91\frac{1}{8}$.

$91\frac{1}{8}$. *Ans.*

Ex. 153.

1. Find the cost of \$5000 stock, at 98.
2. Find the cost of \$7800 stock, at $78\frac{1}{8}$.
3. Find the cost of \$20,000 stock, at $109\frac{7}{8}$.
4. Find the cost of \$5000 United States 4% bonds, at 121.
5. Mr. Jones owns 20 United States 4% bonds of \$1000 each. The interest on these bonds is paid quarterly. How much interest does Mr. Jones receive every quarter?
6. Find the cost of 20 shares of Boston and Maine Railroad stock, at 174.
7. How much of United States 4% bonds may be bought for \$6305, at $121\frac{1}{4}$?
8. How much of Northern Pacific 6% bonds, selling at $102\frac{3}{4}$, may be bought for \$10,275?
9. How many shares (\$100 each) of Old Colony Railroad, at $137\frac{1}{2}$, may be bought for \$1650?

10. How many shares of railroad stock, at $91\frac{1}{8}$, may be bought for $\$8474.62\frac{1}{2}$?

11. What must be the price of stock in order that \$9200 stock may be bought for \$8970?

12. What must be the price of stock in order that \$11,600 stock may be bought for \$8729?

13. If \$3000 stock is bought for \$2748.75, what is the price of the stock?

14. What income will be derived from \$15,000 of 5% bonds?

15. Find the income from \$9000 of 6% stock.

16. How much will a person receive from \$18,800 railroad stock, if a dividend of 7% be declared?

17. What income will be derived from \$30,000 of 4% bonds?

Ex. How much 4% stock must be bought to give an income of \$320?

Since \$0.04 is derived from \$1 stock, \$320 will be derived from as many times \$1 as \$0.04 is contained in \$320. $\$320 \div \$0.04 = 8000$.

\$8000. *Ans.*

18. How much 4% stock must be bought to give an income of \$2400?

19 A person receives \$343 as his quarterly dividend from a 7% stock. How much stock does he hold?

20. Find the entire income of a person whose property consists of \$6000 of 6% stock and \$16,400 of 7% stock.

21. Find the rate of dividend paid by a railroad when a holder of 246 shares receives \$1722.

22. Find the rate per cent at which \$22,200 will yield a semi-annual return of \$999.

Ex. If \$5125 is invested in 6% stock, at $102\frac{1}{2}$, what income will be obtained?

\$1 stock costs 1.025 of \$1.

Hence \$5125 will be the cost of $\$5125 \div 1.025 = \$5000$ stock. And 6% of $\$5000 = \$300$.

\$300. *Ans.*

23. Find the income on \$39,000 invested in 4% stock, at 91.

24. Find the income on \$7000 invested in 4% stock, at $103\frac{1}{4}$.

25. Find the income on \$13,600 invested in 7% stock, at 130.

26. A person invests \$14,280 in railroad stock, at $127\frac{1}{2}$. What will he receive if a dividend of $3\frac{1}{4}$% be declared?

27. Find the income on \$14,000 when invested in 8% stock, at $103\frac{1}{4}$.

Ex. If a person buys 5% stock at 120, what rate of interest does he receive on his money invested?

\$100 stock costs \$120. \$100 stock pays \$5. Hence the \$120 invested yields \$5.

Therefore, the rate of interest is $\frac{5}{120} = 0.04\frac{1}{6}$, or $4\frac{1}{6}$%.

$4\frac{1}{6}$%. *Ans.*

28. If an 8% stock is worth 150, what rate of interest will a purchaser receive on his money?

29. If a 10% stock is worth 175, what rate of interest will a purchaser receive on his money?

30. If a 9% stock is worth 170, what rate of interest will a purchaser receive on his money?

31. If a 4% stock is worth 70, what rate of interest will a purchaser receive on his money?

32. If a 3% stock is worth 65, what rate of interest will a purchaser receive on his money?

Ex. Find the sum required for an investment in a 4% stock, at $98\frac{1}{2}$, to produce an income of $200 a year.

$4 are received from $100 stock.

Hence $200 will be received from $\frac{200}{4} \times \$100$ stock = $5000 stock.

$100 stock costs $\$98\frac{1}{2}$.

Therefore $5000 stock will cost $50 \times \$98\frac{1}{2} = \$4925$.

$4925. *Ans.*

33. How much money must be invested in 8% stock, at 92, to produce $400 income?

34. How much money must be invested in a 3% stock, at $87\frac{1}{2}$, to produce an income of $250?

35. A person bought some bank stock at 107, and received $265 when a 5% dividend was declared by the bank. How much money had he invested?

36. A person buys some 6% railroad stock at 75, and receives $750 income. How much money has he invested?

Ex. What must be the price of a 5% stock in order that a buyer may receive 6% on his investment?

$100 must be invested to produce $6.
Hence $\frac{5}{6}$ of $\$100 = \$83\frac{1}{3}$ must be invested to produce $5.
Therefore the price of the 5% stock must be $83\frac{1}{3}$.

$83\frac{1}{3}$. *Ans.*

37. What must be the price of a 6% stock in order that a buyer may receive 7% on his investment?

38. What must be the price of an 8% stock in order that a buyer may receive 6% on his investment?

39. A person invested $5710 in bank stock when the stock was at 142¾. What per cent dividend is declared, if he receives $300?

40. A person receives 5% interest on his money by investing in some six per cent stock. At what price did he buy it?

41. What must be the price of a 7% stock in order that a buyer may receive 6% on his investment?

Exchange.

242. A **draft** or **bill of exchange** is a written order directing one person to pay a specified sum of money to another.

243. A **time draft** is a draft payable at a specified time after sight (or date).

When the person on whom a draft is drawn accepts a draft, he writes the word "Accepted," with the date, across the *face*, and signs his name. The draft is then called an **acceptance,** and the acceptor is responsible for its payment.

An acceptance is of the nature of a promissory note, the acceptor and maker having respectively the same responsibility for payment as the maker and indorser of a promissory note.

244. The system of paying debts due to persons living at a distance by transmitting drafts instead of money is called **exchange.**

When a draft can be bought for its face, it is said to be *at par.* When the cost is less than the face, it is said to be *at a discount;* and when the cost is more than the face, it is said to be *at a premium.*

Ex. 154.

Ex. Find the cost of a draft on New York for \$1000, at $\frac{1}{4}$ of 1% premium.

$\frac{1}{4}$% of \$1000 = \$2.50 (premium).
\$1000 + \$2.50 = \$1002.50 (cost).

\$1002.50. *Ans.*

1. Find the cost of a draft on New York for \$1200, at $\frac{1}{4}$ of 1% discount.
2. Find the cost of a draft on St. Louis for \$2000, at $\frac{1}{4}$ of 1% premium.
3. Find the cost of a draft on New Orleans for \$2400, at $\frac{1}{8}$% premium.
4. Find the cost of a draft on Chicago for \$3200, at $\frac{3}{8}$% discount.

Ex. Find the cost of a draft on Cincinnati for \$1000, payable in 30 dys. after sight, exchange being $\frac{1}{2}$% premium, and interest 6%.

	\$1000.00	
0.0055 of \$1000 =	\$5.50	discount for 33 dys.
	\$994.50	cost of draft at par.
0.005 of \$1000 =	5.00	premium.
	\$999.50	cost of draft.

5. Find the cost of a draft for \$800, payable 30 dys. after sight, when exchange is $\frac{1}{4}$% premium, and interest 6%.
6. Find the cost of a draft for \$1900, payable in 30 dys., when exchange is at par, and interest $4\frac{1}{2}$%.
7. Find the cost of a draft for \$1450; payable in 60 dys., when exchange is $\frac{1}{4}$% discount, and interest 5%.
8. Find the cost of a draft for \$1000, payable 60 dys. after sight, when exchange is $\frac{1}{2}$% discount, and interest 7%.

CHAPTER XII.

PROPORTION.

245. The *relative magnitude* of two numbers is called their **ratio**, and is expressed by the fraction which has the first number for numerator, and the second number for denominator.

Thus the ratio of 2 to 3, commonly written $2:3$, is expressed by the fraction $\frac{2}{3}$.

246. The first term of a ratio is called the **antecedent**, and the second term the **consequent**.

247. If both terms of a ratio be multiplied or divided by the same number, the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2}:3\frac{1}{3}$ be multiplied by 6, the resulting ratio is $15:20$, and the two ratios are equal, for $\frac{2\frac{1}{2}}{3\frac{1}{3}} = \frac{15}{20}$. Since $\frac{15}{20} = \frac{3}{4}$, the simplest expression for $2\frac{1}{2}:3\frac{1}{3}$ is $3:4$.

248. If the numerator and denominator are interchanged, the fraction is said to be *inverted;* likewise, if the antecedent and consequent of a ratio are interchanged, the resulting ratio is called the *inverse* of the given ratio.

Thus, if the fraction $\frac{4}{5}$ is inverted, the resulting fraction is $\frac{5}{4}$, and the inverse of the ratio $4:5$ is $5:4$.

249. If two *quantities* are expressed in the *same unit,* their ratio will be the same as the ratio of the two *numbers* by which they are expressed.

Thus the quantity \$7 is the same fraction of \$9 as 7 is of 9.

250. Since ratio is simply relative magnitude, two quantities *different in kind* cannot form the terms of a ratio; and two quantities of the same kind must be expressed in a *common unit* before they can form the terms of a ratio.

Thus no ratio exists between $5 and 20 dys.; and the ratio of 3 t. to 5000 lbs. can be expressed only when *both* quantities are written as tons or pounds.

251. When two ratios are equal, the four terms are said to be **in proportion,** and are called **proportionals.**

Thus 6, 3, 18, 9 are proportionals; for $\frac{6}{3} = \frac{18}{9}$.

252. A proportion is written by putting the sign = or a double colon between the ratios.

Thus $6 : 3 = 18 : 9$, or $6 : 3 :: 18 : 9$, means, and is read, the ratio of 6 to 3 is equal to the ratio of 18 to 9.

253. The *first* and *last* terms of a proportion are called the **extremes,** and the two *middle* terms are called the **means.**

254. *Test of a proportion.* When four numbers are proportionals, the product of the extremes is equal to the product of the means.

This is seen to be true by expressing the ratios in the form of fractions, and multiplying both by the product of the denominators.

Thus the proportion $5 : 3 :: 15 : 9$ may be written $\frac{5}{3} = \frac{15}{9}$; and, if both be multiplied by 3×9, the result will be $5 \times 9 = 3 \times 15$.

255. Either extreme, therefore, will be equal to the product of the means divided by the other extreme; and either mean will be equal to the product of the extremes divided by the other mean. Hence, if three terms of a proportion be given, the fourth may be found.

(1) What number is to 4 as 3 is to 6?

This may be written $\frac{\text{What number}}{4} = \frac{3}{6}?$

Multiply both sides of the equation by 4.

The result is, $\text{What number} = \frac{4 \times 3}{6}?$

2. *Ans.*

(2) 20 is to 24 as what number is to 30?

This may be written $\frac{20}{24} = \frac{\text{What number}}{30}?$

Multiply by 30, $\frac{20 \times 30}{24} = \text{What number?}$

25. *Ans.*

(3) 18 is to 32 as 45 is to what number?

This may be written $\frac{18}{32} = \frac{45}{\text{What number}}?$

As these fractions are equal, their reciprocals are equal;

that is, $\frac{32}{18} = \frac{\text{What number}}{45}?$

Multiply by 45, $\frac{32 \times 45}{18} = \text{What number?}$

80. *Ans.*

256. When three terms of a proportion are given, the method of finding the fourth term is called the **Rule of Three.**

It is usual to arrange the quantities (that is, to *state* the question) so that the quantity required for the answer may be the fourth term. Hence the quantity which *corresponds* to that of the required answer must be the third term.

(1) If 5 t. of hay cost $87.50, what will 21 t. cost?

Since the *cost* of 21 t. is required, $87.50 is the third term.

Since 21 t. will cost *more* than 5 t., 21 t. is the second term and 5 t. the first term.

That is, 5 t. : 21 t. : : $87.50 : What quantity?

A difficulty presents itself here, inasmuch as no meaning can be given to the product of the means ($87.50 multiplied by 21 t.). Since, however, the ratio of 5 t. : 21 t. = the ratio of 5 : 21, the ratio 5 : 21 may be substituted for 5 t. : 21 t.

Then 5 : 21 : : $87.50 : What quantity?

That is, What quantity $= \frac{21 \times \$87.50}{5}$?

$367.50. *Ans.*

(2) When a post 11.5 ft. high casts a shadow on level ground 17.4 ft. long, a neighboring steeple casts a shadow 63.7 yds. long. How high is the steeple?

Height is required; the height 11.5 ft. is therefore the third term.

Since the *shadow* of the steeple is the longer, the *height* of the steeple must be the greater; therefore the second term must be the greater of the two remaining quantities expressed in the same unit. 63.7 yds. = 191.1 ft.

	Shadow.	Shadow.	Height.	Height.
	17.4 ft. :	191.1 ft. : :	11.5 ft. :	What?
or,	17.4 :	191.1 : :	11.5 ft. :	What?

That is, height of steeple $= \frac{191.1 \times 11.5 \text{ ft.}}{17.4} = 126.3$ ft.

126.3 ft. *Ans.*

257. In solving problems by the **Rule of Three,**

Make that quantity which is of the same kind as the required answer the third term.

The *numbers* by which the other two quantities are expressed, when expressed in a *common unit,* will be the first and second terms.

If, from the nature of the question, the answer will be *greater* than the third term, make the *greater* of these two numbers the *second* term; if *less,* make the *smaller* of these numbers the *second* term, and the other the first term.

Divide the product of the second and third terms by the first term, and the quotient will be the answer required.

Ex. 155.

1. An express-train runs 40 mi. in 64 min. At the same rate, how many miles will it run in 24 min.?

2. If 110 A. produce 200 hhds. of sugar, how many hogsheads will 176 A. produce?

3. If 48 reapers cut 20 A. in a given time, how many acres will 156 reapers cut in the same time?

4. If 20 reapers can cut a field in 6 dys., in how many days will 30 reapers do it?

5. The number of copies in the first edition of the "Lady of the Lake" was 2050, and was to the number in the second as 41 to 69. Find the number in the second edition.

6. The length of the steamer-track from Liverpool to Quebec is 2502 mi., and is to that from Liverpool to Boston as 139 is to 155. Find the length of the track from Liverpool to Boston.

7. If a steamer from Liverpool to Portland makes the passage of 2750 mi. in $5\frac{5}{8}$ dys., in how many days, at the same rate, would the passage of 2980 mi. from Liverpool to New York have been made?

8. If a person can walk $8\frac{4}{7}$ mi. in $2\frac{1}{2}$ hrs., how many miles can he walk in $3\frac{1}{4}$ hrs.?

9. If the shadow of a staff 3 ft. 7 in. high is 4 ft. 9 in., find the height of a steeple whose shadow is 158 ft. 4 in.

10 A train, at the rate of $25\frac{3}{8}$ mi. an hour, goes a certain distance in $3\frac{1}{2}$ hrs. In how many hours will one at the rate of $24\frac{1}{2}$ mi. an hour go the same distance?

11. The ratio of the diameter to the circumference of a circle was given by Metius as 113 : 355. Find the circumference of a fly-wheel 10 ft. in diameter.

12. Find the horse-power of an engine that can raise 11,200 lbs. of coal in an hour from a pit whose depth is 396 ft.

NOTE. The labor necessary to raise 1 lb. through 1 ft. is called the **unit of work;** and a horse can do 33,000 units of work a minute. Therefore one horse-power = 33,000 units of work, and $\frac{396 \times 11200}{33000 \times 60}$ = the horse-power required.

13. If 1000 sq. yds. of a field produce a load of hay, how many such loads will 25 A. of the field produce?

14. If a train runs 177 mi. 120 rds. in 3 hrs. $56\frac{1}{2}$ min., what is the rate per hour?

15. If 136 masons can build a fort in 28 dys., how many men will be required to build it in 8 dys.?

16. There are provisions in a fort sufficient to support 4000 soldiers for 3 mos. How many must be sent away to make them last 8 mos.?

17. A coach travels $7\frac{1}{2}$ mi. an hour. How many miles will it go between a quarter past ten A.M. and a quarter to six P.M.?

18. The expense of making the hay on 5 A. 135 sq. rds. is $29.08. What is the expense per acre?

19. If 300 laborers can make an embankment in 48 dys., how many *more* days would be required if the number of men is diminished by 60?

20. If 2.45 tons of straw cost $22.75, how many tons can be bought for $11.70?

COMPOUND PROPORTION.

258. A ratio is said to be *compounded* of two or more given ratios, when it is expressed by a fraction which is the product of the fractions representing the given ratios.

Thus the ratios 2 : 3 and 7 : 11 are represented by the fractions $\frac{2}{3}$ and $\frac{7}{11}$; and the ratio 14 : 33, which is represented by $\frac{14}{33}$ (the product of $\frac{2}{3}$ and $\frac{7}{11}$), is said to be compounded of the ratios 2 : 3 and 7 : 11.

259. A proportion which has one of its ratios a compound ratio is called **a compound proportion.**

In stating problems in compound proportion the quantity which corresponds to the answer required is made the third term. Each *pair* of the remaining quantities is then considered *separately* with reference to the answer required. The process will be understood by the following example:

If 4 men mow 15 A. in 5 dys. of 14 hrs., in how many days of 13 hrs. can 7 men mow $19\frac{1}{2}$ A.?

As the answer is to be in days, make 5 dys. the third term.

I. *Will* it require *more* days for 7 men to mow 15 A. than it *did* for 4 men? Evidently less.

Therefore make 7 the first term and 4 the second.

II. *Will* it require *more* days for the same number of men to mow $19\frac{1}{2}$ A. than it *did* to mow 15 A.? Evidently more.

Therefore make 15 the first term and $19\frac{1}{2}$ the second.

III. *Will* it require *more* days of 13 hrs. to mow the same number of acres than it *did* of 14 hrs.? Evidently more.

Therefore make 13 the first term and 14 the second.

Hence the statement is

$$\left.\begin{array}{l} 7:4 \\ 15:19.5 \\ 13:14 \end{array}\right\} :: 5 \text{ days} : \text{what?}$$

or

$$\frac{4 \times 19.5 \times 14 \times 5 \text{ days}}{7 \times 15 \times 13}.$$

This, simplified by cancellation, gives 4 days.

Ex. 156.

1. If 13 bu. of oats serve 3 horses for 11 dys., how many bushels will serve 7 horses for 12 dys.?
2. If a traveller walks 140 mi. in 8 dys., walking 7 hrs. a day, how many miles can he walk in 12 dys. of 8 hrs. each?
3. If 4 masons build 27 yds. of wall in 5 dys., working 9 hrs. a day, in how many days will 32 masons build 81 yds. of a similar wall, if they work 10 hrs. a day?
4. A bootmaker who employs 15 men fills an order for 25 doz. pairs of boots in 4 wks. In how many days can he make 45 pairs if he employs 18 men?
5. If a family, by using 2 gas-burners $7\frac{1}{2}$ hrs. a day, pays \$6 a quarter when gas is \$2.40 per 1000 cu. ft., what will a family using 3 burners 4 hrs. a day pay per quarter when gas is \$1.80 per 1000 cu. ft.?
6. If 330 slices $\frac{3}{14}$ of an inch thick are obtained from 12 rounds of beef, how many similar rounds will be required for 495 slices $\frac{1}{6}$ of an inch thick?
7. If 5 horses eat 8 bu. 14 qts. of oats in 9 dys., how many days, at the same rate, will 66 bu. 30 qts. last 17 horses?
8. If a man walks 600 mi. in 25 dys., walking 8 hrs. a day, in how many days will he walk 330 mi., walking 10 hrs. a day?
9. If a pane of glass 18 in. long and $12\frac{1}{2}$ in. wide costs 20 cts., what will be the cost, at the same rate, of a pane $22\frac{1}{2}$ in. long and 15 in. wide?
10. If 18 men can dig a trench 200 yds. long, 3 yds. wide, and 2 yds. deep, in 6 dys. of 10 hrs. each, in how many days of 8 hrs. each will 10 men dig a trench 100 yds. long, 4 yds. wide, and 3 yds. deep?

Proportional Parts.

260. If it be required to divide a quantity into parts proportional to 3, 4, 5, the *numbers* 3, 4, 5 may be taken as *representatives* of the parts, and then the whole quantity will be represented by $3 + 4 + 5$; that is, by 12.

(1) Divide $391 into parts proportional to 5, 7, and 11.

The whole quantity will be represented by $5 + 7 + 11 = 23$.
Therefore the respective parts will be $\frac{5}{23}$, $\frac{7}{23}$, $\frac{11}{23}$ of $391.

$85, $119, $187. *Ans.*

(2) Divide $248 into parts proportional to $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{25}$.

Multiply the fractions by 150, the L.C.M. of their denominators. The results are 15, 10, 6. Hence the parts will be *represented* by the numbers 15, 10, 6, and the whole by 31.
Therefore the respective parts will be $\frac{15}{31}$, $\frac{10}{31}$, $\frac{6}{31}$ of $248.

$120, $80, $48. *Ans.*

Ex. 157.

1. Divide 1200 into parts proportional to 11, 12, 13, 14.
2. Divide 390 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
3. Divide a profit of $689 among 3 partners, of whom the first owns $\frac{2}{13}$, the second $\frac{5}{13}$, and the third $\frac{6}{13}$ of the joint stock.
4. Four men invest $450, $230, $190, $110 respectively in a joint business. Find their respective liabilities in a loss of $313.60.
5. Three partners claim respectively $\frac{1}{3}$, $\frac{11}{18}$, and $\frac{7}{18}$ of $1260. Give to each his proportional share.
6. An analysis of dissolved bones gives the following results for every 100 parts. Water, 13.97; organic matter, 15.71; soluble phosphates, 21.63; insoluble phosphates, 11.43; sulphate of lime, 15.83; sulphuric acid, 15.63; alkaline salts, 1.10; silica, etc., the remainder. Find the number of pounds of each in a ton of dissolved bones.

Partnership.

261. Partnership is separated into *simple* and *compound.* In simple partnership the capital of each partner is invested for *the same time.* In compound partnership the time for which the capital of each partner is invested is taken into account, as well as the amount of the capital; and the division of profits and losses is made proportionally to the amount of the capital and the time it is invested.

A and B enter into partnership. A puts in $2000 for 2 yrs., and B puts in $3000 for 1 yr. Their profits are $1400. What is the share of each?

The use of $2000 for 2 yrs. is equivalent to 2 × $2000 for 1 yr. Hence their profits must be divided in the ratio $4000 to $3000; that is, 4 : 3.

Ex. 158.

1. Three drovers rent a field of 9 A., at $5 an acre. A puts in 6 cows for 2 mos.; B, 9 cows for 1 mo.; and C, 12 cows for 2 mos. How much should each pay?
2. In a co-partnership A contributed $400 for 9 mos.; B, $350 for 8 mos.; and C, $600 for 2 mos. Divide a gain of $570 among them.
3. At the end of 12 mos. A, B, and C, having a joint capital of $6000, find they have lost $625. A's capital of $2500 has been in the business for 12 mos., B's of $1500 for 8 mos., and C's of $2000 for 4 mos. Divide the loss among them.
4. A and B enter into partnership, A with $1800, and B with $900. At the end of 8 mos. B adds $300 to his capital. Divide a profit of $840 between them, at the end of the year.

AVERAGES.

262. If a dozen eggs weigh 1 lb. 8 oz., what is their average weight?

Since the 12 eggs weigh 1 lb. 8 oz., that is, 24 oz., the average weight of an egg will be $\frac{1}{12}$ of 24 oz. = 2 oz.

Ex. 159.

1. A merchant mixes 3 lbs. of coffee worth 27 cts. a pound, 2 lbs. worth 35 cts., and 1 lb. worth 41 cts. What is the mixture worth a pound?

2. What is the cost of a gallon of a mixture containing 7 gals. worth $1.35 a gallon, 5 gals. worth $1.05 a gallon, and water enough to make the whole mixture 15 gals.?

3. Of 32 candidates for office, 3 were 20 yrs. old, 4 were 21, 12 were 22, 12 were 23, and 1, 24. What was the average age of the candidates?

4. A bankrupt owes A $962.50, B, $3487, and C, $12,686.50. His estate, after paying expenses of settlement, is $3427.20. How much can he pay on a dollar?

5. A grocer buys 106 lbs. of tea, at 80 cts. per pound, 75 lbs., at $1.24 per pound, and 94 lbs., at $1.30 per pound, and mixes the three lots together. At what price per pound must he sell the mixture so as to make 10% on his outlay?

6. In what proportions must oils worth $1.25 a gallon and 80 cts. a gallon be mixed to make a mixture worth $1.00 a gallon?

   HINT. The loss on the $1.25 oil is 25 cts. a gallon. The gain on the 80 ct. oil is 20 cts. a gallon. Therefore there must be more of the 80 ct. oil taken than of the $1.25 oil, and in the ratio of 25 : 20 or 5 : 4.

7. In what proportion must oils worth \$1.20 and 60 cts. a gallon be mixed, so that the mixture may be worth 70 cts. a gallon?

8. Solder is composed of tin and lead. If a solder weighs 10.44 times as much as an equal bulk of water, while tin weighs 7.29, and lead 11.35 as much, find the weight of each metal in a pound of solder.

Average of Payments.

A has given to B notes as follows: \$250, due in 3 mos.; \$400, due in 6 mos.; \$700, due in 8 mos. He wishes to pay them all at one time. In how many months shall the entire payment be made?

The use of \$250 for 3 mos. equals the use of \$750 for 1 mo.
The use of \$400 for 6 mos. equals the use of \$2400 for 1 mo.
The use of \$700 for 8 mos. equals the use of \$5600 for 1 mo.
\$1350 \$8750 for 1 mo.

The question is, for how many months is the use of \$1350 equal to the use of \$8750 for 1 mo.?

The answer required is $\frac{8750}{1350}$ mos. $= 6\frac{13}{27}$ mos.

$6\frac{13}{27}$ mos. *Ans.*

9. Find the equated time for the payment of \$300 due in 3 mos., \$500 due in 6 mos., \$200 due in 9 mos.

10. A owes B \$50 payable in 6 mos., \$60 payable in 8 mos., and \$90 payable in 4 mos. Find the equated time of payment.

11. A owes B \$1000, payable at the end of 9 mos. He pays \$200 at the end of 3 mos. and \$300 at the end of 8 mos. When is the balance due?

12. On the first day of January, A purchases of B \$200 worth of goods on 3 mos. credit, and \$500 worth on 4 mos. credit, and gives one note in payment. When does the note become due?

CHAPTER XIII.

POWERS AND ROOTS.

263. The *square* of a number is the product of *two* factors, each equal to this number.

Thus the squares of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
are	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

264. The **square root** of a number is one of the *two equal factors* of the number.

Thus the square roots of	1,	4,	9,	16,	25,	36,	49,	64,	81,	100,
are	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.

265. The square root of a number is indicated by the *radical sign* $\sqrt{\ }$, or by the fraction $\frac{1}{2}$ written above and to the right of the number.

266. Since $35 = 30 + 5$, the square of 35 may be obtained as follows:

$$\begin{array}{l}
30 + 5 \\
\underline{30 + 5 \qquad\qquad} \\
30^2 + \ (30 \times 5) \\
\underline{\qquad\quad (30 \times 5) + 5^2} \\
30^2 + 2(30 \times 5) + 5^2
\end{array}
\qquad\qquad
\begin{array}{rr}
30^2 = & 900 \\
2(30 \times 5) = & 300 \\
5^2 = & \underline{\ \ 25} \\
= & 1225
\end{array}$$

267. Hence, since every number consisting of two or more figures may be regarded as composed of tens and units,

The square of a number will contain the square of the tens + twice the tens × the units + the square of the units.

SQUARE ROOT.

268. The first step in extracting the square root of a number is to mark off the figures of the number in groups.

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any number expressed by *one* or *two* figures is a number of *one* figure; of any number expressed by *three* or *four* figures is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of two figures each, from the right to the left, the number of figures in the root will be equal to the number of groups of figures. The last group to the left may consist of only one figure.

Find the square root of 1225.

```
   12 25(35
    9
65)3 25
   3 25
```

The first group 12, contains the square of the tens' number of the root.

The greatest square in 12 is 9, and the square root of 9 is 3. Hence 3 is the tens' figure of the root.

The square of the tens is subtracted, and the remainder, contains twice the tens $\times$ the units + the square of the units. Twice the 3 tens is 6 tens, and 6 tens is contained in the 32 tens of the remainder 5 times. Hence 5 is the units' figure of the root. Since twice the tens $\times$ the units + the square of the units is equal to (twice the tens + the units) $\times$ the units, the 5 units are annexed to the 6 tens, and the result, 65, is multiplied by 5.

269. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as so many tens with respect to the next figure.

Extract the square root of 7890481.

```
      7 89 04 81(2809
      4
   48)3 89
      3 84
 5609)5 04 81
      5 04 81
```

When the third group, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 56 is not contained in 50. Therefore, 0 is placed both in the root and the divisor, and the next two figures, 81, are brought down.

270. If the square root of a number have decimal places, the number itself will have *twice* as many.

Thus, if 0.11 be the square root of some number, the number will be $(0.11)^2 = 0.11 \times 0.11 = 0.0121$. Hence, if a given square number contain a decimal, and if it be divided into groups of two figures each, by beginning at the decimal-point and marking toward the left for the integral number, and toward the right for the decimal, the number of groups to the *left* of the decimal-point will show the number of *integral* places in the root, and the number of groups to the *right* will show the number of *decimal* places in the root.

Extract the square root of 52.2729.

```
      52.27 29(7.23
      49
142)3 27
     2 84
1443)43 29
      43 29
```

It will be seen from the groups of figures that the root will have one integral and two decimal places.

271. If a number is not a perfect square, ciphers may be annexed, and an *approximate* value of the root found.

Extract to six places of decimals the square root of 19.

```
     19 00 00 00(4.358899
     16
  83)3 00
     2 49
 865) 51 00
      43 25
 8708)7 75 00
      6 96 64
  8716)78 360
       69 728
        8 6320
        7 8444
         78760
```

In this example, after finding four figures of the root, the other three are found by common division. The rule in such cases is, that one less than the number of figures already obtained may be found without error by division, the divisor to be employed being twice the part of the root already found.

272. The square root of a common fraction is found by extracting the square roots of the numerator and denomi-

nator. But, when the denominator is not a perfect square, it is best to reduce the fraction to a decimal and then extract the root.

Ex. 160.

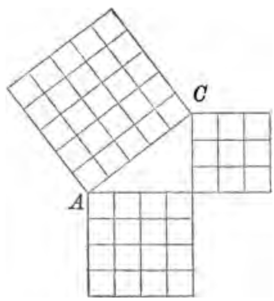
Find the square roots of:

1. 4225.	**5.** 15.7609.	**9.** 0.025.	**13.** $\frac{64}{169}$.
2. 31.36.	**6.** 0.180625.	**10.** 28.75.	**14.** $\frac{225}{361}$.
3. 50625.	**7.** 0.001296.	**11.** 0.009.	**15.** $\frac{3}{4}$.
4. 401956.	**8.** 0.042849.	**12.** 0.081.	**16.** $\frac{1}{2}$.

The side of a square is found by extracting the square root of its area.

17. A rectangle is 972 yds. long and 432 yds. wide. Find the side of a square which has the same area as the rectangle.

18. Find in yards the length of the side of a square field containing 27 A. 12 sq. rds. 1 sq. yd.



In a right triangle, the square on the hypotenuse (AC) is equal to the sum of the squares on the two legs.

Hence hypotenuse = square root of sum of squares on the legs; and one leg = square root of difference of squares on the other two sides.

19. Base = 39, perpendicular = 52; find hypotenuse.

20. Base = 35, hypotenuse = 91; find perpendicular.

21. Perpendicular = 72, hypotenuse = 75; find base.

22. A cord 287 ft. long is stretched from the top of a flag-pole 63 ft. high; find the distance of the end in contact with the ground from the base of the pole.

The length of the diagonal of a room is the square root of the sum of the squares of the length, breadth, and height.

23. Find the diagonal of a room 28 ft. long, 21 ft. wide, and 12 ft. high.

24. Find the diagonal of a hall 50 ft. long, 30 ft. wide, and 15 ft. high.

CUBE ROOT.

273. The *cube* of a number is the product of *three* factors, each equal to the number.

The cubes of	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,
are	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

274. The **cube root** of a number is one of the *three equal factors* of the number.

Thus the cube roots of	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000,
are	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.

275. The cube root of a number is indicated by $\sqrt[3]{\ }$, or by the fraction $\frac{1}{3}$ written above and to the right of the number.

Thus, $\sqrt[3]{343}$, or $343^{\frac{1}{3}}$, means the cube root of 343,

276. Since $35 = 30 + 5$, the cube of 35 may be obtained thus:

$$\begin{array}{l}
30 + 5 \\
\underline{30 + 5} \\
30^2 + \ \ (30 \times 5) \\
\underline{\qquad + \ \ (30 \times 5) + 5^2} \\
30^2 + 2(30 \times 5) + 5^2 \\
\underline{30 + 5} \\
30^3 + 2(30^2 \times 5) + \ \ (30 \times 5^2) \\
\underline{\qquad\qquad (30^2 \times 5) + 2(30 \times 5^2) + 5^3} \\
30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3
\end{array}$$

$$\begin{array}{r}
30^3 = 27{,}000 \\
3(30^2 \times 5) = 13{,}500 \\
3(30 \times 5^2) = \ \ 2{,}250 \\
5^3 = \underline{\quad 125} \\
42{,}875
\end{array}$$

Hence the cube of any number composed of tens and units contains four parts:

I. *The cube of the tens.*

II. *Three times the product of the square of the tens by the units.*

III. *Three times the product of the tens by the square of the units.*

IV. *The cube of the units.*

277. In extracting the cube root of a number, the first step is to mark off the figures of the number in groups.

Since $1 = 1^3$, $1000 = 10^3$, $1{,}000{,}000 = 100^3$, and so on, it follows that the cube root of any number between 1 and 1000, that is, of any number that has *one*, *two*, or *three* figures, is a number of *one* figure; and that the cube root of any number between 1000 and 1,000,000, that is, of any number that has *four*, *five*, or *six* figures, is a number of *two* figures, and so on.

If, therefore, an integral number be divided into groups of three figures each, from right to left, the number of figures in the root will be equal to the number of groups. The last group to the left may consist of one, two, or three figures.

Extract the cube root of 42875.

$$
\begin{array}{rr|l}
 & & 42\,875(35 \\
 & & \underline{27} \\
3 \times 30^2 = & 2700 & 15\,875 \\
3 \times (30 \times 5) = & 450 & \\
5^2 = & \underline{25} & \\
 & 3175 & \underline{15\,875}
\end{array}
$$

Since 42875 consists of two groups, the cube root will consist of two figures.

The first group, 42, contains the cube of the tens' number of the root.

The greatest cube in 42 is 27, and the cube root of 27 is 3. Hence 3 is the tens' figure of the root.

The remainder, 15875, resulting from subtracting the cube of the tens, will contain three times the product of the square of the tens by the units + three times the product of the tens by the square of the units + the cube of the units.

Each of these three parts contains the units' number as a factor.

Hence the 15875 consists of two factors, one of which is the units' number of the root; and the other factor is three times the square of the tens + three times the product of the tens by the square of the units + the square of the units. The larger part of this second factor is three times the square of the tens.

And, if the 158 hundreds of the remainder be divided by the $3 \times 30^2 = 27$ hundreds, the quotient will be the units' number of the root.

The second factor can now be completed by adding to the 2700 $3 \times (30 \times 5) = 450$ and $5^2 = 25$.

278. The same method will apply to numbers of more than two groups of figures, by considering the part of the root already found as so many tens with respect to the next figure of the root.

Extract the cube root of 57512456.

$$
\begin{array}{rr|l}
 & & 57\,512\,456\,(386 \\
 & & 27 \\
\hline
3 \times 30^2 = & 2700 & 30\,512 \\
3 \times (30 \times 8) = & 720 & \\
8^2 = & \underline{64} & \\
 & 3484 & \underline{27\,872} \\
 & & 2\,640\,456 \\
3 \times 380^2 = & 433200 & \\
3 \times (380 \times 6) = & 6840 & \\
6^2 = & \underline{36} & \\
 & 440076 & \underline{2\,640\,456}
\end{array}
$$

279. If the cube root of a number have decimal places, the number itself will have *three times* as many.

Thus, if 0.11 be the cube root of a number, the number is $0.11 \times 0.11 \times 0.11 = 0.001331$. Hence, if a given number contain a decimal, and if the figures of the number be divided into groups of three figures each, by beginning at the decimal-point and marking toward the left for the integral number and toward the right for the decimal, the number of groups toward the *left* from the decimal-point will show the number of *integral* places in the root, and the number of groups toward the *right* will show the number of *decimal* places in the root.

Extract the cube root of 187.149248.

$$
\begin{array}{rr|l}
 & & 187.149\,248(5.72 \\
 & & 125 \\
3 \times 50^2 = & 7500 & 62\,149 \\
3 \times (50 \times 7) = & 1050 & \\
7^2 = & 49 & \\
\hline
 & 8599 & 60\,193 \\
\hline
 & & 1\,956\,248 \\
3 \times 570^2 = & 974700 & \\
3 \times (570 \times 2) = & 3420 & \\
2^2 = & 4 & \\
\hline
 & 978124 & 1\,956\,248 \\
\hline
\end{array}
$$

It will be seen from the groups of figures that the root will have one integral and two decimal places, and therefore the decimal-point must be placed in the root as soon as one figure of the root is obtained.

280. If the given number be not a perfect cube, ciphers may be annexed, and a value of the root may be found as near to the *true* value as we please.

Extract the cube root of 1250.6894.

$$
\begin{array}{rr|l}
 & & 1\,250.689\,400(10.77 \\
 & & 1 \\
3 \times 10^2 = & 300 & 250
\end{array}
$$

Since 300 is not contained in 200, the next figure of the root will be 0.

$$
\begin{array}{rr|l}
3 \times 100^2 = & 30000 & 250\,689 \\
3 \times (100 \times 7) = & 2100 & \\
7^2 = & 49 & \\
\hline
 & 32149 & 225\,043 \\
\hline
 & & 25\,646\,400 \\
3 \times 1070^2 = & 3434700 & \\
3 \times (1070 \times 7) = & 22470 & \\
7^2 = & 49 & \\
\hline
 & 3457219 & 24\,200\,533 \\
\hline
 & & 1\,445\,867
\end{array}
$$

281. The following method very much shortens the work in long examples.

Extract the cube root of 5 to five places of decimals.

	5.000(1.70997
	1
$3 \times 10^2 = 300$	4 000
$3(10 \times 7) = 210$	
$7^2 = 49$	
559	3 913
259	87 000 000
$3 \times 1700^2 = 8670000$	
$3(1700 \times 9) = 45900$	
$9^2 = 81$	
8715981	78 443 829
45981	8 556 1710
$3 \times 1709^2 = 8762043$	7 885 8387
	670 33230
	613 34301

After the first two figures of the root are found, the next trial divisor is obtained by bringing down the sum of the 210 and 49 obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

It is seen at a glance that, when the trial divisor is increased by 3 times the 17 tens of the root, it will be greater than 87000; so that 0 is placed in the root, and 3×1700^2 is obtained by annexing two ciphers to the 86700. Again: the trial divisor is obtained by bringing down the sum of the 45900 and 81, which was obtained in completing the preceding divisor, then adding the three lines connected by the brace, and annexing two ciphers to the result.

The last two figures of the root are found by division. The rule in such cases is, that two less than the number of figures already obtained may be found without error by division, the divisor to be employed being three times the square of the part of the root already found.

282. The cube root of a common fraction is found by taking the cube roots of the numerator and denominator; but, if the denominator be not a perfect cube, it is best to reduce the fraction to a decimal, and then extract the root.

Ex. 161.

Find the cube roots of:

1. 29791.	**5.** 53157376.	**9.** 12396.8834.	**13.** $\frac{64}{729}$.
2. 357911.	**6.** 62099136.	**10.** 0.00027.	**14.** $\frac{125}{343}$.
3. 148877.	**7.** 41.421736.	**11.** 0.00008.	**15.** $\frac{16}{27}$.
4. 103823.	**8.** 12.812904.	**12.** 277.2738.	**16.** $\frac{7}{8}$.

17. The liter contains 61.027 cu. in. Find the side of a cube containing a liter.

18. The edges of a rectangular solid are 154 ft. 11 in., 70 ft. 7 in., 53 ft. 1 in. Find the edge of a cube equivalent to it.

The square of $(30 + 5) = 30^2 + 2(30 \times 5) + 5^2$. § 266.

The 30^2 may be represented by a square (Fig. 1) 30 in. on a side.

The $2(30 \times 5)$ may be represented by two strips 30 in. long and 5 in wide, of Fig. 2, which are added to two adjacent sides of Fig. 1.

The 5^2 may be represented by the small square of Fig. 3 required to make Fig. 2 a complete square.

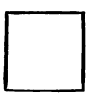


Fig. 1.

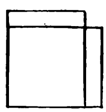


Fig. 2.

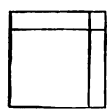


Fig. 3.

In extracting the square root of 1225, the large square, which is 30 in. on a side, is first removed, and a surface of 325 sq. in. remains.

This surface consists of two equal rectangles, each 30 in. long, and a small square whose side is equal to the width of the rectangles.

The width of the rectangles is found by dividing the 325 sq. in. by the sum of their lengths, that is, by 60, which gives 5 in.

Hence the entire length of the surfaces added is 30 in. + 30 in. + 5 in. = 65 in., and the width is 5 in.

Therefore the total area is $(65 \times 5) = 325$ sq. in.

The cube of $(30 + 5) = 30^3 + 3(30^2 \times 5) + 3(30 \times 5^2) + 5^3$. § 392.

The 30^3 may be represented by a cube whose edge is 30 in. (Fig. 1).

The $3(30^2 \times 5)$ may be represented by three rectangular solids, each 30 in. long, 30 in. wide, and 5 in. thick, to be added to three adjacent faces of Fig. 1.

The $3(30 \times 5^2)$ may be represented by three equal rectangular solids, 30 in. long, 5 in. wide, and 5 in. thick, to be added to Fig. 2.

The 5^3 may be represented by the small cube required to complete the cube of Fig. 3.

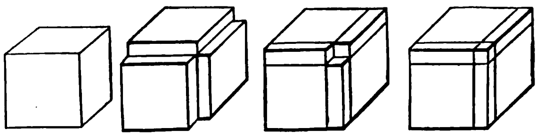


Fig. 1. Fig. 2. Fig. 3. Fig. 4.

In extracting the cube root of 42875, the large cube (Fig. 1), whose edge is 30 in., is first removed.

There remain $(42875 - 27000)$ cu. in. = 15875 cu. in.

The greater part of this is contained in the three rectangular solids which are added to Fig. 1, and which are each 30 in. long and 30 in. wide.

The thickness of these solids is found by dividing the 15875 cu. in. by the sum of the three faces, each of which is 30 in. square; that is, by 2700 sq. in. The result is 5 in.

There are also the three rectangular solids which are added to Fig. 2, and which are 30 in. long and 5 in. wide; and a cube which is added to Fig. 3, and which is 5 in. long and 5 in. wide.

Hence the sum of the products of two dimensions of all these solids is

For the larger rectangular solids,	$3(30 \times 30)$ sq. in. =	2700 sq. in.
For the smaller rectangular solids,	$3(30 \times 5)$ sq. in. =	450 sq. in.
For the small cube,	(5×5) sq. in. =	25 sq. in.
		3175 sq. in.

This number multiplied by the third dimension gives (5×3175) cu. in. = 15,875 cu. in.

A surface, therefore, has only two dimensions, length and breadth.

313. A line is no part of a surface. It is simply a boundary or limit of the surface. So that, if any number of straight lines be put together, they will have no thickness and no width, but will coincide and form only one line.

A line, therefore, has only one dimension, length.

314. A point is no part of a line. It is simply the limit of the line. So that, if any number of points be put together, they will have no length, breadth, or thickness, but will coincide and form a single point.

A point, therefore, has no dimension, but denotes position simply.

315. A point is *represented* to the eye by a fine dot, and named by a letter, as A (Fig. 2); a line is named by two letters, placed one at each end, as BF; a surface is represented and named by the lines which bound it, as $BCDF$; a solid is represented by the faces which bound it.

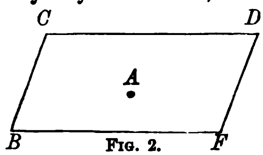


FIG. 2.

316. A line which has the same direction throughout is called a *straight* line, as AB (Fig. 3).

317. A line, no part of which is straight, is called a *curved* line, as CD.

318. A line composed of several straight lines lying in different directions is called a *broken* line, as EF.

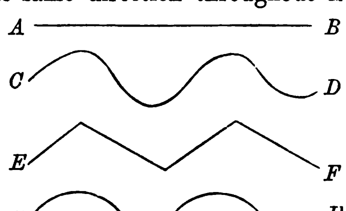


FIG. 3.

319. A line composed of straight and curved lines is called a *mixed* line, as GH.

320. A *plane surface* or a *plane* is a surface in which, if any two points be taken, the straight line joining these two points will lie wholly within the surface.

321. A *curved surface* is a surface no part of which is plane.

322. Since all straight lines which pass through the same point in the same direction coincide, *the position of a straight line is known if its direction and one of the points are known.*

Since all straight lines connecting two points coincide, *the position of a straight line is known if two points of the line are known.*

323. Of all lines between two points, the *shortest* is the straight line; and the straight line is called the *distance* between the two points.

324. When two straight lines cross each other, they are said to *intersect*, and the point of crossing is called their point of *intersection.*

325. To draw a straight line on paper between two points, we place the straight edge of a ruler so that it touches the two points; then draw the line with a pencil, keeping the pencil in contact with the ruler.

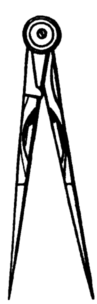


FIG. 4.

326. To draw a straight line on wood, we chalk a cord, and stretch it across two points, through which the line is to pass. While the cord is held firmly at two points, it is pulled away from the wood, and then let go. It strikes the wood, and leaves a white trace, which is a straight line.

327. To measure the distance between two points on paper, we employ an instrument called a pair of compasses (Fig. 4), or a divided rule.

ANGLES.

328. The opening between two straight lines, limited at their point of meeting, is called an *angle*, as BAC (Fig. 5).

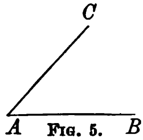


FIG. 5.

The point of meeting A is the *vertex* of the angle, the lines AB and AC are the *sides* of the angle.

An angle is designated by placing a letter at its vertex and one at each of its sides. In naming the angle, we name the letter at the vertex, or the three letters, putting the letter at the vertex between the other two, as BAC. An angle is also designated by putting a small letter within the angle, and close to the vertex.

329. The *magnitude* of an angle depends wholly upon the *extent of opening* of its sides, and not upon their length. Thus, if the sides of the angle BAC, namely AB and AC, be prolonged, their extent of opening will not be altered, and the size of the angle will not be altered; but, if the side AB remains fixed, and the side AC is made to turn about the vertex A, the angle increases or decreases according as the moving side is turned from or towards AB.

330. *Two angles are equal* when they can be applied the one to the other so as to coincide. Thus, the two angles BAC and EDF are equal, if, when the vertex A is placed upon D, and the line AB upon DE, the line AC falls upon the line DF. If AC falls below DF, the angle BAC is less than the angle EDF. If AC falls above DF, the angle BAC is greater than the angle EDF.

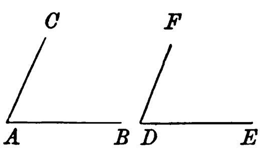


FIG. 6.

331. When two angles have a common vertex and a common side, they are called *adjacent angles.* Thus, the angles FCB and FCE (Fig. 7) are adjacent angles.

332. A line is *perpendicular* to another when it makes with this other *two equal adjacent angles,* called *right angles.*

Suppose the line CF (Fig. 7) can turn freely about the point C. Suppose also that CF lies upon CB. If we make CF turn towards the left about C, the angle BCF at first will be less than the angle ACF. But the angle BCF will increase, and the angle ACF diminish as CF moves to the left about the point C. There will be one position CE in which the two angles BCE and ACE will be equal. In this position EC is said to be *perpendicular* to AB, and the angles ECB and ECA are called *right angles.*

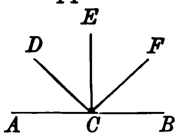


FIG. 7.

333. The sum of all the angles about a point in a straight line, situated on the same side of the straight line, is equal to two right angles (180°), and therefore the sum of all the angles about a point in a plane (on both sides of a straight line drawn through the point) is equal to four right angles (360°).

334. An *acute angle* is an angle less than a right angle, as BCF (Fig. 7).

335. An *obtuse angle* is an angle greater than a right angle, as ACF (Fig. 7).

336. If the sum of two angles is equal to a right angle (90°), each angle is called the *complement* of the other. Thus the angles BCF and FCE (Fig. 7) are complementary angles.

337. If the sum of two angles is equal to two right angles (180°), each angle is called the *supplement* of the other. Thus the angles *BCF* and *ACF* (Fig. 7) are *supplementary* angles.

338. Acute and obtuse angles in distinction from *right angles* are called *oblique angles*, and intersecting lines not perpendicular to each other are called *oblique lines*.

339. When a cord with a weight attached at one end is freely suspended from the other end and is at rest, the line of direction of the cord is a *vertical line* (generally called *a plumb line*).

340. A line perpendicular to a vertical line is called a *horizontal line*.

341. If two angles have the same vertex, and if the sides of the one prolonged coincide with the sides of the other, they are called *vertical angles*. Thus, the angles a and b (Fig. 8) are called vertical angles.

342. THEOREM. *Two vertical angles are equal.*

It is required to show that a equals b.

Now, $a + c = 180°$;

and $b + c = 180°$.

(The sum of the angles about a point in a straight line on the same side of the lines equals 180°.)

Therefore $a + c = b + c$.

(When two magnitudes are each equal to a third magnitude, they are equal to each other.)

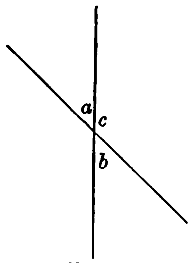


FIG. 8.

Taking away the angle c from each side of the equality, we have,

The angle a = the angle b.

343. THEOREM. *A perpendicular is the shortest distance from a point to a straight line.*

It is required to show that the perpendicular CD (Fig. 9) is less than any oblique line CE drawn from C to AB.

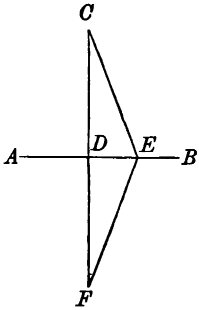


FIG. 9.

On AB as an axis turn the figure CDE until DC takes the position of DF, and EC the position of EF; then turn the figure CDE back to its original position. CF is less than $CE + EF$ (§ 323).

That is, $2\,CD$ is less than $2\,CE$, or CD is less than CE.

By regarding BA as perpendicular to CF, since EC is equal to EF, and CD equal to DF, it follows that,

344. *Two oblique lines drawn from a point in a perpendicular, and cutting off equal distances from the foot of the perpendicular, are equal.*

345. *If two equal oblique lines are drawn from a point in a perpendicular, they cut off equal distances from the foot of the perpendicular.*

346. *If a perpendicular is drawn through the middle point of a straight line, every point in the perpendicular is equally distant from the extremities of the straight line.*

347. *Two points each equidistant from the extremities of a straight line determine the perpendicular to the middle of that line.*

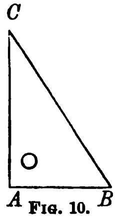


FIG. 10.

348. A set-square (Fig. 10) is a piece of wood with three straight edges, two of which contain a right angle A. AB is the base edge, AC the perpendicular edge, and BC the hypotenuse.

349. PROBLEM. *At a given point in a straight line to erect a perpendicular to this line, with the aid of a set-square.*

Place a ruler so that its edge will coincide with the straight line, and then place the base edge of the square against the ruler, and slide the square along until the point is reached; then draw a line, keeping the pencil in contact with the perpendicular edge of the square. The line thus drawn is the line required.

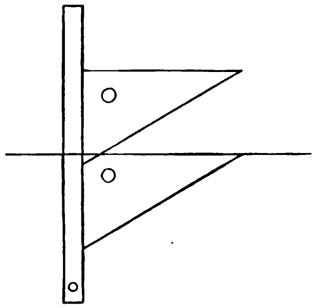


FIG. 11.

350. PROBLEM. *To let fall a perpendicular to a given straight line from a point without the line, with the aid of a set-square.*

Place the ruler and square in the positions described in the last problem, and slide the square along until its perpendicular edge touches the point. Draw a line from the point to the given straight line, keeping the pencil in contact with the perpendicular edge of the square. The line thus drawn is the line required.

351. PROBLEM. *At a given point in a given straight line to erect a perpendicular to this line, with the aid of compasses.*

Let AB be the given line, and P the given point.

It is required to erect a perpendicular at the point P.

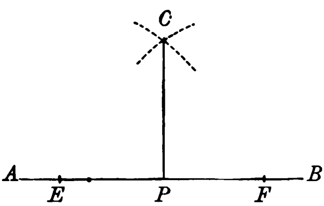


FIG. 12.

Take the equal distances PE and PF, and from E and F as centres with the same opening of the compasses describe two arcs intersecting at some point, as C. The line drawn through C, P is the perpendicular required (§ 347).

352. *At a given point in a given line only one perpendicular to the line can be erected.*

353. PROBLEM. *From a given point to let fall a perpendicular to a given straight line, with the aid of compasses.*

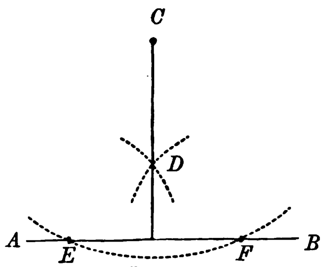


FIG. 13.

Let AB (Fig. 13) be the given line, and C the given point.

It is required to let fall a perpendicular from C to the straight line AB.

From C as a centre, with an opening of the compasses sufficiently great, describe an arc, cutting AB at E and F. From E and F as centres, with the same opening of the compasses, describe two arcs intersecting at D. A line drawn through the points C and D to the given straight line is the line required. For C and D, being two points equally distant from E and F, determine the position of the perpendicular to the middle of EF, and consequently perpendicular to the line AB, § 347.

354. *From a given point without a straight line only one perpendicular can be let fall to that line.*

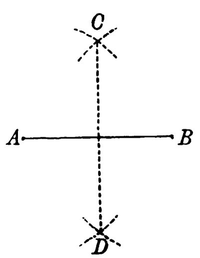


FIG. 14.

355. PROBLEM. *To divide a straight line into two equal parts, that is, to bisect a straight line.*

It is required to bisect AB (Fig. 14).

From A and B as centres, with the same opening of the compasses, describe arcs intersecting at C and D. Draw CD. Now C and D, being two points equally distant from A and B, determine the perpendicular to the middle of AB, § 347.

CIRCUMFERENCES.

356. A *circumference* is a curved line all points of which are equally distant from a point within called the *centre*.

357. To trace a circumference, fix upon paper one of the points of the compasses open, and turn the instrument about the fixed point in such a way that the tracing point does not leave the paper. The line thus formed is called a circumference, since all points are at the same distance from the immovable point which is the centre.

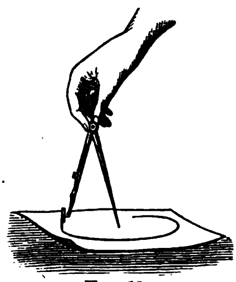


FIG. 15.

358. The *radius* is the distance OA from the centre O to any point of the circumference (Fig. 16).

359. The *diameter* is a straight line passing through the centre and terminated by the circumference, as BOC. The diameter is double the radius, and divides the circumference into two equal parts.

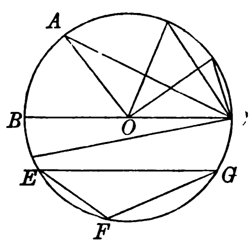


FIG. 16.

360. An *arc* is a portion of the circumference, as EFG.

361. A *chord* is a straight line which joins the extremities of an arc, as EG. A chord is said to *subtend* its arc, and an arc is said to be subtended by its chord.

362. A *circle* is a portion of the plane which is bounded by the circumference.

363. A *sector* is a part of a circle contained by two radii and the intercepted arc, as AOB.

364. A *segment* is the part of a circle comprised between an arc and its chord, as $EFGE$.

365. It follows from the definition of a circumference that the arc of a sector does not cease to coincide with the circumference when the sector is made to turn about its centre.

366. THEOREM. *In the same circle, or in equal circles, equal arcs are subtended by equal chords.*

Let the arcs AMB and CND be equal (Fig. 17).

Turn the arc AMB about the centre O until it coincides with CND. The two arcs will then have the same extremities, and the chords AB and CD will coincide and be equal.

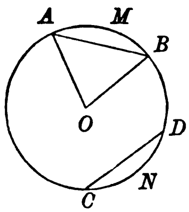


FIG. 17.

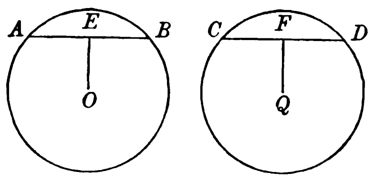


FIG. 18.

367. THEOREM. *In the same circle, or in equal circles, equal chords are equally distant from the centre.*

Let AB and CD be two equal chords in the two equal circles whose centres are O and Q. It is required to show that the perpendicular OE let fall from the centre O upon AB is equal to the perpendicular QF let fall from the centre Q upon CD.

Place the centre O upon Q. The two circles will coincide. Turn the circle whose centre is O about Q until the chord AB falls exactly upon its equal CD; then the perpendicular OE will coincide with, and be equal to, QF; for, from a point without a straight line, only one perpendicular to that line can be drawn.

368. THEOREM. *The diameter perpendicular to a chord divides the chord and each of the arcs which it subtends into two equal parts.*

Let AB be a chord, and MN a diameter perpendicular to the chord which passes through the centre O and intersects the chord in D.

It is required to show

$$AD = DB,$$
$$\text{arc } AM = \text{arc } MB,$$
$$\text{arc } AN = \text{arc } NB.$$

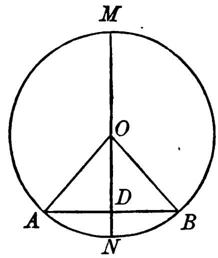


FIG. 19.

Turn the semicircle NAM about MN as an axis until it falls upon the semicircle NBM.

Then, since ODA and ODB are right angles, DA will take the direction of DB, and the point A will fall somewhere in the line DB; but the point A, being in the semi-circumference MAN, will fall somewhere in the semi-circumference MBN. That is, the point A, falling at the same time in the line DB and the semi-circumference MBN, must fall upon a point common to them, namely, B. So that DA will coincide with DB, arc MA with arc MB, and arc NA with arc NB.

369. *A perpendicular to a chord at its middle point passes through the centre of the circle, and bisects each of the arcs which the chord subtends.*

DIVISION OF THE CIRCUMFERENCE.

370. From § 178 it is seen that the whole angular magnitude about a point in a plane is divided into 360 equal parts, called *degrees*. Each degree is subdivided into 60 equal parts, called *minutes*, and each minute into 60 equal parts, called *seconds*.

Likewise the circumference of a circle is divided into 360

equal arcs, each corresponding to an angle of 1° at the centre. These arcs are called *degrees*, and are each subdivided like the angle-degree into minutes and seconds. Hence,

371. *An angle is said to be measured by the arc described from its vertex as centre, and included between its sides.*

The meaning of this is that the angle is such a part of 360° of angular magnitude as the corresponding arc is of 360° of circumference.

372. *If the two arcs described from the vertices of two angles as centres with equal radii and included by their sides are equal, the angles are equal.*

373. Having given a circumference divided into equal parts, if the several points of division are joined to the centre, the radii thus obtained will divide likewise into equal parts every circumference that has the same centre as the given circumference.

Consider for example the two concentric circumferences *ABCD* and *abcd*. If the arcs *AB*, *BC*, *CD* are equal,

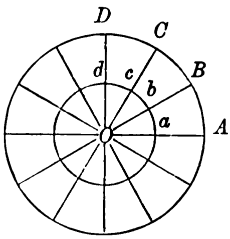


FIG. 20.

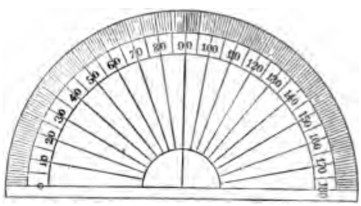


FIG. 21.

the radii *OA*, *OB*, *OC* will cut off equal arcs *ab*, *bc*, *cd*, upon the circumference *abcd*.

374. If we have then a circle marked off in degrees, we can easily divide into degrees a circle of any radius. This divided circle is called a *protractor*.

The protractor (Fig. 21) is a semicircle made of horn or hard rubber, and divided into degrees from 0° to 180°.

375. PROBLEM. *Having given an angle AOB, find with the protractor the number of degrees it contains.*

Place the diameter of the protractor upon the line OA, with its centre upon the vertex O. Read from the protractor the degrees contained between OA and OB. The number of degrees comprised between the sides OA and OB is the number required.

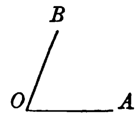


FIG. 22.

376. The *ratio of two angles* is found by finding the number of degrees in each angle, and taking the ratio of the two numbers thus found.

377. PROBLEM. *At a given point in a given straight line, to construct an angle equal to a given angle, by means of compasses.*

Let E be the given point in the line EF, and C the given angle (Fig. 23).

It is required to construct an angle at E equal to the angle C.

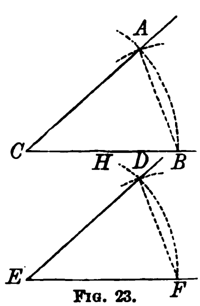


FIG. 23.

From C as a centre, with any radius, as CB, describe the arc BA, terminating in the sides of the angle; and from E as a centre, with a radius equal to CB, describe the arc FH.

From F as a centre, with a radius equal to the distance BA, describe an arc intersecting the arc FH at D.

Draw DE; and the angle $DEF =$ angle ACB.

For the chords AB and DF are equal, and, therefore, the arcs AB and DF are equal. Hence the angles E and C are equal (§ 372).

378. PROBLEM. *To bisect a given arc.*

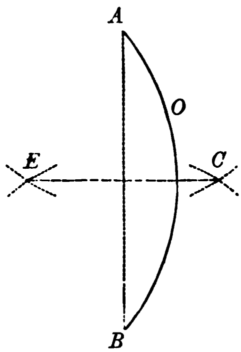


FIG. 24.

It is required to bisect the arc AOB (Fig. 24).

Draw the chord AB.

From A and B as centres, with equal radii, describe arcs intersecting at E and C.

Draw EC.

EC bisects the arc AOB.

For, E and C, being two points at equal distances from A and B, determine the position of the perpendicular to the middle of chord AB; and a perpendicular erected at the middle of a chord passes through the centre of the circle, and bisects the arc of the chord (§ 369).

PARALLEL LINES.

379. Two straight lines lying in the same plane are *parallel* when they cannot meet, however far produced.

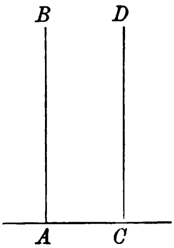


FIG. 25.

To obtain two straight parallel lines, it is only necessary to draw two perpendiculars to the same straight line.

Thus, the straight lines AB and CD, being two perpendiculars drawn in a plane to the same straight line AC, are parallel; for, if they could meet, we could, from their point of intersection, let fall two distinct perpendiculars upon AC, but this is impossible; for, from a point without a straight line, only one perpendicular can be drawn from the point to the straight line.

380. Problem. *To draw through a given point a line parallel to a given straight line, by means of a ruler and set square.*

Let M be the given point, and AB the given straight line. Apply to the line AB the side ab of the right angle of a set-square, then apply to the side ac the edge of the ruler. Hold the ruler firmly pressed upon the paper, and make the square glide along the ruler until the edge ab comes to the given point M. Hold the square in this position, and trace with a pencil the line MN. It is the parallel required; for AB and MN are two perpendiculars to the same straight line, which straight line in this case is the edge of the ruler.

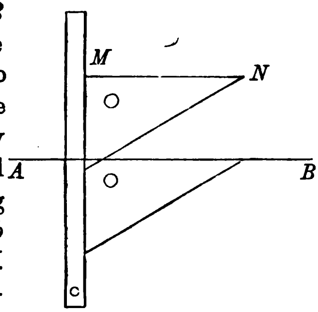


Fig. 26.

381. *Two parallel straight lines are everywhere equally distant.*

This property is often useful in drawing parallel lines of great length. If perpendiculars are erected at different points, G, H, ..., of one line AB, and if these perpendiculars are taken of the same length, their extremities E, F, ..., are all in a line parallel to AB, and this parallel is the line drawn through the extremities E, F, ... of these perpendiculars.

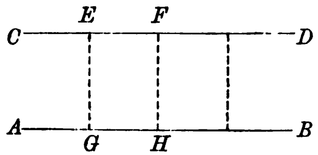


Fig. 27.

POLYGONS.

382. A *polygon* is a portion of a plane bounded by straight lines, as $ABCDE$ (Fig. 28). A polygon has as many angles as sides, and the vertices of the angles are the vertices of the polygon.

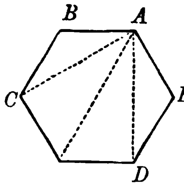


FIG. 28.

383. The *diagonal* of a polygon is a straight line which connects two vertices not adjacent, as AC and AD.

384. A polygon of three sides is called a *triangle.*

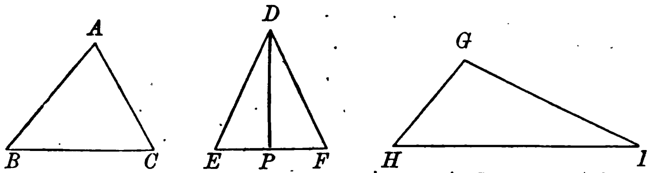


FIG. 29.

385. An *equilateral* triangle is a triangle which has its three sides equal, as ABC (Fig. 29). A triangle which has its three sides unequal is called a *scalene* triangle, as GHI.

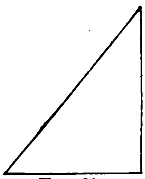


FIG. 30.

386. An *isosceles* triangle is a triangle in which two sides are equal, as DEF, in which DE equals DF.

387. In every isosceles triangle, *the angles opposite the equal sides are equal.* The side which is not one of the two equal sides, is called the *base* of the triangle, and the vertex opposite the base is called the *vertex* of the triangle.

388. A *right triangle* (Fig. 30) is a triangle which has a right angle. The side opposite the right angle is called the *hypotenuse*, and the other two sides are called the *legs.*

389. The *altitude* of a triangle is the perpendicular let fall from a vertex to the opposite side, as DP (Fig. 29).

The altitude of an isosceles triangle falls at the middle of the base, *and divides the triangle into two equal parts.*

390. A polygon of four sides is called a *quadrilateral* (Fig. 31).

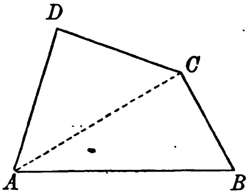


FIG. 31.

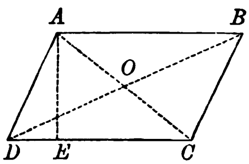


FIG. 32.

391. A quadrilateral of which the sides are parallel, two and two, is called a *parallelogram.* Thus the quadrilateral $ABCD$ (Fig. 32) is a parallelogram. The sides AB and CD are parallel, and likewise the sides AD and BC.

392. We verify, by means of a pair of compasses, that *in a parallelogram the parallel sides are equal.* Thus, AB equals DC, and AD equals BC.

393. We can also verify, by means of a protractor, that *the opposite angles of a parallelogram are equal.* Thus, BAD equals BCD, and ABC equals ADC.

394. The diagonals of a parallelogram are, in general, unequal, *but they always bisect each other.* Thus, AO equals OC, and BO equals OD.

395. *Every diagonal divides the parallelogram into two equal triangles.* Thus, if we turn the triangle BDC about O, through an angle of 180°, the point C will fall upon A, the point B upon D, and the triangles will coincide.

396. The *altitude* AE (Fig. 32) of a parallelogram is the distance between the two parallel sides, called the *bases*.

397. A *rectangle* is a parallelogram whose angles are right angles (Fig. 33). The side AB is the base of the

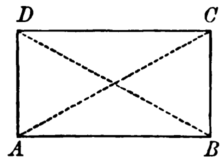


Fig. 33.

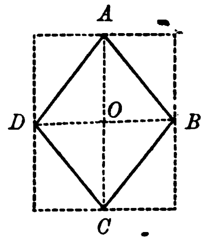


Fig. 34.

rectangle, and AD is its altitude. These two lines are called the *dimensions* of the rectangle.

398. *The diagonals of a rectangle bisect each other*, since the rectangle is a parallelogram. Moreover, *the diagonals are equal;* thus, AC equals BD.

399. A *rhombus* $ABCD$ (Fig. 34) is a parallelogram whose four sides are equal, and whose angles are oblique angles. The diagonals, in general, are unequal, and bisect each other. Moreover, *the diagonals cut each other at right angles.* Thus, the angle AOB is a right angle.

400. The *square* is a rectangle whose four sides are equal (Fig. 35). The diagonals of a square are equal, and bisect each other at right angles. *Hence, if a quadrilateral has its four sides equal, and its diagonals equal, it is a square; if it has its four sides equal, and its diagonals unequal, it is a rhombus.*

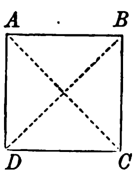


Fig. 35.

401. A *trapezoid* is a quadrilateral which has two sides parallel, and only two (Fig. 36). The

two parallel sides AB and CD are the bases of the trapezoid, and the altitude AE is the distance between the bases.

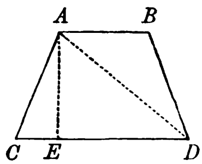


Fig. 36.

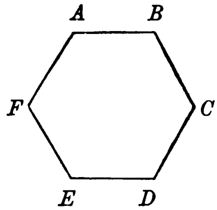


Fig. 37.

402. A polygon is regular when it has its sides equal and its angles equal. Thus, the polygon $ABCDEF$ (Fig. 37) is a regular polygon of six sides, or a regular hexagon.

403. The *perimeter* of a polygon is the sum of the lengths of its sides.

404. Problem. *To construct a triangle when its three sides are given.*

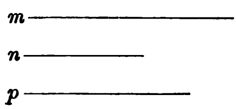


Fig. 38.

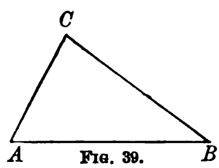


Fig. 39.

Let m, n, and p denote the lengths of the sides. Make AB equal to m. From A and B as centres, with radii equal to n and p, respectively, describe arcs intersecting at C. Join AC and AB, and the triangle ABC is the triangle required.

405. A polygon is said to be *inscribed* in a circle when its vertices lie in the circumference of the circle, and the circle is said to be *circumscribed* about the polygon.

406. PROBLEM. *To inscribe a square in a circle.*

Draw two diameters AE, CG (Fig. 40) perpendicular to each other, and join the extremities of these diameters. We have the square $ACEG$ (§ 400), for its sides are equal, since they are chords subtending equal arcs, and its diagonals are equal, since they are diameters of the same circle.

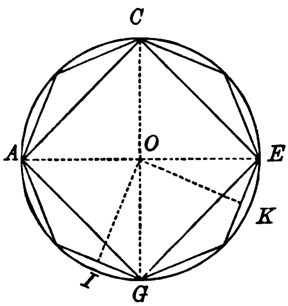


FIG. 40.

407. By bisecting the four arcs, and joining the points of division, we shall have a regular octagon inscribed.

All the sides of the regular octagon are equally distant from the centre O (§ 367), and therefore the perpendiculars OI and OK let fall from the centre upon the sides have the same length, and each of them is called the *apothem* of the polygon.

By bisecting these eight arcs, and joining the points of bisection, we shall have a regular polygon of 16 sides, and by continuing this process we may inscribe regular polygons of 32 sides, 64 sides, and so on.

408. PROBLEM. *To inscribe in a given circle a regular hexagon.*

The side of a regular hexagon is equal to the radius. A regular hexagon is obtained then by applying six times

upon the circumference an opening of the compasses equal to the radius. By joining the alternate vertices B, D, F of a regular hexagon, we have an equilateral triangle inscribed. By bisecting the arcs AB, BC, a regular polygon of 12 sides may be inscribed, and by continuing the process regular polygons of 24 sides, 48 sides, 96 sides, may be inscribed.

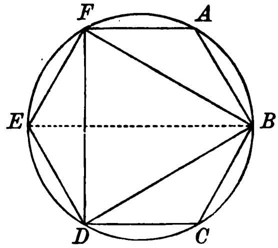


FIG. 41.

LENGTH OF THE CIRCUMFERENCE.

409. In inscribing in the same circle polygons of a greater and greater number of sides, for instance, polygons of 12, 24, 48, 96, etc., sides, we see that the lengths of their perimeters differ less and less from the length of the circumference.

If then we wish to measure the length of a circumference of a circle, we inscribe in it a regular polygon of a great number of sides, and measure its perimeter. We shall then have a value a little less than, but approximating very closely to, the length of the circumference. The error in this approximation will be diminished as the number of sides of the regular inscribed polygon is increased.

410. Below are given results of very exact computations which are employed instead of direct measuring.

Number of sides.	Perimeter of the inscribed polygons.
6	3.00
12	3.1058
24	3.1326
48	3.1393
96	3.1410
192	3.1414
384	3.1415

411. It is found that the length of the circumference whose diameter is 1 yd. is 3.1416 yds. The Greek letter π is used to designate the number 3.1416; that is, $\pi = 3.1416$. Hence the rule.

412. *To find the length of a circumference, multiply its diameter by the number* 3.1416.

413. If C denotes the circumference, R the radius, $2R$ will denote the diameter, and we shall have the formula,

$$C = 2\pi R.$$

414. The letter π is called the *ratio* of the circumference to the diameter.

APPLICATION. Find the circumference of a circle whose radius is 20 ft.

$$\begin{aligned} \text{Circum.} &= 2\pi R \\ &= 40\pi \\ &= 125.664 \text{ ft.} \end{aligned}$$

Ex. 172.

Find the circumference of a circle when the radius is equal to:

1. 9 ft.	3. 2.5 mi.	5. 75 yds.
2. 256 ft.	4. 8 mi.	6. 3.25 yds.

Find the circumference when the diameter is equal to:

7. 60 ft.	9. 8 in.	11. 100 rds.
8. 80 ft.	10. 12 in.	12. 160 rds.

Find the diameter when the circumference is:

13. 162.4 ft.	15. 11 in.	17. 64 ft.
14. 2.16 mi.	16. 5 in.	18. 40 mi.

NOTE. Since the reciprocal of 3.1416 is 0.31831, the diameter is found by multiplying the circumference by 0.31831.

19. If the driving wheels of a locomotive have a diameter of 6 ft., how many revolutions will they make in going from Portland to Boston, 106 miles?

20. What is the diameter of a circular reservoir, if a man in walking round it takes 100 steps, each $2\frac{1}{2}$ ft.?

21. The diameters of two circles having the same centre are 8 ft. and 20 ft.; find the length of the circumference lying midway between them.

22. If the diameter of a circle is 3 yds., find the length of an arc of 160°.

23. The length of an arc of 40° is 10 ft.; find the diameter.

24. Find the distance traversed in an hour by a point in the circumference of a water-wheel 7 ft. in diameter, if the wheel makes 1400 revolutions in one hour.

25. If the radius of a circle is 2 ft., find the number of degrees in an arc of the circle 2 ft. in length.

Areas.

415. The area of a surface is the number of square units (square inches, feet, yards, or rods) which it contains.

416. Two figures are *equal* when they can be applied to each other so as to coincide; two figures are *equivalent* when they have the same area, but cannot be applied to each other so as to coincide. Thus, if we cut the square $ABCD$ into two equal parts by the straight line EF, which joins the middle points of the sides AC and BD, and place the two rectangles as represented in the right-hand figure, we obtain a rectangle equivalent to the square, but which cannot be made to coincide with it.

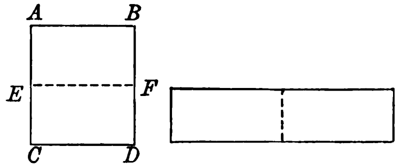


Fig. 42.

417. Theorem. *The area of a rectangle is equal to the product of its base by its altitude* (§ 160).

418. Theorem. *The area of a square, therefore, is equal to the square of the number which measures its side.*

419. Theorem. *The area of a parallelogram is equal to the product of its base by its altitude.*

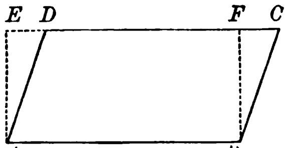


Fig. 43.

Upon the base AB of the parallelogram $ABCD$ construct the rectangle $ABEF$.

The parallelogram $ABCD =$ trapezoid $ABFD +$ triangle BCF.

The rectangle $ABEF =$ trapezoid $ABFD +$ triangle AED.

Triangles BCF and AED can be applied to each other, so as to coincide.

Then $ABCD$ is equivalent to $ABEF$.

Therefore, the area of the parallelogram is equal to AB multiplied by BF; that is, it is equal to the product of its base by its altitude.

420. Theorem. *The area of a triangle is equal to one-half of the product of its base by its altitude.*

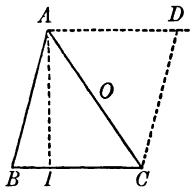


Fig. 44.

For a triangle is half of a parallelogram of the same base and altitude. Thus, if through the point C of the triangle ABC (Fig. 44) we draw CD parallel to AB, and through the point A, AD parallel to BC, we shall have a parallelogram $ABCD$, of which AC will be the diagonal.

We can make the two triangles ABC and ACD coincide by turning ABC 180° about the point O, the middle of AC.

Hence, $ABC = ACD$, and $ABC = \frac{1}{2} ABCD$.

Now, the area of the parallelogram is equal to $BC \times AI$.
Therefore, area of triangle $ABC = \frac{1}{2} BC \times AI$.

421. *Two triangles, therefore, of the same base and same altitude are equivalent.*

422. THEOREM. *The area of a trapezoid is equal to the half-sum of its parallel bases multiplied by its altitude.*

Draw the diagonal AC (Fig. 45) in the trapezoid $ABCD$. This diagonal decomposes the trapezoid into two triangles ACB, DAC, which have for altitudes CF and AE. These altitudes are equal, since each of them represents the distance between the bases of the trapezoid. We have then,

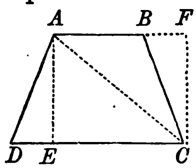


FIG. 45.

$$ACB = \tfrac{1}{2} AB \times CF.$$
$$ACD = \tfrac{1}{2} CD \times AE.$$

Adding and noticing that CF and AE are equal,

$$ACB + ACD = ABCD = \tfrac{1}{2}(AB + CD) \times CF.$$

423. PROBLEM. *To find the area of a polygon by decomposing the polygon into triangles.*

Draw the diagonals AC, AD, AE (Fig. 46) from the same vertex A. They decompose the polygon into as many triangles as the figure has sides less two. The sum of the areas of all these triangles will equal the area required.

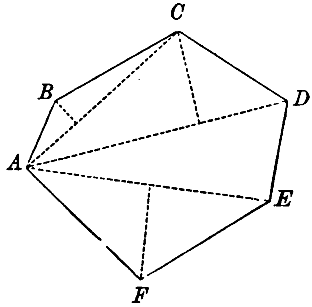


FIG. 46.

Upon the land the lengths of the diagonals are measured by a *surveyor's chain*, and the altitudes of the triangles are determined by means of an instrument called a *surveyor's cross;* they are measured afterwards with the chain.

424. *To find the area of a polygon by decomposing the polygon into trapezoids and right triangles.*

Thus, in the polygon $ABCDEFG$, draw the longest diagonal AE of the polygon, and let fall from the vertices perpendiculars upon this diagonal. Find the areas of the triangles and trapezoids thus formed, and their sum will be the area required of the polygon.

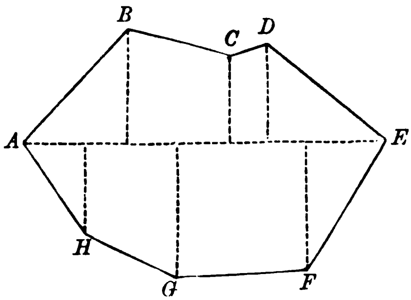


FIG. 47.

425. THEOREM. *The area of a regular polygon is equal to one-half the product of its perimeter by its apothem.*

Thus, in the regular octagon ABC, etc., if we draw from the centre O lines to the vertices of the polygon, we have eight equal triangles, and the sum of their areas is $\frac{1}{2}(8EF \times OI)$.

Since $8EF$ is the perimeter, and OI is the apothem, the area is one-half of the perimeter multiplied by the apothem.

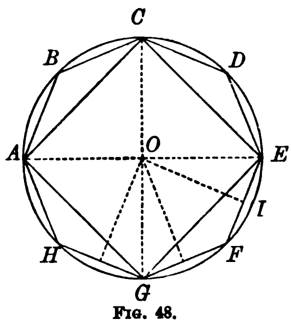


FIG. 48.

426. THEOREM. *The area of a circle is equal to one-half the product of the circumference by the radius.*

The area, circumference, and radius of a circle will differ very little from the area, perimeter, and apothem, respectively, of a regular inscribed polygon of a great number of sides. And, as the area of this regular polygon, however great the number of its sides, is equal to one-half the prod-

uct of its perimeter and apothem, the area of a circle is equal to one-half the product of its circumference and radius.

427. If R denotes the radius of a circle, $2R$ will denote the diameter, and $2\pi R$ will denote the circumference. Hence, the area of a circle $= \frac{2\pi R \times R}{2} = \pi R^2$.

428. *The area of a circle therefore is found by multiplying the square of the radius by the number* 3.1416 ($= 3\frac{1}{7}$ nearly).

429. The area of a sector is such a part of the area of its circle as the arc of the sector is of the circumference of the circle. Hence,

The area of a sector is equal to the product of one-half the arc by the radius.

Thus, if the arc AB of the sector AOB (Fig. 49) is $\frac{1}{12}$ of the circumference, the area of the sector is $\frac{1}{12}$ of the area of the circle.

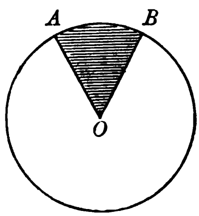


FIG. 49.

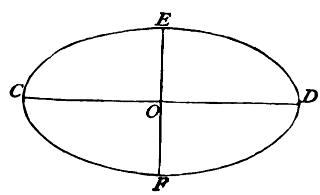


FIG. 50.

430. The oval made by the shadow of a circular plate is called an *ellipse*. (Fig. 50.)

The area of an ellipse is $\frac{\pi}{4}$ of the product of its longest and shortest diameters.

Ex. 173.

1. In the parallelogram $ABCD$ (Fig. 43), the angle A is 45° and the sides which include this angle are $AB = 48^m$, and $AD = 37^m$. Construct this figure to the scale of one millimeter to a meter. Measure its altitude and compute its area.

NOTE. The pupil is expected to obtain only a rough approximation to the true answer.

2. The lengths of the sides of the triangle ABC (Fig. 44) are respectively $AC = 72^m$, $AB = 64^m$, and $BC = 60^m$. Construct this triangle to the scale of one millimeter to a meter. Draw the altitude to the side BC, measure this altitude, and compute the area of the triangle. Verify the result by the rule: When the three sides of a triangle are known the area is found

By subtracting each side separately from half the sum of the three sides, and extracting the square root of the continued product of the half-sum and of the three remainders.

3. Find, by the rule just given, the area of a triangle whose three sides are 20, 30, and 40 ft., respectively.

4. Find the area of a triangle whose base is 150 ft. and altitude 42 ft.

5. The diagonals of a rhombus (Fig. 34) are 4 yds. and 3 yds. Find its area.

6. A trapezoid $ABCD$ (Fig. 45) has for its bases $AB = 29^m$, $CD = 57^m$, and for its non-parallel sides $AD = 25^m$, $BC = 25^m$. Construct this trapezoid to the scale of $\frac{1}{1000}$. Measure its altitude and find its area.

7. In a quadrilateral $ABCD$ (Fig. 31) the diagonal AC is 61^m, and the perpendiculars let fall from B and D upon AC are 32^m and 29^m respectively. Find the area of the quadrilateral.

8. Find the area of a regular hexagon whose sides are 20 ft. each and apothem 17.3205 ft.

9. Find the area in square inches of a circle whose circumference is 1 yd.

10. Find the radius in inches of a circle whose area is 1 sq. yd.

11. A square is inscribed in a circle whose radius is 42 in. Find the area of the four segments of the circle outside the square.

12. The long and short diameters of an ellipse are 48 ft. and 40 ft. respectively. Find its area.

Surfaces and Volumes of Bodies.

431. A body bounded on all sides by polygons is called a **polyhedron.** These polygons are the *faces* of the polyhedron; the sides and vertices of the polygons are the *edges* and *vertices* of the polyhedron. All the faces together constitute the *surface* of the polyhedron.

432. The most simple of the polyhedrons has four triangular faces, six edges, and four vertices. It is called a **tetrahedron** (Fig. 51).

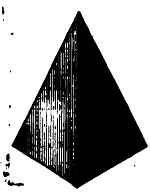


Fig. 51.

433. Polyhedrons of 6, 8, 12, and 20 faces are called *hexahedron*, *octahedron*, *dodecahedron*, *icosahedron*, respectively.

434. A **prism** is a polyhedron (Fig. 52) bounded by parallelograms and by two equal and parallel polygons. These two polygons are the *bases* of the prism. The distance between the planes of their bases (HL) is the *height* or *altitude* of the prism. The parallelograms are the *lateral faces.*

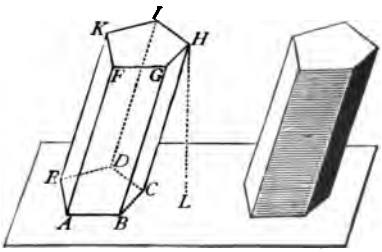


FIG. 52.

435. A prism is *right* or *oblique* according as the lateral edges are *perpendicular* or *oblique* to the planes of the bases. In a *right prism* all the lateral faces are rectangles, and its altitude is equal to one of the lateral edges.

436. A prism is *triangular*, *quadrangular*, etc., according as its base is a *triangle* or *quadrilateral*, etc. A *right section* of a prism is the section made in a prism by a plane perpendicular to the lateral edges.

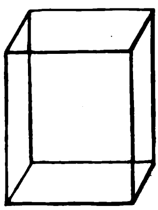


FIG. 53.

437. A **parallelopiped** (Fig. 53) is a prism whose bases are parallelograms; that is, it is a body bounded by six parallelograms. If the lateral faces are rectangles, and the bases parallelograms, it is a *right parallelopiped.* If all the faces are rectangles, it is a *rectangular parallelopiped.* If all the faces are equal squares, it is a *cube.*

438. A **cylinder of revolution** is the body generated by the revolution of a rectangle $ABCD$ revolving about one side CD regarded as fixed. The circles described by the sides AC and BD are the *bases* of the cylinder. The fixed side CD is the *altitude* or *axis*. The side AB generates the *lateral surface* of the cylinder.

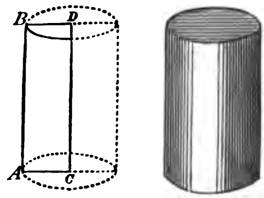


FIG. 54.

439. The lateral surface (or convex surface) of a cylinder, if unrolled, will form a rectangle. The base of this rectangle will be equal to the circumference of the base of the cylinder, and the altitude of the rectangle will be equal to the altitude of the cylinder.

440. *The area of the lateral surface of a prism is the sum of the areas of its lateral faces.*

The area of the total surface of a prism is the sum of the areas of its lateral faces and of its two bases.

441. *The area of the lateral surface of a cylinder is the product of the circumference of the base by the altitude of the cylinder.*

442. *The area of the total surface of the cylinder is the sum of the areas of its lateral surface and of its two bases.*

443. *The volume of a prism or cylinder is the product of its base by its altitude.*

444. A **pyramid** (Fig. 55) is a polyhedron bounded by triangles that have a common vertex, and by a polygon whose sides are the bases of the triangles. The vertex

common to the triangles is the *vertex* of the pyramid. The polygon is the *base* of the pyramid, and the *triangles* are the lateral faces. The perpendicular let fall from the vertex to the plane of the base is the *altitude* of the pyramid.

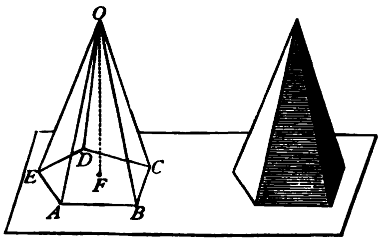


FIG. 55.

445. A pyramid is *triangular*, *quadrangular*, etc., according as its base is a *triangle*, a *quadrilateral*, etc.

446. A pyramid is *regular* when its base is a regular polygon, and its altitude falls at the centre of the base. The perpendicular from the vertex to a side of the base is the *slant height*.

447. A **right cone** (Fig. 56) with a circular base is the body generated by the rotation of a right triangle about one of its sides regarded as fixed. The point O is the *vertex* of the cone. The circle described by AB is the *base* of the cone. The fixed side OB is the *altitude* or *axis* of the cone. The side OA generates the lateral surface and is called the *slant height* of the cone.

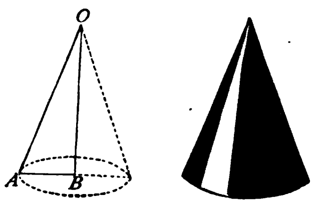


FIG. 56.

448. The lateral surface of a cone, if unrolled, will form a sector of a circle whose radius will be the slant height of the cone, and whose arc will be equal to the circumference of the base of the cone.

449. *The area of the lateral surface of a regular pyramid or of a right circular cone is one-half the product of the perimeter of the base by the slant height.*

450. *The area of the total surface of a pyramid or cone is the sum of the areas of the lateral surface and of the base.*

451. *The volume of a pyramid or cone is equal to one-third of the product of its base by its altitude.*

452. The **frustum** (Figs. 57 and 58) of a pyramid or cone is the part remaining after the top has been cut off by a plane parallel to the base.

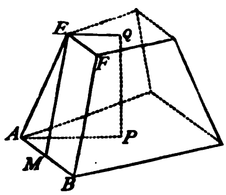


FIG. 57.

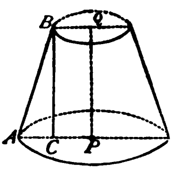


FIG. 58.

The distance between the bases PQ is the *altitude* of the frustum.

453. *The lateral surface of the frustum of a pyramid or cone is equal to half the product of the sum of the perimeters by the slant height.*

454. *The area of the total surface of a frustum is the sum of the areas of the lateral surface and of the two bases.*

455. *The volume of a frustum is equal to the sum of the areas of the two bases and the square root of their product multiplied by one-third of the altitude.*

456. A **sphere** (Fig. 59) is a body generated by the revolution of a semi-circle (or circle) about its diameter regarded as fixed. Hence, it is a body bounded by a surface all points of which are equally distant from a point within called the *centre.*

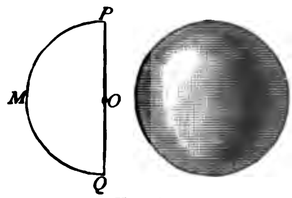


FIG. 59.

457. The straight line joining the centre to any point on the surface is a *radius* of the sphere; and a straight line passing through the centre and terminated by the surface is a *diameter* of the sphere.

458. All sections of a sphere made by a plane passing through the centre are circles which have the same centre and the same radius as the sphere, and are called *great circles.*

459. All sections of a sphere made by a plane that does not pass through the centre are circles, but they have a smaller radius than that of the sphere, and are called *small circles.*

460. If the circle $PAQB$ (Fig. 60) turns about its diameter PQ, the point A, which is 90° from P and also from Q, describes a great circle $AGBH$, called *the equator;* and the point C describes a small circle $CEDF$ parallel to the equator, and called a *parallel of latitude.* The circle $PAQB$, which represents any position of the generating circle during its rotation, is called a *meridian of longitude.*

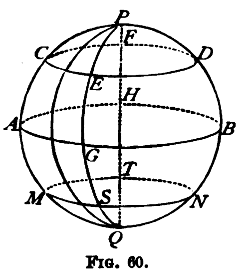


FIG. 60.

461. *The area of the surface of a sphere is equal to four times the area of a great circle of a sphere.*

462. *The volume of a sphere is equal to one-third of the product of the area of the surface by the radius.*

FORMULAS.

463. If V stands for volume, S for lateral surface, B for base, T for total surface, P for perimeter of base, R for radius of base, H for altitude, and L for slant height, the formulas for surfaces and volumes will be as follows:

Prism.

$S = P \times H.$
$T = S + 2B.$
$V = B \times H.$

Cylinder of Revolution.

$S = 2\pi R \times H.$
$T = 2\pi R \times H + 2\pi R^2.$
$V = \pi R^2 \times H.$

Pyramid.

$S = \frac{1}{2} P \times L.$
$T = S + B.$
$V = \frac{1}{3} B \times H.$

Cone of Revolution.

$S = \pi R \times L.$
$T = \pi R \times L + \pi R^2.$
$V = \frac{1}{3}\pi R^2 \times H.$

Frustum of Pyramid.

Perimeters of bases P and P', bases B and B'.

$S = \frac{1}{2}(P + P') \times L.$
$T = S + B + B'.$
$V = \frac{1}{3} H(B + B' + \sqrt{BB'}).$

Frustum of Cone.

Radii of bases R and R'.

$S = \pi L(R + R').$
$T = \pi L(R + R') + \pi(R^2 + R'^2).$
$V = \frac{1}{3}\pi H(R^2 + R'^2 + RR').$

Sphere.

Surface $= 4\pi R^2.$
Volume $= \frac{4}{3}\pi R^3.$

Ex. 174.

1. Find the total surface of a rectangular parallelopiped whose length is 15 ft., breadth 9 ft., height 6 ft.

2. Find the total surface of a triangular prism whose height is 6 ft., side of base 3 ft.

3. Find the volume of a prism whose base is a square 8 ft. on a side, and whose length is 40 ft.

4. Find the volume of a right prism 32 ft. long, if its ends are trapezoids the parallel sides of which are 12 ft. and 8 ft., and the perpendicular distance between these parallel sides is 6 ft.

5. A right cylinder is 10 ft. high, and measures 7 ft., 4 in. around the base. Find the lateral surface and the volume. ($\pi = 3\frac{1}{7}$.)

6. Reckoning $7\frac{1}{2}$ gallons to the cubic foot, if 375 gals. are pumped out of a cylindrical cistern 7 ft. in diameter, how many inches will the surface of the water fall in consequence?

7. A marble column measures 7 ft. 4 in. in circumference and is 15 ft. long. Find the expense of polishing the entire surface at $1.50 a square foot.

8. Find the total surface of a regular pyramid when each side of its triangular base is 6 ft. and its slant height is 18 ft.

9. Find the volume of a regular pyramid whose base is an equilateral triangle measuring 4 ft. on a side, and whose height is 15 ft.

10. How many square feet of canvas is required for a conical tent the altitude of which is 8 ft. and the diameter of the base is 7 ft.? ($\pi = 3.1416$.)

11. A conical tent whose slant height is 12 ft. requires 132 sq. ft. of canvas to cover it. Find how many feet of ground the tent covers. ($\pi = 3\frac{1}{7}$.)

12. Find the volume of a right cone the height of which is 15 ft. and the circumference of the base 14 ft.

13. Find the total surface of the frustum of a pyramid whose bases are 18 ft. sq. and 15 ft. sq. respectively, and whose slant height is 30 ft.

14. How many square feet of tin will be required to make a funnel if the diameters of the top and bottom are to be 28 in. and 14 in. respectively, and the height is 24 in.?

15. Find the volume of the frustum of a regular square pyramid whose height is 24 ft., and the sides of its square ends are 9 ft. and 4 ft. respectively.

16. Find the volume of the frustum of a right cone if the radii of the circular ends are 3 ft. and 3 ft. 10 in. respectively, and the slant height 2 ft. 2 in.

17. Find the surface of a sphere whose diameter is 10 in.

18. The circumference of a dome in the shape of a hemisphere is 66 ft. How many square feet of lead are required to cover it?

19. The ball on the top of St. Paul's cathedral in London is 6 ft. in diameter. What will it cost to gild it at 7 cts. a square inch?

20. Find the volume of a sphere if the diameter is 14 in.

21. Find the volume of a sphere whose circumference is 45 ft. ($\pi = 3.1416$.)

22. How many gallons of water will a hemispherical bowl hold whose diameter is 21 in., reckoning $7\frac{1}{2}$ gals. to the cubic foot?

CHAPTER XVI.

MISCELLANEOUS PROBLEMS.

Decimal Fractions.

1. Four men together paid \$20,000 for some land. The first puts in \$2350, the second \$5820.35, the third \$7640.75. How much must the fourth man pay?

2. What will be the cost of uniforms for a base-ball nine at \$2.87 for each uniform?

3. At \$15.87 a ton, what will be the value of 637 tons of hay?

4. If peaches are worth \$1.25 a basket, and it takes 3 dozen for a basket, what is the value of 2892 dozen peaches?

5. If 964 baskets of peaches are sold for \$1301.40, what is the price per basket?

6. If 324 men contribute together \$2647.08, what is the contribution of each?

7. A boy picks blueberries in a pasture, giving to the owner of the pasture for the privilege 1 quart out of every 8 quarts. In 2 days he picks 48 quarts, and sells his share of the berries for \$3.78. What did he get a quart?

8. If 150 men work on a railroad at the same price per day, and if, at the end of the week, they all together receive \$1575, what price per day does each man receive?

9. A boy has 3 pieces of twine; one is 58.74 ft. long, another is 97.86 ft., and a third 57.26 ft. How long a kite-string can he make?

10. If a kite-string is 213.86 ft. long, and the kite breaks away and carries off 94.38 ft. of the string, how much will be left? How much more must be bought to make up 1000 ft.?

11. One man has in his purse 3 hundred-dollar bills, 16 ten-dollar bills, 12 one-dollar bills, and 19 ten-cent pieces. Another man has in his purse 13 ten-dollar bills, 63 five-dollar bills, 8 one-dollar bills, 2 half-dollars, 3 quarters, 4 five-cent pieces, and 17 cents. How much more money has one man than the other?

12. If 12 boys play hare and hound, and run 2.876 miles, find the whole number of miles run by all the boys.

13. A boy sold 19 dozen eggs at 26 cts. a dozen, 25 lbs. of wool at 35 cts. a pound, and 13 chickens at half a dollar apiece. He bought two sheep at $5.75 each. How much money had he left?

14. If 1964.52 bu. of corn are to be put into bags holding 2.14 bu. each, how many bags will it take?

15. At $9.17 a barrel, how many barrels of flour can be bought for $876.35, and how much money will be left over?

16. Boys in playing hare and hound run 3.876 miles. The hares drop a piece of paper every 4.75 feet on the average. How many pieces do they drop?

17. If a man earns $3.25 a day, how many days will it take him to earn $1964.87?

18. For \$7624.13 how many tons of hay can be bought at \$18.75 a ton?

19. The large wheel of a bicycle is 14.37 feet around. How many times does it turn in going a mile?

COMMON FRACTIONS.

20. Find the prime factors of 41,580.

21. Find the G.C.M. of 144, 126, 108.

22. Find the L.C.M. of 18, 90, 60, 24.

23. Find the L.C.M. of 14, 35, 343.

24. At $16\frac{1}{4}$ cts. a yard. what will $3\frac{1}{2}$ yds. of cloth cost?

25. A man has $376\frac{3}{8}$ quarts of berries, which he wishes to put into boxes holding $2\frac{1}{4}$ qts. each. How many boxes will be required, and what part of a box will be left over?

26. If a man earns $\$2\frac{3}{4}$ a day, how many days will it take him to earn \$100?

27. A lady has $37\frac{1}{2}$ qts. of berries to can. If each can holds $2\frac{3}{4}$ qts., how many cans of berries will she have, and what part of another can will there be over?

28. If a man walks $4\frac{3}{8}$ miles an hour, how many hours will it take him to walk $40\frac{3}{4}$ miles?

29. Some boys wanted a long rope to use on the ice. They made the rope by taking off their sled-ropes and tying them together. The first sled-rope was $2\frac{3}{4}$ yds. long, the second $3\frac{1}{2}$ yds., the third $2\frac{7}{8}$ yds., the fourth $5\frac{3}{4}$ yds., and the fifth $3\frac{2}{15}$ yds. If the whole length was shortened $1\frac{1}{3}$ yds. by the knots, from

tying the sled-ropes together, how long was the rope?

30. A lady bought $3\frac{7}{8}$ yds. of cotton cloth, $4\frac{1}{8}$ yds. of calico, $16\frac{2}{5}$ yds. of flannel, and $12\frac{1}{8}$ yds. of gingham. How many yards did she buy in all?

31. A boy went to a store with $5.75 in his purse. He bought $3\frac{1}{4}$ lbs. of butter at 28 cts. a pound, $13\frac{1}{2}$ lbs. of sugar at 11 cts. a pound, and $1\frac{1}{2}$ lbs. of coffee at 35 cts. a pound. How much money did he have left?

32. Four boys went fishing, and caught 40 trout; the first caught $\frac{2}{5}$ of the whole, the second $\frac{1}{8}$, and the third $\frac{1}{4}$. How many did the fourth boy catch?

33. George has his choice to be one of 3 boys to receive 8 oranges, or one of 4 boys to receive 11 oranges. Which shall he choose?

34. Five girls pick blueberries together; the first picks $7\frac{2}{3}$ qts., the second $5\frac{9}{10}$ qts., the third $12\frac{3}{4}$ qts., the fourth $8\frac{1}{6}$ qts., and the fifth $3\frac{7}{8}$ qts. How much will they all together get for their berries, at $12\frac{1}{2}$ cts. a quart?

35. A farmer puts the following lots of apples into 6 bins: namely, $6\frac{3}{4}$ bu., $18\frac{1}{2}$ bu., $25\frac{7}{8}$ bu., $19\frac{3}{8}$ bu., $143\frac{3}{4}$ bu., $976\frac{1}{4}$ bu., $25\frac{1}{2}$ bu. How many bushels will there be for each bin?

Compound Quantities.

36. How many rods are there in 4379 ft.?

37. Reduce 9,627,834 ft. to yards, rods, etc.

38. Reduce 96,284 sq. in. to square feet.

39. Reduce 15 sq. rds. 3 sq. yds. 18 sq. ft. 3 sq. in. to square inches.

40. What will 1000 sq. ft. of land cost at $67 an acre?

41. What will 20 sq. yds. of land cost at 75 cts. a square foot?

42. How much less will 15 acres of land cost, at $16 an acre, than 96,342.42 sq. ft. at 5 cts. a foot?

43. How many acres in a rectangular piece of land 9634 ft. long and 3840 ft. wide?

44. A pile of four-foot wood is 4 ft. high and 75 ft. long. How many cords of wood are there in the pile?

45. In a woodshed there is a pile of wood 12 ft. long and 10 ft. high. If the sticks average a foot in length, what part of a cord is there in the pile?

46. What will 7 bu. 3 pks. of blueberries bring at 9 cts. a quart?

47. How many gallons of milk, at 8 cts. a quart, can be bought for $7.37?

48. How many quarts of water will a tin box hold that is 13 in. long, 6 in. wide, and 7 in. deep?

49. The total net weight of several loads of hay is 63,782 lbs. How many tons in all the loads of hay?

50. If an ounce of candy is worth 5 cts., what will 5 lbs. cost at the same rate?

51. Reduce 9 dys. 5 hrs. 16 min. to seconds.

52. Reduce 948,741 min. to higher denominations.

53. How many weeks between Jan. 1 and Nov. 1?

54. A boy has 10 mi. to go. After he has gone 6 mi. 48 rds. 12 ft., how much of his journey has he still to go?

55. A lady bought 4 remnants of cloth; the first contained $9\frac{1}{4}$ yds., the second 4 yds. 11 in., the third $6\frac{1}{3}$ yds., and the fourth $5\frac{1}{2}$ yds. How much cloth did she buy in all?

56. A certain basket holds 1 bu. 3 pks. 7 qts. A farmer raises enough of yellow-eyed beans to fill this basket 7 times. How many bushels does he raise?

57. A farmer cuts 26 loads of hay, which average 1 t. 436 lbs. How many tons does he cut in all?

58. What is $\frac{1}{25}$ of 9 mi. 5 rds. 13 ft.?

59. Three men in company buy 175 t. 19 cwt. 36 lbs. of hay. What is each man's share?

60. Seven boys together pick 4 bu. 3 pks. 7 qts. of berries. What is each boy's share?

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61. Bought 9 lbs. of sugar at 13 cts. a pound, 18 yds. of cloth at 33 cts. a yard, 4 doz. eggs at 29 cts. a dozen, and 5 lbs. of butter at 32 cts. a pound. What change should I receive from a ten-dollar bill given in payment?

62. How many quarts of berries, at 12 cts. a quart, will it take to pay for 8 yds. of cloth, at $16\frac{1}{2}$ cts. a yard?

63. A basket of peaches is half a bushel; how many bushels are there in 250 car-loads of 500 baskets each?

64. A fast railway train in England went 186 mi. 240 rds. in 3 hrs. What was the rate per hour?

65. If a man could proceed to the moon at the same rate per hour as the train went in example 64, how many hours would it take him, reckoning the distance 239,000 miles?

66. In one bin there are 23 bu. 2.48 pks. of wheat, and in another 141 bu. 2 pks. If $\frac{3}{4}$ of the wheat in the first bin is put into the second, how much wheat will there be in the second bin?

67. A load of four-foot wood is $3\frac{1}{2}$ ft. high and 7 ft. long. What is it worth at the rate of $6.40 a cord?

68. In one field there are $17\frac{1}{8}$ A., in a second there are 49 sq. rds., and a third field is 1740 ft. long and 927 ft. wide. What is the area of the three fields together?

69. A bin contains 164 bu. 3 pks. 2 qts. of oats. How long will these oats last if there are taken out 3 qts. of oats three times a day?

70. From a barrel containing $27\frac{3}{4}$ gals. of oil, 3 qts. a day were taken out for 3 weeks. How many gallons were left in the barrel at the end of that time?

71. If from a barrel of oil holding 27 gals. 2 qts. 1 pt. there is drawn out a can full, holding 1 gal. 2 qts. 1 pt., every day, how many days will the oil last?

72. Reduce $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{14}{19}$ of a mile to rods.

73. Reduce $\frac{4}{5}$ of $\frac{7}{8}$ of $3\frac{1}{2}$ in. to the fraction of a yard.

Special Problems.

If a man can do a piece of work in 5 dys., in one day he can do $\frac{1}{5}$ of the work; and if another man can do the same work in 4 dys., in one day he can do $\frac{1}{4}$ of it.

Therefore, both men together can do $\frac{1}{5} + \frac{1}{4} = \frac{9}{20}$ in one day.

Hence they will do $\frac{1}{20}$ in $\frac{1}{9}$ of a day, and therefore the whole work in $\frac{20}{9}$ days, that is, in $2\frac{2}{9}$ days.

74. If A can do a piece of work in 4 dys., B in 5 dys., and C in 7 dys., in how many days will they do it, all working together?

75. A can do a piece of work in 2 hrs., B in $2\frac{1}{2}$ hrs., and C in $3\frac{1}{2}$ hrs. How much of the work can they do in 20 min., all working together?

76. If A and B can do a piece of work in 18 dys., A and C in 12 dys., and B and C in 9 dys., find the number of days that it will take them, all working together.

77. A can do a piece of work in 6 dys., B in 8 dys., and C in 10 dys. How much of it can they do in 2 dys. together?

78. A cistern can be filled by means of a water-pipe in 30 min., and can be emptied by a waste-pipe in 20 min. If the cistern is full, and both pipes are open, in what time will it be emptied?

79. From Paris to Berlin by railway it is 1308^{km}. A kilometer is 1093.63 yds. Express the distance between Paris and Berlin in miles and yards.

80. Mercury revolves around the sun in 87.9692580 dys. Express the period of revolution in days, hours, minutes, and seconds.

81. The Roman foot was 0.97075 of our foot. The Greek foot was $\frac{25}{24}$ of the Roman foot. Find the length in inches of the Greek foot.

82. The radius of a circle is 0.1591549 of its circumference, which contains 360°. Find the angle at the centre whose arc is equal to the radius.

83. Find the L.C.M. of all the multiples of 3, from 6 to 27, inclusive.

84. Arrange $\frac{2}{3}$, $\frac{25}{39}$, and $\frac{58}{89}$ in order of magnitude.

85. Subtract the sum of $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{14}{15}$ from 5.

86. Find the decimal which, when added to the difference of $\frac{9}{200}$ and 0.002775, produces the square of 0.215.

87. A, at the rate of $4\frac{1}{4}$ miles an hour, walks a certain distance in $3\frac{1}{10}$ hrs. In what time will B walk the same distance at the rate of $\frac{3}{4}$ of $5\frac{1}{6}$ miles an hour?

PERCENTAGE.

88. A house worth \$15,000 sustains injury from fire to the amount of \$3840. What is the rate per cent of loss?

89. A and B have each \$350; A spends 16% and B spends 20%. A's expenditure is what per cent of B's?

90. A gentleman having a court 20 ft. by 40 ft. enlarged it 10% in each dimension. Find the per cent of increase in area.

91. A young man buys a farm for \$5200, which sum is 30% more than a legacy received from his grandfather. Required the amount of the legacy.

92. A lady gave to her daughter 25% and to her son 20% of her estate. The difference between the shares of the son and daughter was \$1500. What is the value of the estate?

93. If a quart of Jersey milk is worth 10 cts., and produces 1 gi. of cream worth 25 cts. a pint, what per cent of the value of the milk is the value of the cream?

94. A farmer raised 360 bu. of potatoes, and the crop was 2400% of the seed. How many bushels did he plant?

95. A man received from a bankrupt \$937.50, which was $37\frac{1}{2}$% of the sum due. What was his loss?

96. What per cent of $\frac{3}{4}$ is $\frac{1}{2}$?

97. If 200% of a number is $\frac{1}{5}$% of 70, what is the number?

98. A grocer sold 10% of his stock of sugar, and then 25% of the remainder, after which he had 3 t. 1560 lbs. How much sugar had he at first?

99. A man lost $37\frac{1}{2}$% of his money. He then earned $50, and had 125% of what he had at first. How much did he have at first?

100. A merchant bought a cask of molasses from which 20% of the molasses had been drawn. He sold $30\frac{1}{4}$ gals., and then the cask was one-quarter full. Find the capacity of the cask in gallons.

101. What per cent of a common year is the time from July 1 to November 23, both days included?

102. A horse and chaise together are valued at $225; the horse is worth 25% more than the chaise. Find the value of the horse.

103. A man owning 30% of a mine sold 50% of his share for $3000. What was the value of the mine?

104. For what price per pair must shoes be sold to gain 25%, if 15% is lost when they are sold at $1.275 per pair?

105. If $\frac{1}{5}$ of goods valued at $1500 are sold at a loss of 10%, what must the remainder bring to gain 20% on the whole?

106. A fruit dealer bought 200 apples at the rate of 4 for a cent, and 200 at 5 for a cent. He sold them all at 5 for 3 cents. What per cent did he gain on his investment?

107. If 75% of the price of a bushel of corn is 50% of the price of a bushel of wheat, how many bushels of corn can be bought for $24 when wheat is worth $1.20 a bushel?

108. A horse dealer sold a horse for $90, and lost 25% of the cost of the horse. He sold another horse at an advance of 20% on the cost, and gained as much as he lost on the first horse. What was the selling price of the second horse?

109. If 20 men can build a wall in 9 dys., what per cent of the number of men could build the wall in 12 dys.?

110. If 7% of a ton of butter costs $42, what per cent of a ton can be bought for $57?

111. Five hundred barrels of flour were sold for $4125, at a profit of 10%. Find the cost per barrel.

112. An agent makes 20% by selling a book for 72 cts. If he had sold it for $1, what per cent would he have made?

113. A merchant bought from a shoe dealer 12 cases of shoes, each containing 60 pairs, at 87½ cts. per pair, and sold the whole for $756. Find his gain per cent.

114. If 196 sq. rds. are 40% of the area of a field 30 rds. in length, what is the width of the field?

115. When brooms are $5.50 a dozen, what will be paid for 18¾ gross, if a discount of 10% is allowed on the bill for cash?

116. A grocer bought, at 60 cts. per gallon, 16 hhds. of molasses of 63 gals. each, and sold it at a profit of $120.96. What was his gain per cent?

117. A merchant in his first year of business increased his capital 40%, and increased his capital the second year 30%. He lost $33\frac{1}{3}\%$ of his capital the third year, and had $18,200 left. What was his capital at first?

118. A contractor engaged to build a railroad at $31,200 a mile. The work actually cost $90 per rod. What was his gain per cent?

119. A merchant sold goods at 25% discount and 4% off from the selling price for cash. What was the whole per cent discount?

120. At $1\frac{1}{2}\%$ commission an agent receives $97.29 for selling goods. Find the amount of the sale.

121. A merchant sent $30,750 to his agent in New Orleans, for the purchase of cotton. Find the sum spent for cotton, if the agent charges $2\frac{1}{2}\%$ commission for buying.

122. Find the sum paid for insurance, at $\frac{1}{2}\%$, on a house worth $8000, and at $\frac{3}{4}\%$ on furniture worth $2000, if the insurance is on $\frac{7}{8}$ of the value of the property insured.

123. A sea captain paid $345, at $1\frac{1}{2}\%$, for insuring $\frac{2}{3}$ of the value of a ship. Find the value of the ship.

124. A town has to raise $192,000 for expenses. If 4% is allowed for collecting, how much money must be raised?

125. A merchant sends $24,600 to his agent at St. Louis, for the purchase of flour at $5 a barrel. How many barrels can be bought if the agent charges $2\frac{1}{2}\%$ commission for buying?

126. A paper-mill worth \$30,000 was insured for an annual premium of $1\frac{3}{8}\%$ on 90% of its value. In the second year it was injured by fire to the amount of \$1780. How much did the mill owner save by insuring?

127. A city voted a tax of \$74,500; the poll-tax was \$1.25 on 2000 polls; the assessed value of city property was \$6,000,000. What was the tax on \$1000?

128. What insurance must be placed upon a store and its contents, valued at \$20,085, that the entire value of the goods and store and of a premium of $2\frac{1}{2}\%$ may be recovered in case of loss by fire?

129. A premium of \$88.14 is paid upon a cargo of wheat insured at $2\frac{3}{5}\%$ on $\frac{3}{4}$ of its value. Find the number of bushels shipped, if the average price is 80 cts. a bushel.

130. A 30% duty of \$5594.40 was paid on 252 watches. What was the invoice price of each watch?

131. Eleven and one-half yards of cloth $1\frac{1}{8}$ yds. wide are required for a dress. How many yards must be bought if the shrinkage in sponging is 10% in length and 8% in width?

132. If 30% of a merchant's sales is profit, what is his gain per cent?

133. A merchant insured a ship and cargo at $4\frac{3}{8}\%$. If \$158,650 cover both property and premium, what is the value of the ship and cargo?

134. How much money must be sent to purchase 10,000 bbls. of sugar, at \$8.50 per barrel, if the commission for buying is 3%, and the sum prepaid for freight is \$315?

INTEREST.

Find the interest of:

135. \$1000 for 2 yrs. 7 mos. 18 dys., at 6%.

136. \$1496 for 7 mos. 21 dys., at 6%.

137. \$582 for 1 yr. 7 mos. 15 dys., at 6%.

138. \$168 for 1 yr. 5 mos. 12 dys., at $2\frac{2}{3}$%.

139. \$548 for 7 mos. 18 dys., at $6\frac{1}{2}$%.

140. \$1272 from July 12, 1880, to Feb. 24, 1882, at $3\frac{1}{4}$%.

141. \$1975.30 for 60 dys., at 6%.

142. \$1675 for 90 dys., at 6%.

143. \$976 for 3 yrs. 6 mos., at 1% a month.

Find the rate per cent:

144. When the interest on \$3000 for 3 yrs. is \$630.

145. When the interest on \$1500 for 2 yrs. is \$172.50.

146. When the interest on \$1278.50 for 3 yrs. 6 mos. is \$178.99.

147. When a sum of money is doubled in 8 yrs.

148. When \$1758 amount to \$1869.34 in 8 mos.

Find the time:

149. When the interest on \$278.40, at $7\frac{1}{2}$%, is \$100.92.

150. When \$600, at $3\frac{1}{3}$%, amount to \$660.

151. When the interest on \$78, at $1\frac{1}{2}$% a month, is \$28.08.

152. When the principal, at 5%, is doubled.

Find the principal that will:

153. Produce $424.94 interest in 3 yrs., at $5\frac{1}{2}\%$.

154. Produce $285.60 interest, at 7%, in 1 yr. 8 mos. 12 dys.

155. Produce $81.37 interest, at $3\frac{3}{4}$ %, in 2 yrs. 9 mos. 18 dys.

What principal will amount to:

156. $88.80, at 6%, in 3 yrs. 4 mos.

157. $308.10, at $5\frac{3}{8}\%$, in 6 mos.

158. $570.475, at 6%, in 3 yrs. 4 mos. 6 dys.

159. $661.32, at $\frac{3}{8}\%$ a month, in 3 yrs. 6 mos.

160. Find the interest on $1825 from Jan. 1 to June 25, at $5\frac{1}{2}\%$, counting the exact number of days, and allowing 365 dys. for a year.

BANK DISCOUNT.

Find the proceeds of the following notes:

161. $300. SPRINGFIELD, Ill., Aug. 12. 1884.

Sixty days after date I promise to pay Nicholas Welsh, or order, $300, value received.

Discounted at 6%, Sept. 1. JOHN BRYCE.

162. $700. BOSTON, Nov. 13, 1880.

Ninety days after date I promise to pay to the order of David Morrison seven hundred dollars, value received.

Discounted at 7%, Jan. 1, 1881. GEORGE BROWN.

163. $217.40. NEW YORK, July 30, 1884.

Ninety days after date I promise to pay to the order of Seth Jay two hundred seventeen and $\frac{40}{100}$ dollars, value received.

Discounted at 6%, Aug. 10, 1884. JAMES BENT.

164. $500. CHICAGO, July 9, 1883.

Ninety days from date, for value received, I promise to pay to the order of John Hogan five hundred dollars, with interest at 9%.

Discount at 6%, July 9, 1883. JOHN FOSTER.

165. $5897.50. TROY, June 24, 1881.

Four months from date, for value received, I promise to pay to the order of Aaron Reed five thousand eight hundred ninety-seven and $\frac{50}{100}$ dollars, with interest at 6%.

Discounted at 5%, Aug. 15. JAMES CAREY.

Find the face of a note which:

166. Discounted at 6% for 90 dys. yields $344.57.

167. Discounted at 9% for 46 dys. yields $493.87.

168. Discounted for 6% for 3 mos. yields $984.50.

PARTIAL PAYMENTS.

169. A note for $680, dated June 15, 1884, payable on demand, with interest at 6%, bears the following endorsement: May 15, 1885, $425. What is due June 15, 1885?

170. On a note of $1400, dated March 1, 1880, there was received Oct. 19, 1880, $700; Jan. 1, 1881, $400. What is due March 1, 1881, reckoning interest at 6%?

171. A note of $900, dated Jan. 1, 1884, and bearing interest at 5%, has the following endorsements: May 13, $240; Aug. 19, $300; Oct. 25, $180. Required the balance due Jan. 1, 1885.

172. A note of \$1800, dated Jan. 1, 1880, and bearing interest at 5%, has the following endorsement: June 1, 1881, \$400. Find the balance due June 1, 1884.

173. A note of \$600, dated Aug. 13, 1881, and bearing interest at 6%, has the following endorsements: Jan. 1, 1882, \$200; April 1, 1882, \$110. Find the balance due Aug. 13, 1883.

174. A note of \$1150, dated June 30, 1878, and bearing interest at 6%, has the following endorsements: Jan. 30, 1879, \$15; April 30, 1880, \$570; July 30, 1881, \$420. Find the balance due Dec. 30, 1882.

Compound Interest.

Find the compound interest of:

175. \$300, at 6%, for 3 yrs. 4 mos. 18 dys.

176. \$350, at 6%, for 3 yrs. 5 mos. 24 dys.

177. \$840, at 8%, from June 13, 1880, to Aug. 1, 1881, interest being payable quarterly.

178. \$400, at $4\frac{1}{2}$%, from Jan. 1, 1881, to Feb. 13, 1884.

179. \$1100, at 6%, for 2 yrs. 7 mos. 6 dys., interest being payable semi-annually.

180. \$1000, at 8%, for 2 yrs. 3 mos. 18 dys., interest payable quarterly.

Stocks.

181. Find the cost of \$2400 stock, at $97\frac{1}{2}$.

182. Find the cost of \$2785 stock, at $105\frac{1}{8}$.

183. Find the cost of \$5680 stock, at $103\frac{1}{4}$.

184. How much stock, at $85\frac{3}{8}$, including brokerage, can be bought for \$2376.84?

185. How much 6% stock will produce an income of \$840?

186. Find the price of stock, when \$4647.50 will pay for \$5200 worth of stock.

187. How many hundred-dollar shares of 7% stock will yield a yearly income of \$686?

188. A gentleman gave his daughter \$25,700 of $4\frac{1}{2}\%$ bonds. What yearly income from them will she receive?

189. What amount of 8% stock will yield a yearly income of \$8000?

190. What is the rate of dividend when the sum of \$300 is received from \$7500 stock?

191. Find the rate of dividend when the sum of \$1603.80 is received from \$35,640.

192. How much stock, at $121\frac{1}{2}$, can be bought for \$6318?

193. How much stock, at $97\frac{1}{2}$, can be bought for \$1755?

194. Find the sum paid for \$5600 stock, at $112\frac{3}{4}$, and brokerage $\frac{1}{8}$.

195. What income will be obtained from \$5125, invested in 6% stock, at $102\frac{1}{2}$?

196. Find the income from \$8190, invested in 5% stock, at 91.

197. Find the income on \$1935, invested in 8% stock, at $107\frac{1}{2}$.

198. Find the income from \$6750 invested in $4\frac{1}{5}$ stock, at 75.

199. If \$7656 be invested in stock, at $63\frac{4}{5}$, and the stock pays a dividend of $3\frac{1}{4}\%$, how much will be received on the money invested?

200. If \$7000 be invested in stock, at $87\frac{1}{2}$, and the stock pays a dividend of $7\frac{1}{2}\%$, how much will be received?

201. If 9% stock is bought at 150, what rate of interest will be received on the investment?

202. What rate of interest will be received on 5% stock at 75?

203. What rate of interest will be received on 4% stock, at $62\frac{1}{2}$?

204. How much money must be invested in 5% stock, at 80, to produce \$400 income?

205. How much money must be invested in 4% stock, at 90, to produce \$320 income?

206. How much money must be invested in 6% stock, at 75, to produce \$200 income?

207. A man received \$240 from his 6% dividend, on stock bought at 105. How much money did he have invested in the stock?

208. What should be paid for a 4% stock, that 5% interest may be realized on the investment?

209. What should be paid for a 6% stock, that 8% interest may be realized on the investment?

210. If 4% stock, which produces an income of \$180, is sold at 90, what sum will be realized from the sale?

211. What increase of income will there be, if \$3600 of 4% stock is sold at 90, and the proceeds invested in 7% stock, at 108?

212. Find the increase of income, if $3900 of 3% stock is sold at 88, and the proceeds invested in 4½% stock, at par.

213. Find the increase of income, if $6000 of 8% stock is sold at 120, and the proceeds invested in 6½% stock, at 90.

SIMPLE PROPORTION.

214. If 16 bbls. of apples cost $28, what will 129 bbls. cost?

215. If 15 workmen can do a piece of work in 25 dys., in how many days can 25 men do the same work?

216. If 8 horses eat a certain quantity of hay in 2 mos., how long will the same quantity last 12 horses?

217. A meadow can be mowed by 40 men in 10 dys. How many days will it take 30 men to mow it?

218. If 30 men can build a wall in 18 dys., how many men will be required to build it in 12 dys.?

219. A bankrupt owes $3000, and his assets amount to $850. How much on a dollar will his creditors receive?

220. What does a bankrupt pay on a dollar, if his creditors receive $376.275 on $2076?

221. A bankrupt's effects amounted to $2675.40, and his debts to $3057.60. What did his creditors receive on a dollar?

222. If 4 men reap 5 A. 159 sq. rds. in 1 week, how many men at the same rate will reap 35 A. 154 sq. rds.?

223. A wall whose height is 9.1875 ft. casts a shadow of 10.5 ft. Find the length of the shadow of a steeple 93.8 ft. high.

224. A cistern can be filled in 54 min. by a pipe running $3\frac{7}{8}$ gals. a minute. In how many minutes can it be filled by another pipe, running $4\frac{1}{2}$ gals. a minute?

225. A watch set on Saturday, at half-past eight in the evening, loses $1\frac{1}{2}$ min. in 30 hrs. What time does it show the next Thursday, at 4 o'clock in the afternoon?

226. When do the hour and minute-hands of a watch coincide between 5 and 6 o'clock?

NOTE. Since the hour-hand moves through 5 minute-spaces while the minute-hand traverses 60, the minute-hand moves 12 times as fast as the hour-hand. The minute-hand, therefore, in moving through 12 minute-spaces, traverses 11 minute-spaces more than the hour-hand.

When the hour-hand is at V., the minute-hand, being at XII., is 25 minute-spaces behind it. The question, therefore, is, if the minute-hand, to gain 11 spaces, must move through 12 spaces, how many spaces must it move through to gain 25 spaces?

$$11 : 12 :: 25 : ?$$

227. When do the hour and minute-hands of a watch coincide between 8 and 9 o'clock?

228. When do the hour and minute-hands of a watch coincide between 3 and 4 o'clock?

229. When do the hour and minute-hands of a watch coincide between 10 and 11 o'clock?

The true weight of a body weighed successively in the scales of a false balance is the square root of the product of the apparent weights.

230. A body appears to weigh $5\frac{1}{16}$ lbs. in one scale, and $5\frac{4}{9}$ lbs. in the other scale, of a false balance. Find its true weight.

The times in which bodies fall are proportional to the square roots of the distances traversed. Since a body falls 16.1 *ft. the first second, to find the time a body is falling, divide the distance by* 16.1, *and extract the square root of the quotient.*

231. In how many seconds will a stone fall to the bottom of a coal-pit 420 ft. deep?

Compound Proportion.

232. If 60 bu. of corn feed 6 horses for 50 dys., in how many days will 15 horses consume 75 bu.?

233. If 20 cwt. are carried 50 miles for $5, how much will be the cost of carrying 40 cwt. 40 miles?

234. If 20 men can perform a piece of work in 12 dys., required the number of men who can perform another piece of work three times as great in $\frac{1}{5}$ of the time.

235. If 12 horses, in 5 dys., draw 44 loads of stone, how many horses will draw 132 loads the same distance in 18 dys.?

236. If a footman travels 130 mi. in 3 dys., of 14 hrs. each, in how many days, of 7 hrs. each, will he travel 390 mi.?

237. If 50 men dig a cellar in 7 dys., working 11 hrs. a day, how many days will 24 men require, working 8 hrs. a day?

238. A garrison of 1500 men has provisions for 12 wks., at the rate of 20 oz. per day to each man. How many men will the same provisions maintain for 20 wks., allowing each man only 8 oz. per day?

239. If 12 candles, of which 8 weigh a pound, serve 4 winter evenings, from five to eleven, how many candles, of which 6 weigh a pound, will serve 3 spring evenings, from seven to eleven?

240. A contractor, having engaged to lay 10 mi. of railway in 150 dys., finds that 90 men have finished 3 mi. in 80 dys. How many more men must he engage to finish the work in the given time?

241. If 200 men in 12 dys., of 8 hrs. each, can dig a trench 160 yds. long, 6 yds. wide, and 4 yds. deep, in how many days, of 10 hrs. each, will 90 men dig a trench 450 yds. long, 4 yds. wide, and 3 yds. deep?

242. If 120 men make an embankment $\frac{3}{4}$ of a mile long, 30 yds. wide, and 7 yds. high, in 42 dys., how many men will it take to make an embankment 1000 yds. long, 36 yds. wide, and 22 ft. high, in 30 dys.?

Powers and Roots.

Find the square root of:

243. 30976.	**247.** 2052.09.
244. 106929.	**248.** 4795.25731.
245. 622521.	**249.** 24674.1264.
246. 1234321.	**250.** $\frac{196}{1369}$.

Find the cube root of:

251. 373248.

252. 54872.

253. 389017.

254. 1092727.

255. 84604519.

256. 52734.375.

257. 7834.87438.

258. 0.053157376.

259. $\frac{4}{7}$.

260. $7\frac{1}{5}$.

METRIC SYSTEM.

261. How many liters will a box hold which is 75^{cm} long, 15^{cm} wide, and 12^{cm} deep?

262. Add 3.473^{m}, 50^{cm}, 83^{mm}, 4.5^{m}, and 16^{cm}.

263. There are 4 measuring lines: the first is 7.5^{m}, the second is 3^{m} 75^{cm}, the third is 4^{m} 80^{cm}, and the fourth is 8^{m} 6^{cm}. Express in meters the total length of the four lines.

264. On the same railroad are four stations, between which the consecutive distances are as follows: 7^{km} 249^{m}, 3^{km} 200^{m}, and 5.007^{km}. Find in kilometers the distance between the first and fourth stations.

265. A goldsmith has sold jewels of the following weights respectively: 27^{g} 9^{mg}, 30^{g} 7^{dg}, 7^{g} 4^{cg}, and 19^{g} 3^{dg} 4^{cg} 7^{mg}. Find the total weight in grams.

266. From 17^{km} 6^{m} take 243^{m} 691^{mm}.

267. From a farm containing 340^{ha} 7^{a} there are sold 119^{ha} 29.03^{a}; how many hektars are left?

268. A liter of mercury weighs 13^{kg} 598^{g}. Find the weight in kilograms of 3.69 liters.

269. If 16.94^{l} of olive oil weigh $15^{kg}\ 500^{g}$, find the weight of one liter.

270. Into how many lots of 3.75^{a} may $8^{ha}\ 40^{a}$ be divided?

271. If 122.6^{g} of chlorate of potassa yield 48^{g} of oxygen, what weight of oxygen may be obtained from 1^{kg} of the chlorate?

272. A field of wheat containing $8\frac{1}{2}^{ha}$ has furnished 600 sheaves per hektar; 2 sheaves of wheat have furnished a bundle of straw weighing 5^{kg}. What will the whole straw bring, at $15 a ton?

273. A farmer shears 620 sheep and 180 lambs; the sheep give on an average 4^{kg} of wool each, and the lambs $\frac{1}{4}$ of a kilogram each. The wool is sold for 50 cts. a kilogram. How much money does the farmer receive?

274. A vessel full of water weighs 5.25^{kg}; the weight of the vessel when empty is 250^{g}. How many liters will the vessel hold?

275. Wheat weighs 80^{kg} a hektoliter. A field of 4.6^{ha} has produced 9.2 t. of wheat. If a sheaf of the wheat on the average gives 4^{l} of grain, find the average number of sheaves produced per hektar.

276. A piece of zinc weighs in the air 343^{g}, and in water 293^{g} only. What is its volume?

277. A piece of zinc weighs in the air 343^{g}, and in water 293^{g}. What is its specific gravity?

278. A jug empty weighs 1.02^{kg}; full of water it weighs 3.8^{kg}. Find the capacity of the jug in liters.

279. The price of 8 casks of olive oil, containing each 9.05^{hl}, is $1072. What will be the price of 20^{l}?

280. How many hektoliters of wheat, weighing 80^{kg} to the hektoliter, is it necessary to grind to obtain 15 bags of flour, weighing 159^{kg} a bag, if the wheat furnishes in flour $\frac{3}{4}$ of its own weight?

MENSURATION.

NOTE. The side of the hexagon is equal to the radius of the ...

Paper 1.

for the walls is 21 cts., and for the ceiling 32 cts. a square yard.

305. A circular plot of ground contains $\frac{3}{4}$ of an acre.

283. A square court, whose side is 42 yds., is paved with 28,224 square tiles. Find the dimensions of each tile.

284. Find the area of a triangle whose base is 9 ft. 8 in., and whose altitude is 5 ft. 3 in.

285. How many yards of carpeting, 1 yd. wide, will be required for a room 27 ft. long, and 21 ft. 3 in. wide, if the strips run across the room?

286. How many yards in the side of a square field containing 3 A. 44 sq. rds. 25 sq. yds.?

287. The longest diagonal of a quadrilateral is 49 yds., and the perpendiculars let fall on it from the remaining angles are 9 ft. 6 in. and 13 ft. 10 in. Find the area. ... Find the length of the path in yards.

313. Find the capacity of a round basket 20 in. in dia...

289. How many cubic feet in a piece of timber 18 ft. long, 15 in. wide, and 10 in. thick?

290. The sides of a triangular garden are 52, 64, and 72 yds. respectively. Find the area.

291. Find the circumference and area of a circle whose radius is 2 ft. 4 in.

292. Find the length of an arc of 72° in a circle whose diameter is 56 ft.

272. A field of wheat containing $8\frac{1}{2}^{ha}$ has furnished 600 sheaves per hektar; 2 sheaves of wheat have furnished a bundle of straw weighing 5^{kg}. What will the whole straw bring, at $15 a ton?

273. A farmer shears 620 sheep and 180 lambs; the

295. How many square yards of paper will cover the walls of a room 22 ft. 10 in. long, 17 ft. 8 in. wide, and 10 ft. 4 in. high?

296. What will be the cost of painting 12 circular pillars, each 38 in. in circumference, and 13 ft. 6 in. high, at 13 cts. a square yard?

297. Find the volume of a cylindrical marble column whose diameter is 16 in., and whose height is 13 ft. 8 in.

298. The long and short diameters of an ellipse are 25 ft. and 18 ft. respectively. Find the area.

299. How many diamond-shaped panes of glass, whose diagonals measure 6 in. and $4\frac{1}{2}$ in., will be required for a window $7\frac{1}{2}$ ft. high and $4\frac{1}{2}$ ft. wide?

278. A jug empty weighs 1.02^{kg}; full of water it weighs 3.8^{kg}. Find the capacity of the jug in liters.

301. How many yards of paper, 22 in. wide, will be required to cover the walls of a room that is 23 yds. 2 ft. 6 in. in circuit, and 9 ft. 9 in. in height?

302. Find the area of a sector, the radius of which is 16 in. and the arc. 27°.

303. A regular hexagon is inscribed in a circle, the diameter of which is 9 ft. Find the area of the hexagon.

NOTE. The side of the hexagon is equal to the radius ...

Paper 1.

for the walls is 21 cts., and for the ceiling 32 cts. a square yard.

305. A circular plot of ground contains $\frac{3}{4}$ of an acre. What is its diameter?

306. Find the volume of a square pyramid, if each side of its base is 4 ft. 6 in., and its altitude is 12 ft.

307. A line 62 ft. long reaches from the top of a house 48 ft. high, to the bottom of a house on the opposite side of the street. Find the width of the street.

308. Construct a triangle whose sides shall measure $5\frac{1}{4}$ in., 4 in., and $2\frac{3}{4}$ in. respectively, and find its area.

309. Find the surface of a sphere whose diameter is 3.5 ft.

310. Find the volume of a sphere whose diameter is 25 in.

311. If the volume of a sphere is 33510.4 cu. in., what is its diameter?

312. A square field, containing $27\frac{1}{2}$ acres, has a diagonal path across it. Find the length of the path in yards.

313. Find the capacity of a round basket 20 in. in diameter and 28 in. deep.

314. Find the volume of a triangular prism whose length is 8 ft. 6 in., and the three sides of the base 2 ft. 6 in., 2 ft., and 1 ft. 6 in. respectively.

315. The diameter of a spherical balloon is 25 ft. How many square yards of silk were required to make it, and how many cubic feet of gas will be required to fill it?

316. Find the weight of an ivory ball 2 in. in diameter, the weight of ivory being 1825 oz. a cubic ft.

272. A field of wheat containing $8\frac{1}{2}^{ha}$ has furnished 600 sheaves per hektar; 2 sheaves of wheat have fur-

318. The circumference of the base of a cone is 33 ft., and its height is 35 ft. Find its volume.

319. How many rolls of paper 12 yds. long and 22 in. wide must be bought to paper the walls of a room $27\frac{1}{2}$ ft. long, $20\frac{1}{8}$ ft. wide, and 11 ft. high?

320. The base of a pyramid is an equilateral triangle, each side of which is 8 ft., and the height of the pyramid is 13 ft. 6 in. Find its volume.

321. If a cylindrical tin canister, 3 in. in diameter and $6\frac{3}{4}$ in. in height, holds a pound of ground coffee, how much will a canister hold that is $4\frac{1}{2}$ in. in diameter and 8 in. in height?

322. If a pipe 2 in. in diameter discharges 20 gals. of water in a minute, how many gallons will be discharged in a quarter of an hour by a pipe 4 in. in diameter, the water flowing at the same uniform speed?

323. The horses employed to turn a threshing-machine move round at the distance of $15\frac{3}{4}$ ft. from the centre, and make 15 circuits in five minutes. At what rate per hour do they walk, reckoning $\pi = 3\frac{1}{7}$?

EXAMINATION PAPERS.

Paper 1.

1. Write in words the sum of 4,300,702, 6,070,506, and 341,318.
2. Divide 19,094,867 by 4009.
3. Express in words and in figures the excess of the value of one 5 over the other in the number 658,457.
4. Multiply seventeen thousand nine hundred forty-three by five thousand seventy-nine.
5. Find the number which subtracted from 80,000 leaves 57,735.
6. Divide five hundred one millions five hundred by three thousand eight hundred fifty.

Paper 2.

1. Add together 0.0023, 2.36, 250, 0.527.
2. Subtract 2.03 from 20.2.
3. Multiply 3.26 by 1.02.
4. Divide 998.824392 by 0.018.
5. Divide 1.8 by 500.
6. Divide 101 by 1.01; and 0.101 by 10.1.

Paper 3.

1. Find the G. C. M. of 203 and 2291.
2. Find the L. C. M. of 12, 21, 28, 30, 35.
3. Find the G. C. M. and L. C. M. of 144, 176, 272.
4. Find the L. C. M. of 8, 10, 12, 25, 30, 36.
5. Find the G. C. M. and L. C. M. of 161, 253, 299.
6. The imperial gallon of Great Britain contains 277.274 cu. in. What is the size of a pint pot in cubic inches? What weight of water will it hold, if a cubic foot of water weighs 1000 oz.?

Paper 4.

1. What number added to the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ will make 4?
2. A field of $3\frac{1}{2}$ acres is divided into 28 equal parts. What fraction of an acre is there in each part?
3. Find the product of $\frac{9}{14}$, $\frac{5}{27}$, $9\frac{4}{5}$, and $2\frac{2}{7}$.
4. Subtract $3\frac{3}{4}$ from $30\frac{7}{19}$.
5. Add $\frac{1}{6}$, $2\frac{1}{7}$, and $13\frac{3}{10}$; divide this sum by $13\frac{1}{14}$; and subtract the result from $5\frac{3}{70}$.
6. Add $\frac{5}{6}$ of a score to $\frac{4}{5}$ of a dozen, and subtract from the result $\frac{2}{9}$ of a hundred. What is the remainder?
7. The upper floor of a house is 18 ft. 4 in. above the ground and is reached by two equal staircases, separated by the landing of the first floor. How many stairs are there in each staircase, if each stair is 11 in. high?

Paper 5.

1. Reduce 125 sq. rds. 23 sq. yds. 6 sq. ft. 108 sq. in. to square yards.

2. How many grains in a pound Avoirdupois? in a pound Troy?

3. The latitude of St. Paul's in London is $51° 30' 49''$, and that of St. Peter's in Rome $41° 53' 54''$. What is one-fifth of the difference of their latitudes?

4. Reduce 3 qts. 1 pt. to the decimal of a gallon.

5. A dozen persons hire an omnibus for \$9.25. How much more must each pay than if the party were fifteen in number?

6. Find the G. C. M. of 3575 and 4719.

7. Express as a fraction of \$5 the difference between \$$7\frac{4}{5}$ and $\frac{4}{5}$ of \$7.

Paper 6.

1. Reduce 5 oz. 12 dwt. 16 grs. to the decimal of a pound.

2. Divide $\frac{2}{3}$ of $\frac{1}{3}$ by $\frac{5}{7}$ of $\frac{3}{5}$.

3. Express 0.0075 as a common fraction in its lowest terms.

4. Find the value of 0.009943 of a mile in rods, etc.

5. A can do a piece of work in 10 dys. B can do the work in 15 dys. In what time can they both do it working together?

6. From 1 lb. Troy of standard gold are coined 46.725 sovereigns. Find the weight in grains of each sovereign.

Paper 7.

1. A man walked $11\frac{2}{3}$ mi. in $3\frac{1}{2}$ hrs. How many hours would he be in going 20 mi. at the same rate?

2. Find the value of 0.2625 of a mile in rods, etc.

3. If a quantity of wheat fills 1155 sacks, each holding 8 bu. 4 qts., how many sacks will it take to contain the wheat when each sack holds 6 bu. 3 qts.?

4. Find the rent of 213 A. 45 sq. rds. at $5.62 an acre.

5. Find the cost of 11 mi. 130 rds. of railway at $32,500 per mile.

6. Reduce 30 rds. 2 yds. 1 ft. to the fraction of 150 rds. 4 yds. 2 ft. 1 in.

Paper 8.

1. Find the wages of a man for 3 wks. 4 dys. 8 hrs. at $8 per week, reckoning 6 dys. to a week and 10 hrs. to a day.

2. Express in the simplest form $3\frac{1}{2} + \frac{7}{8}$ of $10\frac{3}{4} + \frac{3\frac{1}{6}}{\frac{13}{18}}$.

3. What number multiplied by $\frac{4}{5}$ will produce $90\frac{1}{3}$?

4. Divide the sum of 14.4 and 1.44 by their difference, and express the result as a common fraction.

5. Find the L. C. M. of 10, 14, 15, 21, 30, 42.

6. A can build a wall in $3\frac{1}{4}$ dys., and B in $5\frac{1}{2}$ dys. In what time will A and B together do it?

7. Find the average of $12\frac{12}{25}$, 21, $7\frac{3}{4}$, 0.034, 3.125, 0, 24.5, and $12\frac{7}{20}$. Express the fractional part decimally.

Paper 9.

1. Divide 10.8528 by 1.02, and express the quotient as a common fraction.

2. Reduce 2.5 gills to the decimal fraction of $3\frac{1}{3}$ gallons.

3. Find the value of 14.1275 acres.

4. What is the value of 40 A. 132 sq. rds. at $37.50 per acre?

5. A railroad train goes 200 mi. in 8 hrs. In what time will it make a journey of 40 mi.?

6. How long will a journey of 15 mi. 24 rds. 3 yds. take at the rate of 3 mi. 204 rds. 3 yds. in 56 min.?

Paper 10.

1. Sound travels at the rate of 1120 ft. in a second. How far off is a thunder-cloud, when the clap follows the flash of lightning in $10\frac{1}{2}$ seconds?

2. The four sides of a field are 23 chains 19 links, 17 chains 34 links, 6 chains 85 links, 24 chains 62 links. How many yards around the field?

3. Reduce 10,000 sq. yds. to acres, etc.

4. Find the sum of 8 cu. yds. 13 cu. ft. 1234 cu. in., 5 cu. yds. 14 cu. ft. 976 cu. in., 2 cu. yds. 5 cu. ft. 854 cu. in., and 7 cu. yds. 10 cu. ft. 1012 cu. in.

5. Express 1 pk. 6 qts. 1 pt. as the fraction of 6 bu. 1 pk. 3 qts.

6. Express 13 hrs. 15 min. 17 sec. as the fraction of 6 dys. 1 hr. 48 min. 7 sec.

Paper 11.

1. There are 7 mi. of road in a certain district. If the average width of the road is half a chain, how many acres of land consist of roads?

2. Reduce 1012 qts. to bushels.

3. A dishonest milkman mixes a pint of water with every two quarts of milk. How many gallons will he make in this way out of 20 gals. of pure milk?

4. A well is 18 yds. 2 in. deep, and the wheel is 4 ft. 2 in. round. How many turns of the wheel will raise the bucket?

5. Express 1 dy. 1 hr. 50 min. 42 sec. as a day and a decimal of a day.

6. Express in days, hours, etc., the true length of a year, 365.242218 dys.

Paper 12.

1. If 64 gals. of wine and 16 gals. of water are mixed, how much wine is there in 2 qts. 1 pt. of the mixture?

2. Find the total weight of 5792 iron bars, each weighing 23 lbs. 10 oz.

3. Reduce 0.056 of a square rod to the decimal of an acre.

4. Find the value of 0.58 of a common year.

5. The price of gold being £3 17*s.* 10½*d.* an ounce, find the weight of a nugget worth £341.0925.

6. Simplify $\frac{113}{8} + \frac{3}{5}$ of $\frac{10}{9} - \frac{11}{15}$ of $6\frac{1}{4}$.

Paper 13.

1. A piece of cloth, when measured with a yard measure that is $\frac{3}{8}$ of an inch short, appears to be $10\frac{1}{2}$ yds. long. What is the true length?

2. Divide 120,987.2 by 400.

3. Add $\frac{14}{17}$, $3\frac{1}{9}$, $1\frac{5}{51}$, and $\frac{6}{27}$.

4. Simplify $\dfrac{(4\frac{1}{2} + 7\frac{1}{2}) \div 3\frac{1}{2}}{\frac{1}{7} \times 2\frac{2}{7} \times 5\frac{1}{4}}$.

5. A garrison of 1000 men have provisions for 30 dys. At the end of 10 dys. a reinforcement arrives, and then the provisions last only 5 dys. What is the number of the reinforcement?

6. Find the price of 5 A. 72 sq. rds. of land at $47.50 per acre.

Paper 14.

1. Find the greatest common measure of 5522 and 6006.

2. Divide $\frac{1}{12} - \frac{1}{15}$ by $3\frac{1}{75} - 2\frac{24}{25}$.

3. Express as a common fraction 0.06125.

4. If a room is 27 ft. 5 in. long, 14 ft. 7 in. wide, 12 ft. 10 in. high, how much paper $\frac{7}{8}$ of a yard wide is required to cover the walls?

5. What is the amount at simple interest of $1275 in 5 yrs. 8 mos. at $3\frac{3}{5}$ per cent?

6. If I buy sugar at $7 per cwt., at what rate per pound must I retail it to gain $7\frac{1}{7}$ per cent?

Paper 15.

1. Divide \$154 among 4 persons in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
2. In how many days of 12 hrs. will 72 men do a piece of work which 60 men can do in 30 dys. of 9 hrs.?
3. Find the interest on \$354.27 for 3 yrs. 2 mos. 12 dys. at $2\frac{1}{4}$ per cent.
4. \$4500 is divided among A, B, C, and D. A receives $\frac{1}{2}$ of the money, B and C each $\frac{2}{5}$ of the remainder. How much is left for D? [illegible]

   2 in. round. How many turns of the wheel will raise the bucket?

5. Express 1 dy. 1 hr. 50 min. 42 sec. as a day and a decimal of a day.

   Which will win, and by how much time?

Paper 16.

1. If \$1350 at simple interest amount to \$1570.50 in 1 yr. 8 mos., find the rate per cent.
2. If by selling wine at \$3 a gallon I lose 6 per cent, at what price must I sell it to gain $17\frac{1}{2}$ per cent?
3. Find the compound interest on \$750 for 5 yrs. at $2\frac{1}{2}$ per cent.
4. Reduce 347,894,178 sq. in. to acres, etc.
5. If \$850 amounts to \$913.75 at $2\frac{1}{2}$ per cent, find the time.
6. A room is 10 ft. high, $5\frac{1}{3}$ yds. long, and 3 yds. wide. It contains a door 8 ft. by 4 ft., two windows each 5 ft. by 4 ft., and a fireplace 6 ft. by 4 ft. 6 in. How many sq. yds. on its walls require to be painted?

Paper 17.

1. Divide 2054.95 by 0.0563.

2. Find the greatest common measure of 1288, 1736, 104.

3. If 120 men build a house 60 ft. high in 15 dys., how many will build a house 55 ft. high in 10 dys.?

4. Find the difference between the simple and compound interest on $955 at 6 per cent for 4 yrs.

5. A garrison of 1000 men have provisions for 30 dys. At the end of 10 dys. a reinforcement arrives, and then the provisions last only 5 dys. What is the number of the reinforcement?

6. Simplify $\dfrac{(2+\frac{1}{5})\div(3+\frac{1}{7})}{(\frac{1}{2}-\frac{1}{3})\times(4-3\frac{3}{7})}$.

Paper 18.

1. Find the square root of 1,079,521 and 0.1079521.

2. Find the simple interest on $281.60 at 3[illegible] per cent for

3. By selling a carriage for [illegible] per cent. At what price must I sell it to gain 15 per cent?

4. Which is the better investment, 3 per cent stock at $64\frac{1}{4}$, or 5 per cent stock at $102\frac{3}{4}$?

5. A person has a certain capital, half of which is invested in 3 per cent stock at 90, and the other half in 5 per cent at 110. His total income is $6883.50. What is his capital?

Paper 19.

1. A bankrupt has goods worth \$975, and, had they realized their full value, his creditors would have received $81\frac{1}{4}$ per cent of their claims; but $\frac{3}{5}$ of the goods were sold at $17\frac{1}{2}$ per cent, and the remainder at $23\frac{3}{4}$ per cent, below their value. How many cents on the dollar did the bankrupt pay?
2. Find the value of 157 mi. 144 rds. of telegraph wire at \$57.50 per mile.
3. Reduce $\frac{595}{1071}$, $\frac{1547}{5712}$ to their lowest terms.
4. Extract the cube root of 1250.6894.
5. Find the sum of 1871^{ccm}, 541^{l}, 4.51^{hl}, and give the answer in liters.
6. What is the commission on goods worth \$767.73 at $7\frac{1}{2}$ per cent?

Paper 20.

1. A ship, valued at \$14,500, is insured at $3\frac{1}{2}$ per cent, and her cargo, valued at \$32,000, is insured at 5 per cent. Find the whole cost of insurance.

what price must I sell it to gain $1?\frac{1}{2}$ per cent?

3. Find the compound interest on \$750 for 5 yrs. at $2\frac{1}{2}$ per cent.
3. The solid contents of a cube is 37 cu. ft. 64 cu. in. Find the cost of painting it outside at $33\frac{1}{3}$ cts. per square foot.
4. What must be the edge of a cubical cistern, which will contain exactly 1000 imperial gallons of water, if an imperial gallon contains 277.274 cu. in.?
5. If a compound consist of 1185 parts copper, 715 parts tin, and 100 parts zinc, find the percentage of each metal in the compound.

Paper 21.

1. If a horse trots $23\frac{3}{4}$ mi. in $2\frac{1}{2}$ hrs., what is his rate per hour?

2. If 5 per cent be lost by selling an article at \$2.50, find the gain or loss per cent by selling it at $\$3.12\frac{1}{2}$.

3. Find the cost of carpeting a room 18 ft. 6 in. long, 12 ft. 6 in. wide, with carpet $\frac{3}{4}$ of a yd. wide, at 75 cts. per yd., strips running lengthwise, and 3 yds. being

4. How many bricks $8\frac{3}{4}$ in. long, $4\frac{1}{4}$ in. wide, $2\frac{1}{2}$ in. thick can be stored in a building $17\frac{1}{2}$ yds. long, 10 yds. wide, and $8\frac{1}{2}$ ft. high?

5. A bar of gold, weighing 196 lbs. 10 oz. 10 dwts., is cased in lead weighing 24 lbs. 14 oz. Find the weight of the whole in Avoirdupois weight.

6. A plate of iron 137^{cm} long, 643^{mm} wide, 43.1^{mm} thick weighs 277.54^{kg}. What is its specific gravity?

Paper 22.

Find the net amount of a bill of:

1. \$720; discounts 55, 10, and 5.

2. \$600; discounts $12\frac{1}{2}$, $7\frac{1}{2}$, 5, and $2\frac{1}{2}$.

3. \$450; discounts 50, 10, 10, and 10, that is, "50 and three tens."

4. What single discount is equal to $\frac{1}{8}$ and 10?

5. Find the bank discount upon a note of \$587, given Feb. 29, 1880, for 3 mos., and discounted April 30 at $6\frac{1}{2}$ per cent.

Paper 23.

1. A broker receives $6150 to invest in cotton at $10\frac{1}{4}$ cents a pound. His commission is $2\frac{1}{2}$ per cent for buying. How many pounds of cotton can he buy?

2. Find the diameter of a cylinder whose circumference is $16\frac{1}{2}$ ft. ($\pi = 3.1416$).

3. Find in yards the side of a square field containing 15 A. 109 sq. rds. 3 sq. yds.

at $607.50 per mile.

3. Reduce $\frac{595}{[illegible]}$, $\frac{1547}{[illegible]}$ to their lowest terms.

5. The volume of a cube is 2 cu. yds. 14 cu. ft. 145 cu. in. Find in inches its edge, and also its diagonal, to two places of decimals.

Paper 24.

1. Find the number of feet, board measure, in a stick of timber 15 in. square and 32 ft. long.

2. What is the difference in time between Boston and San Francisco, the longitude of the first being W. 71° 3′ 30″, and the second, W. 122° 24′ 40″?

3. If $11.20 worth of paper is required for a room 25 ft. 3 in. long, 19 ft. 9 in. wide, and 12 ft. high, when the paper is $\frac{3}{4}$ of a yard wide, find the cost of each yard of paper.

4. Find within an inch the diagonal of a square field containing $3\frac{1}{2}$ A.

5. A pond whose area is 3 A. is frozen over with ice to the uniform thickness of 6 in. If a cubic foot of ice weighs 896 oz., find the weight of ice on the pond in long tons.